A Study of Normalized Geometric and Normalized Hamming Distance Measures in Intuitionistic Fuzzy Multi Sets

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Abstract: ***The Normalized Geometric and Normalized Hamming distance measures of Intuitionistic Fuzzy Multi sets (IFMS) are presented in depth in this paper. Due to the wide applications in various fields, the distance measure plays a vital role in Intuitionistic Fuzzy sets (IFS). We extend the distance measure of IFS to IFMS as there are possibilities of multi membership, non membership for the same element. To demonstrate the efficiency of the proposed measures, the properties of distance measures are analysed. As the proposed method is mathematically valid, it can be applied to any decision making problems, medical diagnosis, engineering problems, pattern recognition, etc. The application of medical diagnosis and pattern recognition shows that the proposed distance measures are much simpler, well suited one to use with linguistic variables.***

Keywords: Intuitionistic fuzzy set, Intuitionistic Fuzzy Multi sets, Geometric Distance, Normalized Hamming distance

1. Introduction

Lofti A. Zadeh [1] in 1965 introduced the concept of Fuzzy sets (*FS*), was the generalisation of Crisp sets. The fuzzy set allows the object to partially belong to a set with a membership degree (μ) between 0 and 1. Later, the generalization of Fuzzy sets, introduced by **Krasssimir T. Atanassov [2], [3]** was the Intuitionistic Fuzzy sets (*IFS*) represents the uncertainties with respect to membership ($\mu \in [0,1]$) and non membership ($\vartheta \in [0,1]$) such that $\mu + \vartheta \le 1$. The number $\pi = 1 - \mu - \vartheta$ is called the hesitiation degree or intuitionistic index. As they can present the degrees of membership and non membership, the *IFS*s are widely applied in the area of logic programming, decision making, pattern recognition and medical diagnosis. Also *IFS*s defined on the same universe are compared using the Distance Measures. (**Dengfeng and Chuntian [4],** and **Szmidt and Kacprzyk [5],[6],[7][8])**

R. R. Yager [10] introduced the Fuzzy Multi Sets (*FMS*s), as Multi sets **[9]** allow the repeated occurrences of any element. In the *FMS*s, the occurrences are more than one with the possibility of the same or the different membership functions. Later **T.K Shinoj and Sunil Jacob John [11]** in 2012, generalised the new concept of Intuitionistic Fuzzy Multi Sets (*IFMS*s) from the Fuzzy Multi Sets (*FMS*s) consisting of the uncertainties membership, non membership and hesitation functions**.**

In this paper, the Normalized Geometric and Normalized Hamming distance measures of *IFMS*s is applied to examine the capabilities to cope in pattern recognition and medical diagnosis problems. As the Numerical results **[12], [13]** show that the proposed measure is well suited one, we extend this measure to real time application also.

The organization of this paper is as follows: In section 2, the Fuzzy Multi sets, Intuitionistic Fuzzy Multi sets are explained. The distance measures of the Intuitionistic Fuzzy Multi Sets (*IFMSs*) are proposed in Section 3. The section 4, analyses the Pattern Recognition and Medical Diagnosis Application using the Normalized Geometric and Normalized Hamming distance measures of *IFMS*s.

2. Preliminaries

Some basic concepts and definitions used in next section are given here

Definition: 2.1

An ***Intuitionistic fuzzy set*** *(IFS)*, *A* in *X* is given by
A = $\{\langle x,\ \mu_A(x), \vartheta_A(x)\ \rangle / \ x \in X\}$ -- (2.1)
where μ_A : X → [0,1] and ϑ_A : X → [0,1] with the condition $0 \le \mu_A(x) + \vartheta_A(x) \le 1, \forall\, x \in X$ Here $\mu_A(x)\ and\ \vartheta_A(x) \in [0,1]$ denote the membership and the non membership functions of the fuzzy set *A*; For each Intuitionistic fuzzy set in *X*, $\pi_A(x) = 1 - \mu_A(x) - [1 - \mu_A(x)] = 0$ for all $x \in X$ that is $\pi_A(x) = 1 - \mu_A(x) - \vartheta_A(x)$ is the hesitancy degree of $x \in X$ in *A*.
Always $0 \le \pi_A(x) \le 1, \forall\, x \in X$.
The ***complementary set*** A^c of *A* is defined as
$A^c = \{\langle x, \vartheta_A(x),\ \mu_A(x)\ \rangle / \ x \in X\}$ -- (2.2)

