Enhancement of Natural Convection Heat Transfer in a Square Enclosure with Localized Heating from Below

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Abstract: ***This work is focused on the numerical study of natural convection of heat transfer in a three dimensional square cavity. The two opposite vertical walls and the top wall have been considered cold constant temperature with source of heat fixed at the middle bottom surface while the other two vertical walls and the non-heated part of the bottom surface were considered insulated. The aim of the study is to examine the velocity flow and temperature flow. The momentum and energy equations were solved by the finite difference method. The differential equations were discretized using central difference method and forward difference method. The discretized equations with proper boundary conditions of numerical solutions were used. The solutions are presented at the Reynolds number 200 with Prandtl numbers of 0.71.The results are presented in terms of vector plots and contours to show the behavior of the fluid flow and temperature field. The result shows that temperature decreases as the fluid moves from the source of heat towards the cold walls.***

Keywords: Natural convection, buoyancy, flow field, temperature field, boundary conditions

1. Introduction

The study of natural convection in enclosure cavities is very important due to its wide application in engineering fields. It is very important in engineering application, such as solar energy systems, cooling of electronic circuits, air conditioning and many others. The cooling of electronic components is essential for their reliable operation. Therefore, the enhancement of heat transfer is an important subject in an engineering field. The heat transfer from the source may in general be enhanced by increasing the heat transfer coefficient between the source and the surrounding area, by increasing the cooling surface area.

The transfer of heat from one point to another by the movement of fluids is known as convection, which is usually the dominant form of heat transfer in fluids. Convection can be forced by movement of a fluid by means other than buoyancy force (for example, a water pump in an automobile engine). In some cases, natural buoyancy forces alone are entirely responsible for fluid motion when the fluid is heated and this process is called natural convection. In natural convection an increase in temperature processes causes a reduction in density which causes a fluid motion due to pressure and forces when fluids of different densities are affected by gravity the types of heat transfer may be distinguished. Free convection, natural convection and forced convection.

Natural convection is when a fluid motion is caused by buoyancy forces that result from the density variations due to variations of temperatures in the fluid, in the absence of an external force, when the fluid is in contact with the hot surface. The molecules of the fluid separate and scatter causing the fluid to be less dense. The hotter fluid rises and the cooler fluid gets denser and it sinks. Forced convection is when a fluid is forced to flow over the surface by an external source such as fans and pumps creating an artificially induced convection current. Both internal and external flow can be classified as convection. Internal flow occurs when a fluid is enclosed by a solid boundary such as flowing through rectangular enclosure, square enclosure or through pipes. Convection cooling can be sometimes be described by Newton's law of cooling in cases where the heat transfer coefficient is independent of the temperature difference between object and environment. The study of natural convection in enclosures is an area of interest due to its wide application and great importance in engineering. For example, in the solar energy system, the cooling of the electronic circuits, the conditioning of the air and many others.

Turbulence natural convection of air in two dimensional rectangular enclosures with localized heating from below and vertically cooling from the two opposite sides was numerically investigated by Aydin and Yang [2]. Localized heating was placed at the middle on the bottom wall, and different dimensionless heat source length were considered and it was observed from the solutions for Rayleigh number values from 10^3to10^6.Theaverage Nusselt number at the heated part of the lower wall seemed to increase with an increase of the dimensionless heat source length.

In the present work, natural convection of fluid that takes place in a square cavity with localized heating from horizontal bottom surface was numerically solved. The source of heat was placed at the centre of the bottom surface. The ceiling and the two vertical opposite walls were considered cold, while, the other vertical walls and the bottom surface were considered insulated.

2. Literature Review

A long the years, researchers have looked for more flows with the objective to approximate the real case found in industrial means. We can define four basic types of boundary conditions, they are: the natural convection due to a uniformly heated wall, the natural convection induced by a local heat source, the natural convection under multiple heat source with the same strength and type and the natural convection conjugated with inner heat generated conductive body or conductive walls. These boundary conditions are based on a single temperature difference between the differentially heated walls. Most of the studies have addressed natural convection in enclosures due to either a horizontally or vertically imposed temperature difference. These basic situation, are often found in fields of engineering applications such as solar collectors electronic equipment cooling and energy transfer in buildings and nuclear reactors.

