MHD Free Convection Flow past a Vertical Infinite Porous Plate in the Presence of Transverse Magnetic Field with Constant Heat Flux

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Abstract: ***A study of magnetic hydrodynamic free convective flow past an infinite vertical porous plate in an incompressible electrically conducting fluid has been considered. The investigation is of the effect of various parameters (Prandtl, Grashof and Hartman) on the velocity profiles and temperature distribution of the fluid in the presence of a transverse magnetic field subject to a constant heat flux. The partial differential equations governing the flows were analyzed using an explicit finite- difference scheme in computer generated programs. The results has been presented in tabular and graphical form showing the effects of the various parameters (Prantl, Grashof, and Hartman) arising in the flow. The numerical results of the study show that an increase in the Grasshof number causes an increase in the velocity profiles; an increase of Hartman number causes a decrease of velocity profile whereas an increase of Prandtl number causes a decrease in temperature distribution.***

Keywords: Magneto Hydrodynamics, incompressible fluid, transverse, steady state

1. Nomenclature

B	Magnetic field vector (wb/m^2)
$B_X B_Y B_Z$	Magnetic field in xyz direction respectively (wb/m^2)
C_p	specific heat at constant pressure (J/Kg.K)
$\vec{E}$	Electric field intensity vector (vm^{-1})
F	Body force (N)
F_e	Electromagnetic force (Kgm^{-2})
g	Acceleration due to gravity (ms^{-2})
$\bar{J}$	Current density vector (Am^{-2})
k	Thermal conductivity (Wm^{-1}k^{-1})
L	Characteristic length (m)
T	General fluid temperature
T_∞	Characteristic free stream temperature
U	Characteristic velocity (ms^{-1})
U_∞	Free stream fluid velocity (ms-2)
G_r	Grashof Number $G_r = ug\beta (T-T_\infty)/u^3$
M	Hartman Number $M^2 = \alpha\, \mu\, H^2u/U^2\rho$
P_r	Prandt Number $Pr = \mu Cp/k$
V	Kinematic Viscosity (m^{2}s^{-1})
ρ	Fluid Density (Kgm^{-3})
θ	Dimensionless Fluid Temperature

2. Introduction

Magneto hydrodynamics (MHD) is the study of flow of electrically conducting fluid in the presence of magnetic field. The word magneto hydrodynamic (MHD) is derived from: Magneto-meaning magnetic field, Hydro meaning Liquid and Dynamics which means movement. Hydrodynamics is the study of fluid flow and the forces that cause the flow in the absence of the electromagnetic field. In MHD a current is induced when a current conductor moves in a magnetic field. Hence when a viscous conducting fluid flows in the presence of a transverse magnetic field, electromagnetic forces act on the fluid particles thereby altering the geometry of the motion. The concept of MHD is largely perceived to have been initiated by Faraday[1] when he did the first quantitative observation of Magneto hydro dynamics. He did experiments with mercury as a conducting fluid flowing in a glass tube placed in magnetic field and observed that voltage was induced in direction perpendicular to both the direction of flow and magnetic field. He further showed that when an electric field is applied to a conducting fluid in the direction which is perpendicular to magnetic field, a force is exerted on the fluid in the direction perpendicular to both electric field and magnetic field. Since then a lot has been done on MHD and its related fields Rao *et al* [2] studied the heat transfer in porous medium in the presence of transverse magnetic field. The effects of the heat source parameter and Nusselt number were analyzed. They discovered that the effect of increasing porous parameter is to increase the Nusselt Number. Merkin and Mahmood [3] considered the free convection boundary on a vertical plate with a prescribed surface heat flux while Chartuvedi *et al* [4] investigated MHD flow past an infinite porous plate with variable suction. Kinyanjui *et al* [5] investigated MHD Stokes problem for a vertical infinite plate in dissipative rotating fluid with Hall current as Sigey *et al* [6] presented an investigation on the numerical study on natural convection turbulent heat transfer in an enclosure. Kwanza *et al* [7] presented the work on MHD Stokes free convection flow past an infinite vertical porous plate subjected to a constant heat flux with ion-slip and radiation absorption. The concentration velocity and temperature distributions were discussed and results presented in graphs and tables.