Definition: 2.2

Let X be a nonempty set. A ***Fuzzy Multi set (FMS)*** A in X is characterized by the count membership function Mc such that Mc : X → Q where Q is the set of all crisp multi sets in [0,1]. Hence, for any $x \in X$, Mc(x) is the crisp multi set from [0, 1]. The membership sequence is defined as
$(\ \mu_A^1(x)\ , \mu_A^2(x), \dots \dots \dots \mu_A^p(x)\ )$ where
$\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$.
Therefore, A *FMS A* is given by
$A = \{\langle\ x, (\ \mu_A^1(x)\ , \mu_A^2(x), \dots \dots \dots \mu_A^p(x)\ )\ \rangle / \ x \in X\}$
-- (2.3)

Definition: 2.3

Let X be a nonempty set. A ***Intuitionistic Fuzzy Multi set (IFMS)*** *A* in X is characterized by two functions namely count membership function Mc and count non membership function NMc such that

Mc : X → Q and NMc : X → Q where Q is the set of all crisp multi sets in [0,1]. Hence, for any $x \in X$, Mc(x) is the crisp multi set from [0, 1] whose membership sequence is defined as

$(\mu_A^1(x), \mu_A^2(x), \dots \dots \dots \mu_A^p(x))$ where

$\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^p(x)$ and the corresponding non membership sequence NMc (x) is defined as

$(\vartheta_A^1(x), \vartheta_A^2(x), \dots \dots \dots \vartheta_A^p(x))$ where the non membership can be either decreasing or increasing function. such that $0 \leq \mu_A^i(x) + \vartheta_A^i(x) \leq 1, \forall x \in X \text{ and } i = 1,2, \dots p.$

Therefore, An ***IFMS*** *A* is given by

$$A = \{\langle x, (\mu_A^1(x), \mu_A^2(x), \dots \mu_A^p(x)), (\vartheta_A^1(x), \vartheta_A^2(x), \dots \vartheta_A^p(x)) \rangle / x \in X\}$$

-- (2.4)

where $\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^p(x)$

The ***complementary set*** A^c of A is defined as

$$A^c = \{\langle x, (\vartheta_A^1(x), \vartheta_A^2(x), \dots \vartheta_A^p(x)), (\mu_A^1(x), \mu_A^2(x), \dots \mu_A^p(x)) \rangle / x \in X\}$$

-- (2.5)

$$\text{where } \vartheta_A^1(x) \geq \vartheta_A^2(x) \geq \cdots \geq \vartheta_A^p(x)$$

Definition: 2.4

The **Cardinality** of the membership function Mc(x) and the non membership function NMc (x) is the length of an element x in an *IFMS A* denoted as η, defined as η = | Mc(x)| = |NMc(x)|

If A, B, C are the *IFMS* defined on X , then their cardinality η = Max { η(A), η(B), η(C) }.

3.a Geometric Distance Measure

The Geometric distance of the Intuitionistic Multi Fuzzy set is defined as

$$D_g(A, B) = \frac{1}{\eta} \sum_{j=1}^{\eta} \left\{ \frac{1}{n} \sum_{i=1}^{n} \sqrt{(\mu_A^j(x_i) - \mu_B^j(x_i))^2 + (\vartheta_A^j(x_i) - \vartheta_B^j(x_i))^2} \right\}$$

-- (3.1.1)

Where the Normalized Geometric distance is $D_G(A, B) = \frac{1}{\sqrt{2}} D_g(A, B)$ -- (3.1.2)

Proposition: 3.2

The defined distance $D_g(A, B)$ between *IFMS A* and *B* satisfies the following properties

D1. $0 \leq D_g(A, B) \leq 1$

D2. $A = B$ if and only if $D_g(A, B) = 0$

D3. $D_g(A, B) = D_g(B, A)$

D4. If $A \subseteq B \subseteq C$, for A, B, C are *IFMS* then, $D_g(A, B) \leq D_g(A, C)$ and $D_g(B, C) \leq D_g(A, C)$

Proof

D1. $\mathbf{0 \leq D_g(A, B) \leq 1}$

As the membership and the non membership functions of the *IFMSs* lies between 0 and 1, the distance measure based on these function also lies between 0 and 1.