A natural convection heat transfer experiment in a tall vertical rectangular enclosure (aspect ratio 16.5) with an array of eleven discrete flush heaters has been done by Keyhani et al., [7]. It was found that the discrete heating in the enclosure results in a significantly augmented local heat transfer rate over that for an enclosure with the uniformly heated vertical wall. Aydin and Yang [2] numerically investigated the natural connection of air in a vertical square cavity with localized isothermal heating from below and symmetrical cooling from sidewalls. The top wall as well as non-heated parts of the bottom walls was varied. Two counter rotating vertices were formed in the flow domain due to natural convection. The average Nussselt number at the heated part of the bottom wall was shown to increase with increasing Rayleigh number as well as with increasing length of the heat source.

The work done by Sigey et al., [11] who both investigated in detail turbulent flow in a three dimensional enclosure in the form of a room with a convectional heater built into one of the walls and having a window in the same wall. The size of the window was varied to pave way for two cases of study but centre fixed with respect to the heater. The result indicated that the rate of heat transfer is higher for a larger window than for a small window as the Rayleigh number increases. Deng et al., [5] studied numerically a two dimensional lamina natural convection in a rectangular enclosure with discrete heat sources on walls in the unsteady regime. A new combined temperature scale was suggested to non dimensionalize the governing equations of natural convection induces by multiple temperature difference, the Rayleigh numbers used were
Ra = 10 3 to 106.

Ampofo and Karayiannis [3] conducted an experiment study of low-level turbulence natural convection in an air filled vertical square cavity. The cavity was 0.7m high × 1.5 deep giving rise a 2D flow. The hot and cold walls of the cavity were isothermal at 50 and 10 0c respectively, that is, a Rayleigh number equals to 1.58 × 109. The experiments that were carried out on Ampofo work and Karayiannis [3] were conducted with very high accuracy and as such the results formed experimental benchmark data and were useful for validation of computational fluid dynamics codes.

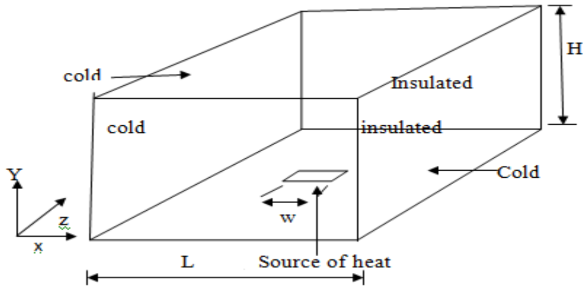
Calcagni et al., [4] studied the natural convection heat transfer numerically and experimentally in square enclosure heated from below. The study was focused on the calculation of heat and average Nusselt number on the heat source. Martorell et al., [8] work dealt with the natural convection flow and heat transfer from horizontal plate cooled from above experiments were carried out for rectangular plated leaving aspect ratios between$\emptyset$ =0.36 and 0.43 and Rayleigh numbers in the range of 290 ≤ Raw ≤ 3.3 × 105. The values of Raw and $\emptyset$ where selected, to the design of printed circuit boards. The results showed that such a low Raw effect could be accounted for in a physically consistent manner by adding a constant term to the heat transfer correlation.

Francese et al., [6] studied the natural convective heat transfer generated by a source with a height located in two different positions inside a square enclosure. A natural convection in a square enclosure with a hot source experiment with three different heights was carried out by Paroncipi M. F. Corvaro [9]. The height of the strip influenced the distribution of the velocity fields, and consequently the heat transfer efficiency. The study shows how the natural convective heat transfer worsens with the increase in the source height. Ahmed W. Mustafa et al., [1] performed experiment on natural convection in a trapezoidal enclosure with partial heating from below and symmetrical cooling from the sides. The heating was simulated by a centrally located heat source on the bottom wall and four different values of the dimensionless source length 0.2, 0.4, 0.6, 0.8 were considered. The results show that the average Nusselt number increases with the increases on the source length.