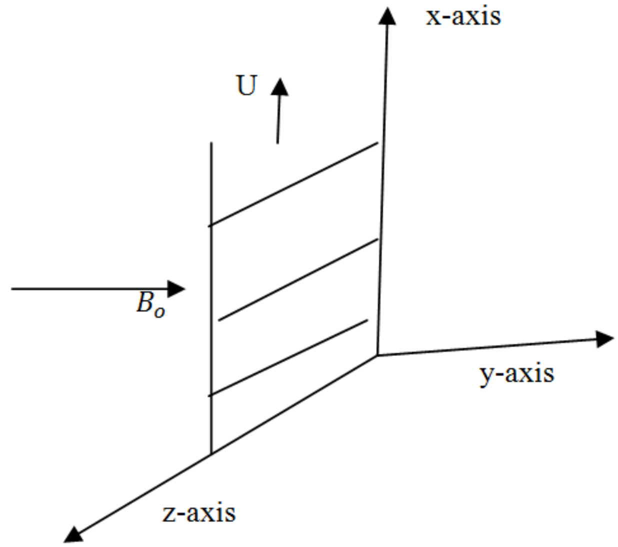
Okello *et al* [8]investigated, unsteady free convection incompressible fluid past a semi infinite vertical porous plate in the presence of a strong magnetic field inclined at an angle α to the plate with Hall and ion-slip current effects.

The effects of modified Grasshof number, suction velocity, the angle of inclination, time, Hall current, ion-slip current, Eckert number, Schmidt number and heat source parameter on the convectively cooled or convectively heated plate restricted to laminar boundary layer were studied. He found that an increase in mass diffusion parameter S_c causes a decrease in concentration profiles, absence of suction velocity or an increase of it causes an increase of concentration profiles, an increase of Eckert number causes an increase in temperature profiles and also an increase of an angle on inclination leads to an increase in primary velocity profiles but a decrease in secondary velocity profiles. Okwoyo J. M. and Sing C. B. [9] presented a paper on steady laminar flow of viscous incompressible fluid between two parallel infinite plates when upper plate is moving with constant velocity and lower plate is held stationary under the influence of transverse magnetic field. The resulting expression was solved by the application of Laplace transform and analytical expression was obtained.

Sigey *et al* [10] carried out a study of magnetic hydrodynamic free convective flow past an infinite vertical porous plate in an incompressible electrically conducting fluid. The investigation of the effect of viscous dissipation on the velocity profiles and temperature distribution of the fluid in the presence of a transverse magnetic field subject to a constant suction velocity was conducted. The partial differential equations governing the flows were analyzed using an explicit finite difference method. The numerical results of the study showed that an increase in the viscous dissipation causes an increase in the velocity profiles and temperature distribution of the fluid. This study finally asserted that an increase in the viscous dissipation parameter or term leads to an increase in velocity and temperature profiles. This increase in the velocity profiles and temperature profile occurred at a distance away from the porous plate. Now for this study we have considered MHD free convection flow past a vertical infinite porous plate in the presence of transverse magnetic field with heat flux.

3. Mathematical Formulation

The x-axis is taken upward along the plate and the y-axis normal to the plate. The flow is taking place along the plate thus the magnetic field applied is along the direction of y-axis. The geometry of the problem may be presented as shown in the figure below.



4. Governing Equations

4.1 Conservation of mass

The equation of conservation of mass is given by

$$\frac{\partial \rho}{\partial t} + \nabla . (\rho u) = 0 \quad (1)$$

Assuming that the density is constant this equation reduces to

$$\nabla u = 0 \quad (2)$$

For a two dimensional flow equation (2) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (3)$$

For this flow, parallel plates are infinite in length and therefore no flow variable is a function of x. hence equation (3) reduces to

$$\frac{\partial u}{\partial y} = 0 \quad (4)$$

This upon integration gives,

$$V = -v_o \quad (5)$$

Where, $-v_o$ is suction velocity.