D2. $\mathbf{A = B}$ **if and only if** $\mathbf{D_g(A, B) = 0}$

(i) Let the two *IFMS* A, B be equal (i.e.) **A = B**. This implies for any $\mu_A^j(x_i) = \mu_B^j(x_i)$ and $\vartheta_A^j(x_i) = \vartheta_B^j(x_i)$ states that $(\mu_A^j(x_i) - \mu_B^j(x_i))^2$ and $(\vartheta_A^j(x_i) - \vartheta_B^j(x_i))^2 = 0$. Hence $\mathbf{D_g(A, C) = 0}$

(ii) Let the $\mathbf{D_g(A, B) = 0}$

The zero distance measure is possible only if $(\mu_A^j(x_i) - \mu_B^j(x_i))^2 = 0$ and $(\vartheta_A^j(x_i) - \vartheta_B^j(x_i))^2 = 0$, as the Geometric distance measure concerns with addition of both membership and non membership difference. This refers that $\mu_A^j(x_i) = \mu_B^j(x_i)$ and $\vartheta_A^j(x_i) = \vartheta_B^j(x_i)$ for all i, j values. Hence **A = B.**

D3. $\mathbf{D_g(A, B) = D_g(B, A)}$

It is obvious that

$\mu_A^j(x_i) - \mu_B^j(x_i) \neq \mu_B^j(x_i) - \mu_A^j(x_i)$ and $\vartheta_A^j(x_i) - \vartheta_B^j(x_i) \neq \vartheta_B^j(x_i) - \vartheta_A^j(x_i)$

But $(\mu_A^j(x_i) - \mu_B^j(x_i))^2 = (\mu_B^j(x_i) - \mu_A^j(x_i))^2$ and $(\vartheta_A^j(x_i) - \vartheta_B^j(x_i))^2 = (\vartheta_B^j(x_i) - \vartheta_A^j(x_i))^2$,

Hence

$$D_g(A, B) = \frac{1}{\eta} \sum_{j=1}^{\eta} \left\{ \frac{1}{n} \sum_{i=1}^{n} \sqrt{\left(\mu_A^j(x_i) - \mu_B^j(x_i)\right)^2 + \left(\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right)^2} \right\}$$

$$= \frac{1}{\eta} \sum_{j=1}^{\eta} \left\{ \frac{1}{n} \sum_{i=1}^{n} \sqrt{(\mu_B^j(x_i) - \mu_A^j(x_i))^2 + (\vartheta_B^j(x_i) - \vartheta_A^j(x_i))^2} \right\}$$

$= D_g(B, A)$

D4. **If** $\mathbf{A \subseteq B \subseteq C}$**, for A, B, C are *IFMS* then,** $\mathbf{D_g(A, B) \leq D_g(A, C)}$ **and** $\mathbf{D_g(B, C) \leq D_g(A, C)}$

Let $A \subseteq B \subseteq C$, then the assumption is

$\mu_A^j(x_i) \leq \mu_B^j(x_i) \leq \mu_C^j(x_i)$ and

$\vartheta_A^j(x_i) \geq \vartheta_B^j(x_i) \geq \vartheta_C^j(x_i)$ for every $x_i \in X$

Case (i)

Let $\mathbf{(\mu_A^j(x_i) - \mu_C^j(x_i))^2 \geq (\vartheta_A^j(x_i) - \vartheta_C^j(x_i))^2}$ Then from the assumption of non membership function, we have

$(\vartheta_A^j(x_i) - \vartheta_B^j(x_i))^2 \leq (\vartheta_A^j(x_i) - \vartheta_C^j(x_i))^2$

$\leq (\mu_A^j(x_i) - \mu_C^j(x_i))^2$ -- (3.2.1)

Also $(\vartheta_B^j(x_i) - \vartheta_C^j(x_i))^2 \leq (\vartheta_A^j(x_i) - \vartheta_C^j(x_i))^2$

$\leq (\mu_A^j(x_i) - \mu_C^j(x_i))^2$ -- (3.2.2)