Patrick H. et al., [10] performed experiment on a numerical study of the effect of a below-window convective heater on the heat transfer rate from a cold recessed window. Convective heaters are often mounted below a cold window in buildings in cold claimants. The presence of the heater alters the flow and temperature distributions in the room near the window and the rate of convective heat transfer to the window. The result shows that when the heater output is low the cold downward flow from the window reaches the floor leading to a cold air layer forming near the floor. At a higher heater output the hot upward flow from the heater is strong to divert the cold flow from the window away from the floor.

3. Problem Description

The problem studied in this paper is a three dimensional lamina buoyancy flow and heat transfer in a square enclosure with localized heating from the bottom. The central bottom wall has a centrally located heat source of length W which is assumed to be isothermally heated at temperature [Th], the side walls and top wall are isothermally cooled at a constant temperature (T_c) , while the bottom surface except for the heated section and in the other two opposite vertical walls are insulted or considered to be adiabatic. Fig. 1 show physical problem.



4. Mathematical Model

The numerical solution is obtained using a finite difference method and the following assumptions are considered in the present analysis.

1. The fluid properties are assumed to be constant except for the density variation in the buoyancy which is treated according to Boussnesq approximation.
2. The flow is considered, laminar, steady and three dimensional.
3. The fluid inside the square enclosure is assumed to be Newtonian and incompressible while viscous dissipation effects are considered negligible.
4. The model is assumed to be subjected to internal natural convection process and not to any external flow.

The flow field and the temperature distribution inside the enclosure are described by the Navier-stokes and the energy equations respectively. Therefore these equations are expressed as follows:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \ (1)$$

$$\rho^*\left(\frac{u^*\partial u^*}{\partial x^*}+\frac{v^*\partial u^*}{\partial y^*}\right)=-\frac{\partial p^*}{\partial x^*}+\mu\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right) \ (2)$$

$$\rho^*\left(\frac{u^*\partial v^*}{\partial x^*}+\frac{v^*\partial v^*}{\partial y^*}\right)=-\frac{\partial p^*}{\partial x^*}+\rho^* g+\mu\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right) \ (3)$$

$$\rho c_p\left(\frac{u\partial T}{\partial x}+\frac{v\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial x^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \ (4)$$

The governing equations were transformed into dimensionless upon incorporating the following non-dimensional variables.

$$\theta=\frac{T^*-T_c^*}{T_h^*-T_c^*},\ u^*=U u,\ U v=v^*\ p^*=p_o p\ ,x^*=Lx,\ y^*=Ly,\ p_r=\frac{c_p\mu}{k}$$

$$E_C=\frac{U^2}{c_p(T_h^*-T_C^*)}\ \rho=\rho\ \ \rho r$$

Where x^* and y^* are the dimensional dimensionless co-ordinates measured along the horizontal and vertical axes, respectively; u and v are the dimensional velocity components along x and y axes and θ is the dimensionless temperature. The dimensionless forms of the governing equations under steady state condition are expressed in the following forms.

$$\left(\frac{u\partial u}{\partial x}+\frac{v\partial u}{\partial y}\right)=-E_u\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \ (5)$$

$$\left(\frac{u\partial v}{\partial x}+\frac{v\partial v}{\partial y}\right)=-E_u\frac{1}{\rho_r}\frac{\partial \rho}{\partial x}+\left(\frac{1}{Fr}\right)^2\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \ (6)$$

$$\left(\frac{u\partial\theta}{\partial x}+\frac{v\partial\theta}{\partial y}\right)=\frac{1}{Pr}\frac{\partial^2\theta}{\partial x^2}+E_C\left(\frac{\partial u}{\partial y}\right)^2 \ (7)$$

5. Boundary Conditions

Boundary conditions which are used in the present study can be arranged as follows:

Bottom Wall

For all wall $\left(U=V=0,\frac{\partial p}{\partial n}=0\right)$
But for adiabatic position $\left(\frac{\partial\theta}{\partial n}=0\right)$
And for source of heat $(\theta=2)$

Top wall

U= V= 0, $\frac{\partial P}{\partial n}=0$

Right wall U= V=$\theta=0,\frac{\partial p}{\partial n}=0$

Left wall U=V=$\theta=0,\frac{\partial P}{\partial n}=0$Where n is the normal direction on the walls.