Now from the assumption of the membership, we have

$(\mu_A^j(x_i) - \mu_B^j(x_i))^2 \leq (\mu_A^j(x_i) - \mu_C^j(x_i))^2$ and $(\mu_B^j(x_i) - \mu_C^j(x_i))^2 \leq (\mu_A^j(x_i) - \mu_C^j(x_i))^2$ -- (3.2.3)

From (3.2.1, 3.2.2, 3.2.3) $D_g(A, B) \leq D_g(A, C)$ and $D_g(B, C) \leq D_g(A, C)$

Case (ii)

Let $\mathbf{\left(\mu_A^j(x_i) - \mu_C^j(x_i)\right)^2 \leq \left(\vartheta_A^j(x_i) - \vartheta_C^j(x_i)\right)^2}$ Then from the assumption of membership function, we have

$(\mu_A^j(x_i) - \mu_B^j(x_i))^2 \leq (\mu_A^j(x_i) - \mu_C^j(x_i))^2$

$\leq (\vartheta_A^j(x_i) - \vartheta_C^j(x_i))^2$ -- (3.2.4)

Also $(\mu_B^j(x_i) - \mu_C^j(x_i))^2 \leq (\mu_A^j(x_i) - \mu_C^j(x_i))^2$

$\leq (\vartheta_A^j(x_i) - \vartheta_C^j(x_i))^2$ -- (3.2.5)

Now from the assumption of the non membership, we have

$(\vartheta_A^j(x_i) - \vartheta_B^j(x_i))^2 \leq (\vartheta_A^j(x_i) - \vartheta_C^j(x_i))^2$ and $(\vartheta_B^j(x_i) - \vartheta_C^j(x_i))^2 \leq (\vartheta_A^j(x_i) - \vartheta_C^j(x_i))^2$ -- (3.2.6)

From (3.2.4, 3.2.5, 3.2.6) $D_g(A,B) \leq D_g(A,C)$ and $D_g(B,C) \leq D_g(A,C)$

3. b Normalized Hamming Distance Measure

In the *IFMS*, the Normalized Hamming distance is

$$N_D^*(A,B) = \frac{1}{\eta}\sum_{j=1}^{\eta}\left\{\frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_A^j(x_i)-\mu_B^j(x_i)\right| + \left|\vartheta_A^j(x_i)-\vartheta_B^j(x_i)\right|\right)\right\}$$

-- (3.3)

Proposition: 3.4

The defined distance $N_D^*(A,B)$ between *IFMS* A and B satisfies the following properties

D1. $0 \leq N_D^*(A,B) \leq 1$

D2. $A = B$ if and only if $N_D^*(A,B) = 0$

D3. $N_D^*(A,B) = N_D^*(B,A)$

D4. If $A \subseteq B \subseteq C$, for A, B, C are *IFMS* then, $N_D^*(A,B) \leq N_D^*(A,C)$ and $N_D^*(B,C) \leq N_D^*(A,C)$

Proof

D1. $0 \leq N_D^*(A,B) \leq 1$

As the membership and the non membership functions of the *IFMSs* lies between 0 and 1, the distance measure based on these function also lies between 0 and 1.

D2. $A = B$ if and only if $N_D^*(A,B) = 0$

(i) Let the two *IFMS* A , B be equal (i.e.) **A = B.**

This implies for any $\mu_A^j(x_i) = \mu_B^j(x_i)$ and $\vartheta_A^j(x_i) = \vartheta_B^j(x_i)$ which states that $\left|\mu_A^j(x_i)-\mu_B^j(x_i)\right|$ *and* $\left|\vartheta_A^j(x_i)-\vartheta_B^j(x_i)\right| = 0$. Hence $N_D^*(A,B) = 0$

(ii) Let the $N_D^*(A,B) = 0$

The zero distance measure is possible only if both $\left|\mu_A^j(x_i)-\mu_B^j(x_i)\right|$ *and* $\left|\vartheta_A^j(x_i)-\vartheta_B^j(x_i)\right| = 0$, as the Hamming distance measure concerns with addition of membership and non membership difference. This refers that $\mu_A^j(x_i) = \mu_B^j(x_i)$ and $\vartheta_A^j(x_i) = \vartheta_B^j(x_i)$ for all i, j values. Hence **A = B.**

D3. $N_D^*(A,B) = N_D^*(B,A)$

It is obvious that

$\mu_A^j(x_i)-\mu_B^j(x_i) \neq \mu_B^j(x_i)-\mu_A^j(x_i)$ and $\vartheta_A^j(x_i)-\vartheta_B^j(x_i) \neq \vartheta_B^j(x_i)-\vartheta_A^j(x_i)$

But $\left|\mu_A^j(x_i)-\mu_B^j(x_i)\right| = \left|\mu_B^j(x_i)-\mu_A^j(x_i)\right|$ and $\left|\vartheta_A^j(x_i)-\vartheta_B^j(x_i)\right| = \left|\vartheta_B^j(x_i)-\vartheta_A^j(x_i)\right|$

Hence

$$\begin{aligned} N_D^*(A,B) &= \frac{1}{\eta}\sum_{j=1}^{\eta}\left\{\frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_A^j(x_i)-\mu_B^j(x_i)\right| + \left|\vartheta_A^j(x_i)-\vartheta_B^j(x_i)\right|\right)\right\} \\ &= \frac{1}{\eta}\sum_{j=1}^{\eta}\left\{\frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_B^j(x_i)-\mu_A^j(x_i)\right| + \left|\vartheta_B^j(x_i)-\vartheta_A^j(x_i)\right|\right)\right\} \\ &= N_D^*(B,A) \end{aligned}$$

D4. If $A \subseteq B \subseteq C$, for A, B, C are *IFMS* then, $N_D^*(A,B) \leq N_D^*(A,C)$ and $N_D^*(B,C) \leq N_D^*(A,C)$

Let $A \subseteq B \subseteq C$, then the assumption is

$\mu_A^j(x_i) \leq \mu_B^j(x_i) \leq \mu_C^j(x_i)$ and

$\vartheta_A^j(x_i) \geq \vartheta_B^j(x_i) \geq \vartheta_C^j(x_i)$ for every $x_i \in X$

Case (i)

Let $\left|\mu_A^j(x_i)-\mu_C^j(x_i)\right| \geq \left|\vartheta_A^j(x_i)-\vartheta_C^j(x_i)\right|$ Then from the assumption of non membership function, we have $\left|\vartheta_A^j(x_i)-\vartheta_B^j(x_i)\right| \leq \left|\vartheta_A^j(x_i)-\vartheta_C^j(x_i)\right| \leq \left|\mu_A^j(x_i)-\mu_C^j(x_i)\right|$ -- (3.4.1)

Also $\left|\vartheta_B^j(x_i)-\vartheta_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i)-\vartheta_C^j(x_i)\right| \leq \left|\mu_A^j(x_i)-\mu_C^j(x_i)\right|$ -- (3.4.2)

Now from the assumption of the membership, we have $\left|\mu_A^j(x_i)-\mu_B^j(x_i)\right| \leq \left|\mu_A^j(x_i)-\mu_C^j(x_i)\right|$ and $\left|\mu_B^j(x_i)-\mu_C^j(x_i)\right| \leq \left|\mu_A^j(x_i)-\mu_C^j(x_i)\right|$ --- (3.4.3)

From (3.4.1, 3.4.2, 3.4.3 **$N_D^*(A,B) \leq N_D^*(A,C)$ and $N_D^*(B,C) \leq N_D^*(A,C)$**

Case (ii)

Let $\left|\mu_A^j(x_i)-\mu_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i)-\vartheta_C^j(x_i)\right|$ Then from the assumption of membership function, we have $\left|\mu_A^j(x_i)-\mu_B^j(x_i)\right| \leq \left|\mu_A^j(x_i)-\mu_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i)-\vartheta_C^j(x_i)\right|$ -- (3.4.4)

Also $\left|\mu_B^j(x_i)-\mu_C^j(x_i)\right| \leq \left|\mu_A^j(x_i)-\mu_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i)-\vartheta_C^j(x_i)\right|$ -- (3.4.5)

Now from the assumption of the non membership, we have $\left|\vartheta_A^j(x_i)-\vartheta_B^j(x_i)\right| \leq \left|\vartheta_A^j(x_i)-\vartheta_C^j(x_i)\right|$ and $\left|\vartheta_B^j(x_i)-\vartheta_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i)-\vartheta_C^j(x_i)\right|$ - (3.4.6)