5.1 Nusselt number

The characteristic of the flow is the rate of heat transfer across the cavity. The Nusselt number on the heated dimensionless length (L= H) is calculated as:

$$Nu_y=\frac{\partial\theta}{\partial x}\frac{L}{(T_h-T_c)}$$

$$\left.\frac{\partial\theta}{\partial x}\right|=\frac{w}{k}$$

The average Nusselt number is given by: Nu =

$$\left.\int_0^H\frac{\partial\theta}{\partial x}\,dy\right|x=0$$

5.2 Numerical Solutions

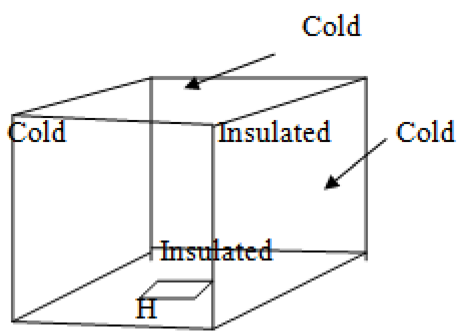
The partial differential equations are replaced by discrete approximation. The temperature and velocity profile are approximated by values at discrete points. This gives a computational mesh.

6. Results and discussion

Enhancement of natural convection heat transfer in a square enclosure with localized heating from below and cooling is considered here. The source of heat is placed at the middle on the bottom surface while the opposite vertical surfaces are at low constant temperature as shown in the geometry of the model infi three. The rest of the surfaces are assumed to be adiabatic. The source of heat has been kept constant and the location fixed. The movement of the fluid in various regions of the enclosure brought about by the temperature difference is our main concern in this chapter. Natural convection plays an important role in engineering applications such as solar engineering system, cooling of electronic circuits, air conditioning, crystal growth and engineering transfer in buildings and nuclear reactors. The velocity and temperature fields depend mainly on the temperature of the source of heat and the cold walls. The enclosure considered here is assumed to have no inlets and outlets, so that the flow is driven buoyancy alone. The temperature difference between the source of heat and the cold walls are in the range of between $0^{\circ C}$- $1^{\circ C}$. The Reynolds number based in the room length is 200 and therefore the flow is laminar.

7. Geometry of the Model

The fig.2 below shows the source of heat and the two opposite vertical cold walls and the top cold surface which were used to obtain the results. Convection was investigated in a room, 1m long, 1m width and 1m height. The heater was placed at the middle, of the bottom surface and opposite vertical walls and top wall kept at cold constant temperature. Computation was carried out for the above problem and the temperature difference between $0^{\circ C}$ and $1^{\circ C}$ the source of heat and the cold surfaces was varied between the results were obtained using the computer code within sources the momentum and energy equations together with various boundary conditions.



7.1 Temperature and Flow Fields

The aim of this study is to understand the structure of the flow due to the temperature differences on the two vertical walls and the top wall. The other walls of the enclosure were assumed to be diabatic. The solutions presented are for Reynolds number Re=200 and Prandtl number Pr=0.71. The results are analyzed by looking at the flow fields and temperature fields. In the velocity profile we have the vector plots while in the temperature profile we have contours on selected planes.