From (3.4.4, 3.4.5, 3.4.6) **$N_D^*(A,B) \leq N_D^*(A,C)$ and $N_D^*(B,C) \leq N_D^*(A,C)$**

4.Medical Diagnosis Using Ifms-Normalized Geometric Distance and Normalized Hamming Distance Measures

Uncertainty is an important aspect of medical diagnosis problems. A symptom is an uncertain indication of a disease and hence the uncertainty characterizes a relation between symptoms and diseases. In most of the medical diagnosis problems, there exist some patterns, and the experts make decision based on the similarity between unknown sample and the base patterns. Situations where terms of membership function alone is not adequate, the Intuitionistic fuzzy set theory consisting of both the terms like membership and non membership function is considered to be the better one. Due to the increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. The proposed distance measures among the Patients Vs Symptoms and Symptoms Vs diseases give the proper medical diagnosis.

The unique feature of these proposed methods are that they consider multi membership and non membership functions. Instead of one time inspection, this multi time inspection taking the samples of the same patient at different times gives best diagnosis

Let P = { P_1, P_2, P_3, P_4 } be a set of Patients,
D = { Fever, Tuberculosis, Typhoid, Throat disease } be the set of diseases and
S = { Temperature, Cough, Throat pain, Headache, Body pain } be the set of symptoms.
Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different membership and non membership function for each patient.

Table 4.1: IFMs Q: The Relation between Patient and Symptoms

Q	Temperature	Cough	Throat Pain	Head Ache	Body Pain
P_1	(0.6, 0.2) (0.7, 0.1) (0.5, 0.4)	(0.4, 0.3) (0.3, 0.6) (0.4, 0.4)	(0.1, 0.7) (0.2, 0.7) (0, 0.8)	(0.5, 0.4) (0.6, 0.3) (0.7, 0.2)	(0.2, 0.6) (0.3, 0.4) (0.4, 0.4)
P_2	(0.4, 0.5) (0.3, 0.4) (0.5, 0.4)	(0.7, 0.2) (0.6, 0.2) (0.8, 0.1)	(0.6, 0.3) (0.5, 0.3) (0.4, 0.4)	(0.3, 0.7) (0.6, 0.3) (0.2, 0.7)	(0.8, 0.1) (0.7, 0.2) (0.5, 0.3)
P_3	(0.1, 0.7) (0.2, 0.6) (0.1, 0.9)	(0.3, 0.6) (0.2, 0) (0.1, 0.7)	(0.8, 0) (0.7, 0.1) (0.8, 0.1)	(0.3, 0.6) (0.2, 0.7) (0.2, 0.6)	(0.4, 0.4) (0.3, 0.7) (0.2, 0.7)
P_4	(0.5, 0.4) (0.4, 0.4) (0.5, 0.3)	(0.4, 0.5) (0.3, 0.3) (0.1, 0.7)	(0.2, 0.7) (0.1, 0.6) (0, 0.7)	(0.5, 0.4) (0.6, 0.3) (0.3, 0.6)	(0.4, 0.6) (0.5, 0.4) (0.4, 0.3)

Let the samples be taken at three different timings in a day (morning, noon and night)

Table 4.2: IFMs R: The Relation among Symptoms and Diseases

R	Viral Fever	Tuberculosis	Typhoid	Throat disease
Temperature	(0.8,0.1)	(0.2, 0.7)	(0.5,0.3)	(0.1,0.7)
Cough	(0.2,0,7)	(0.9, 0)	(0.3,0,5)	(0.3,0,6)
Throat Pain	(0.3,0.5)	(0.7, 0.2)	(0.2,0.7)	(0.8,0.1)
Head ache	(0.5,0.3)	(0.6, 0.3)	(0.2,0.6)	(0.1, .8)
Body ache	(0.5,0.4)	(0.7, 0.2)	(0.4, 0.4)	(0.1,0.8)

Table 4.3: The Geometric distance between IFMs Q and R

$D_g(A,B)$	Viral Fever	Tuberculosis	Typhoid	Throat disease
P_1	0.2475	0.5372	0.2131	0.5710
P_2	0.4227	0.2460	0.3521	0.5126
P_3	0.5194	0.4465	0.4021	0.1924
P_4	0.2536	0.5037	0.1684	0.5286

Table 4.4: The Normalized geometric distance between IFMs Q and R

$D_G(A,B)$	Viral Fever	Tuberculosis	Typhoid	Throat disease
P_1	0.1750	0.3799	0.1507	0.4038
P_2	0.2989	0.1739	0.2490	0.3625
P_3	0.3673	0.3157	0.2843	0.1360
P_4	0.1793	0.3562	0.1191	0.3738

The lowest distance from the table 4.3 gives the proper medical diagnosis. Patient P_1 suffers from **Typhoid,** Patient P_2 suffers from **Tuberculosis,** Patient P_3 suffers from **Throat disease** and Patient P_4 suffers from **Typhoid.**

Table 4.5: The Normalized Hamming distance between IFMs Q and R

P_1	0.1633	0.3067	**0.1430**	0.4067
P_2	0.2607	**0.1833**	0.2533	0.3600
P_3	0.3533	0.3000	0.2600	**0.1200**
P_4	0.1767	0.3333	**0.1033**	0.3667

The lowest distance from the table 4.3 gives the proper medical diagnosis. Patient P_1 suffers from **Typhoid,** Patient P_2 suffers from **Tuberculosis,** Patient P_3 suffers from **Throat disease** and Patient P_4 suffers from **Typhoid.**

Pattern Recognition of the Two Proposed Distance Measures

Example: 4.1

Let X = {A_1, A_2, A_3, A_4........ A_n } with A = { A_1, A_2, A_3, A_4, A_5} and B ={ A_2, A_5, A_7, A_8, A_9} are the *IFMS* defined as

Pattern I = $\{\langle A_1 : (0.6,0.4), (0.5, 0.5) \rangle, \langle A_2 : (0.5,0.3), (0.4, 0.5) \rangle, \langle A_3 : (0.5, 0.2), (0.4, 0.4) \rangle, \langle A_4 : (0.3,0.2), (0.3, 0.2) \rangle, \langle A_5 : (0.2,0.1), (0.2, 0.2) \rangle \}$
Pattern II = $\{\langle A_2 : (0.5,0.3), (0.4, 0.5) \rangle, \langle A_5 : (0.2,0.1), (0.2, 0.2) \rangle, \langle A_7 : (0.7,0.3), (0.4, 0.2) \rangle, \langle A_8, (0.4, 0.5), (0.3, 0.3) \rangle, \langle A_9 : (0.2,0.7), (0.1, 0.8) \rangle \}$
Then the testing *IFMS* Pattern III be { A_6, A_7, A_8, A_9, A_{10}} such that $\{\langle A_6 : (0.8, 0.1), (0.4, 0.6) \rangle, \langle A_7 : (0.7,0.3), (0.4, 0.2) \rangle, \langle A_8, (0.4, 0.5), (0.3, 0.3) \rangle, \langle A_9 : (0.2,0.7), (0.1, 0.8) \rangle, \langle A_{10} : (0.2,0.6), (0, 0.6) \rangle \}$

Here, the cardinality η = 5 as $|Mc(A)| = |NMc(A)| = 5$ and $|Mc(B)| = |NMc(B)| = 5$ then the Normalized Geometric distance between Pattern (I, III) is 0.2411, Pattern (II, III) is **0.2012** and the Normalized Hamming distance between Pattern (I, III) is 0.215, Pattern (II, III) is = **0.185**

The testing Pattern belongs to Pattern II type (As the distance is lesser in both the methods)

Example: 4.2

Let X = {A_1, A_2, A_3, A_4........ A_n } with A = { A_1, A_2 };
B ={ A_4, A_6}; C = { A_1, A_{10}} ; D = { A_4, A_6} ;
E = { A_4, A_6} are the *IFMS* defined as
A = $\{ \langle A_1 : (0.1,0.2) \rangle, \langle A_2 : (0.3, 0.3) \rangle \}$;
B = $\{ \langle A_4 : (0.2,0.2) \rangle, \langle A_6 : (0.3, 0.2) \rangle \}$; $C = \{ \langle A_1 : (0.1,0.2) \rangle, \langle A_{10} : (0.2, 0.3) \rangle \}$;
D = $\{ \langle A_3 : (0.1,0.1) \rangle, \langle A_4 : (0.2, 0.2) \rangle \}$;
E = $\{ \langle A_1 : (0.1,0.2) \rangle, \langle A_4 : (0.2, 0.2) \rangle \}$