7.2 Velocity Profile (Flow Fields)

The velocity profile can be easily seen in the vector plots which have been selected from the planes Y-X and Y-Z in the Y-Z plane, the vector plots are the planes Z=0.1, Z=0.5 and Z=0.9 as shown in the figure 4, 5 and 6. The planes describes that the hot fluid rises above the source, since it gains energy and become less dense. Hot fluid gains in velocity resulting in an upward movement at the centre, while the cold fluid descends. The velocity of the descending fluid is weak near the cold walls. On the sides there is mixing up of warm and cold fluid resulting into low movement of the fluid particles, hence low velocity. Owing to the symmetry, the flow, in the left and right of the enclosure is identical. In figure 4 and 6 shows the same flow patterns in the planes Z=0.1 and Z=0.9. This indicates that in the two planes effects of the temperature difference are the same hence resulting into the same flow pattern.

Fig 3 and fig 5 shows the flow patterns in the planes Z=0.1 and Z=0.9. These are the planes which are far from the source of heat and there is less activity at the ends than at the centre. The concentration of the flow is high at the centre than at the cold walls.

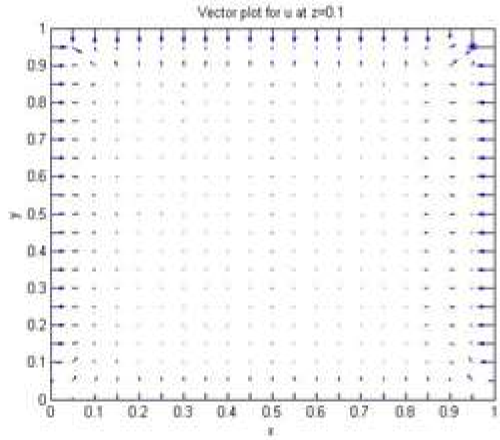


Figure3: Vector plots for u at z=0.1

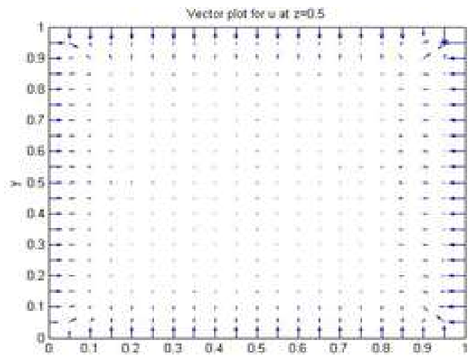


Figure4: Vector plots for u at z=0.5

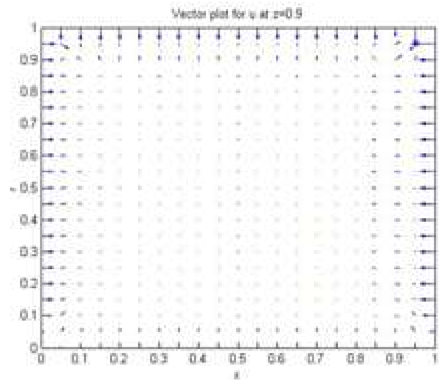


Figure 5: Vector plots for u at z= 0.9

8. Temperature Fields

Fig.6 - 8 illustrates the temperature contours for the typical cases in the square enclosure. Figure7 presents the temperature distribution contours for the square enclosure with the heat source at the centre. From the figure it can be seen that the area near the source of heat records a high temperature but decreases as it moves towards the cold sides and it falls setting off a rotationally motion within the enclosure that enhances heat transfer through the enclosure. Fig.6 and fig.8 show the vertical temperature fields in the plane z = 0.1 and z = 0.9 respectively. The temperature decreases upwards. The center of the enclosure is relatively warm (low temperature). This comes as a result of the hot

fluid rising up from the source of heat mixing up with the cold fluid from the cold surfaces. The two planes are equidistant from the source of heat; hence the two planes record the same temperatures. The position of the source of heat make the spreading area of the temperature distribution less than fig 7 because the heating system is located on the middle of the base wall of the enclosure.

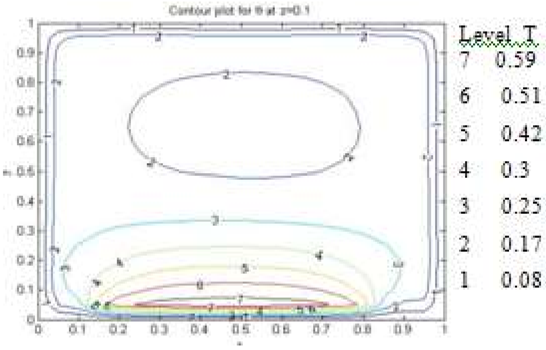


Figure 6: contour for θ at z =0.1

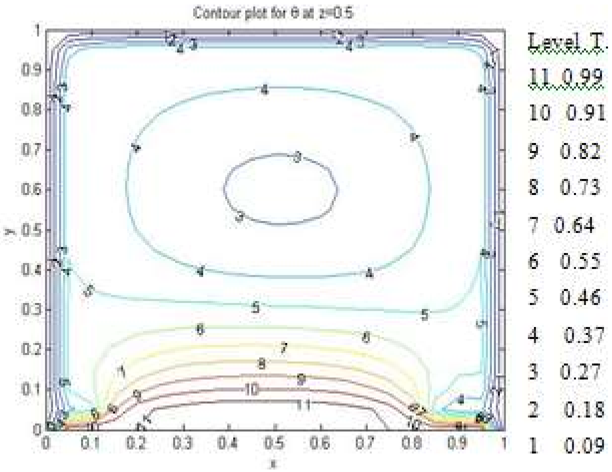


Figure 7: Contour for θ at z =0.5

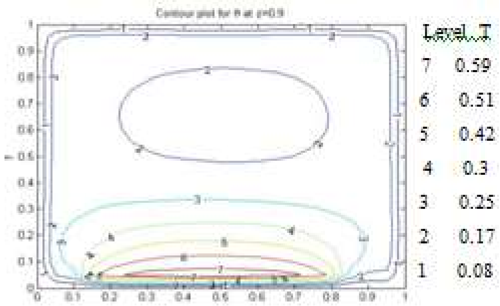


Figure 8: Contour for θ at z =0.9

9. Conclusion

In the present paper a numerical study of natural convection in a square enclosure cavity heated from the bottom wall and cooled from vertical opposite walls and top. The main parameters of interest are Reynolds number 2000, Prandtl number 0.71, Fraude number 0.1 and Eckert number 0.01. Vector plots and contours have been used to study the effects of parameters on fluid flows and heat transfer. A complete set of continuity, Navier-stoke and energy equations were presented in their most general form and were discretized using central difference approximation for a uniform mesh. In view of the obtained results, the following conclusions can be drawn:

1. The position of the source of heat is one of the most important parameter on flow and temperature field.
2. Heat transfer is very weak at the sides of the square enclosure because the source of heat is at the centre of the enclosure.
3. Flow and temperature fields are strongly affected by the cold surfaces hence suitable for machines computers that work in low temperatures.

10. Recommendation

1. Further studies may include experimental investigations and three dimensional turbulent problems
2. Investigations of flow field and temperature at various positions of the heater.

11. Nomenclature

∂v *volume*
P density
Dxdy.1 volume
F_x force in the x-direction
Pf_x body force on the fluid element
$\vec{v}$ velocity flow
H specific enthalpy
K coefficient of thermal conductivity
Re Reynolds number Re= $\frac{\rho L v}{\mu}$
Fr Froude number Fr= $\frac{v}{\sqrt{gL}}$
Eu Euler number Eu=$\frac{P}{\rho v^2}$
P pressure
T temperature
Φ viscous dissipation function
e specific internal energy
β volumetric coefficient of expansion
C_p specific heat at constant pressure
λ thermal conductivity
s entropy
g acceleration due to gravity
∇ gradient operator
x, y, z co-ordinate direction in the i, j and k direction
δx , δx^2 central difference operators denoting $\frac{\partial}{\partial x}$ and $\frac{\partial^2}{\partial x^2}$ respectively
$\emptyset$ aspect ratio
W wall
Ec Eckert number=$\frac{U^2}{C_p \Delta T}$

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