The *IFMS*
Pattern Y = $\{ \langle A_1 : (0.1, 0.2) \rangle, \langle A_{10} : (0.2, 0.3) \rangle \}$
Here, the cardinality η = 2 as $|Mc(A)| = |NMc(A)| = 2$ and $|Mc(B)| = |NMc(B)| = 2$,

then the Normalized Geometric distance between the Patten (A, Y) = 0.05, Patten (B, Y) = 0.085, **Patten (C, Y) = 0** Patten (D, Y) = 0.19, Patten (E, Y) = 0.035 and the Normalized Hamming distance between the Patten (A, Y) =

0.025, Patten (B, Y) = 0.075, **Patten (C, Y) = 0** Patten (D, Y) = 0.1, Patten (E, Y) = 0.025

Thus, the testing Pattern Y belongs to Pattern C type (As the distance is lesser in both the methods)

Example: 4.3

Let X = {A_1, A_2, A_3, A_4........ A_n } with X1 = { A_1, A_2 }; X2 ={ A_3, A_4 }; X3 = { A_1, A_4 } are the *IFMS* defined as

A = {⟨A_1 : (0.4,0.2,0.1), (0.3, 0.1, 0.2), (0.2, 0.1, 0.2), (0.1, 0.4, 0.3)⟩,
⟨A_2 : (0.6,0.3, 0), (0.4, 0.5, 0.1), (0.4, 0.3, 0.2), (0.2, 0.6, 0.2)⟩}
B = {⟨A_3 : (0. 5,0.2,0.3), (0.4, 0.2, 0.3), (0.4, 0.1, 0.2), (0.1, 0.1, 0.6)⟩
⟨A_4 : (0.4,0.6,0.2), (0.4, 0.5, 0), (0.3, 0.4, 0.2), (0.2, 0.4, 0.1)⟩}
C = {⟨A_1 : (0.4,0.2,0.1), (0.3, 0.1, 0.2), (0.2, 0.1, 0.2), (0.1, 0.4, 0.3)⟩,
⟨A_4 : (0.4,0.6,0.2), (0.4, 0.5, 0), (0.3, 0.4, 0.2), (0.2, 0.4, 0.1)⟩}

then the Pattern D of *IFMS* referred as

{⟨A_5 : (0.4,0.6,0.2), (0.4, 0.5, 0), (0.3, 0.4, 0.2), (0.2, 0.4, 0.1)⟩ ,
⟨A_6 : (0.4,0.2,0.2), (0.5, 0.5, 0), (0.2, 0.4, 0.2), (0.2, 0.5, 0.1)⟩}

The cardinality η = 2

as | Mc(A)| = |NMc(A)| = | Hc(A)| = 2 and | Mc(B)| = |NMc(B)| = | Hc(B)| = 2, then the Normalized Geometric distance between the Pattern (A, D) is 0.1959 ; the Pattern (B, D) is 0.2171; the Pattern (C, D) is **0.1846.** Also the Normalized Hamming distance of the Pattern (A, D) is 0.1938; the Pattern (B, D) is 0.2; and the Pattern (C, D) is **0.1563**

Hence, the testing Pattern D belongs to Pattern C type (As the distance is lesser in both the methods)

5. Conclusion

This paper deals the methods to measure the distance between *IFMSs* on the basis of Normalized Geometric distance and Normalized Hamming distance. Both the new proposed – Normalized Geometric and Normalized Hamming measures prove the properties of the distance measure. The specific characteristic of these methods is that they consider the multi membership, multi non membership functions for any element. The application of the two distance measures in medical diagnosis and pattern recognition reveals that the resulting distance values refer the same identification. The **example 4.1 and 4.2** of pattern recognition shows that the two new distance measures perform well in the case of two representatives of *IFMS* – multi membership and non membership function. Whereas the **example 4.3** of pattern recognition depicts that the proposed measures are effective with three representatives of *IFMS* – multi membership, non membership and hesitation functions. It is also clear that the Normalized Geometric distance values are comparatively larger than the Normalized Hamming distance in all cases. (**Medical diagnosis result, Pattern recognition examples 4.1, 4.2, 4.3).** Thus the proposed distance measures are much simpler and well suited to use with linguistic variables.

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