4.2 Conservation of Momentum Equation

$$\rho\left[\frac{\partial u}{\partial t} + \mu\nabla\right] = -\nabla p + \mu\nabla^2 u + J \times B - \rho g \quad (6)$$

Since the electric field is assumed to be negligible the electromagnetic force reduces to $J \times B$. Therefore for a two dimensional flow, the flow component in x-direction becomes

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} + J \times B - \rho g \quad (7)$$

Since the parallel plates are infinite in extent, the flow velocity profiles at various positions depend on y-coordinates and not x, however pressure in this flow is a function of x. thus equation 4.2.2 becomes

$$\rho\left[\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x}+ \mu\frac{\partial^2 u}{\partial y^2}+J \times B-\rho g \ (8)$$

For a steady flow we neglect the term $\frac{\partial u}{\partial t}$(since velocity does not depend on time).

$$\rho\left[v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}+J \times B-\rho g \ (9)$$

Using $p = \rho h g$ where h=x

Then,

$$\frac{\partial p}{\partial x} = \rho g \ (10)$$

To evaluate the pressure gradient term in the momentum equation we consider the edge of the boundary layer where $\rho \rightarrow \rho_\infty$ and u = 0. Thus the pressure term in x-direction is -$\frac{\partial p}{\partial x} = -\rho_\infty g$ (results from change in elevation). Combining with the body force term (-ρg) along x-direction we get

$$-\rho g - \frac{\partial p}{\partial x} = g(\rho_\infty - \rho),$$

Hence equation (9) becomes

$$\rho\left[v\frac{\partial u}{\partial y}\right] = \mu\frac{\partial^2 u}{\partial y^2}+g(\rho_\infty - \rho) + J \times B \ (11)$$

Defining the volumetric coefficient β of thermal expansion by

$$\beta = -\frac{1}{\rho}\left(\frac{\Delta\rho}{\Delta T}\right)p$$

$$\beta = -\frac{1}{\rho}\left[\frac{\rho_\infty - \rho}{T_\infty - T}\right]$$

Implying that $(\rho_\infty - \rho) = \beta g \rho (T_\infty - T)$ (12)

Substituting equation (5), (12) and (13) into (11) our momentum equation reduces to

$$-v_o\rho\frac{\partial u}{\partial y} = \mu\frac{\partial^2 u}{\partial y^2}+ \beta g \rho (T_\infty - T) - \sigma u B_o^2 \ (13)$$

4.3 Conservation of Energy Equation

$$\rho c_p \frac{\Delta T}{\Delta t} = k\nabla^2 T + Q'' + \mu\Phi \ (14)$$

In two dimension the energy equation become

$$\rho c_v\left(\frac{\partial T}{\partial t}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)\right) = k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q''+\mu\left[2\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right)+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]$$
(15)

For a parallel flow with the plates infinite in extent, no flow is a function of x. equation (15) becomes

$$\rho c_p \left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2}+Q''+\mu\left(2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right)$$
(16)

Since we are considering steady flow then we neglect $\frac{\partial T}{\partial t}$, assuming also that $Q'' = 0$ (since there are no surplus electric charges) and that there is no dissipative heat because the plates are non-conducting, then the equation of energy reduces to

$$-\rho c_p \ v_o\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} \ (17)$$

This can be rewritten as

$$-v_o\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \ (18)$$

The final set of equations are given by

$$-v_o\rho\frac{\partial u}{\partial y} = \mu\frac{\partial^2 u}{\partial y^2}+ \beta g \rho (T_\infty - T) - \sigma u B_o^2 \ (19)$$

$$-v_o\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \ (20)$$

With the initial boundary conditions

U (0, 0) =0 u (0, y) =U u (t, 0) =0
T (0, 0) =T T (t, 0) =T T (0, y) = T_∞ T (t, y) = T_∞

5. Method of the Solution

Non linear differential equations were generated and solved using finite differential approximations. In this form equations (13) and (20) are written as;

$$\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} + v_o\frac{\partial\theta}{\partial y} = 0$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{2m} = -\frac{1}{P_r v_o}\left(\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{m^2}\right) \ (21)$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{2m} = -\frac{1}{P_r v_o}\left(\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{m^2}\right) \ (22)$$

$$\frac{\partial^2 u}{\partial y^2} + v_o\frac{\partial u}{\partial y} + G_r - M^2 u = 0$$

$$\frac{u_{i,j+1}-u_{i,j}}{2n} = \frac{1}{v_o}\left(\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{n^2} - G_r + M^2 u_{i,j}\right) \ (23)$$

6. Results and Discussions

Table 1: Effect of Hartman number on velocity profile for different values of Hartman at a distance y away from the leading edge when h = 0.01 and Gr = 0.5

y	0	0.2	0.4	0.6	0.8	1.0	0.8	0.6	0.4	0.2	0
M=0.2	1.0000	0.8473	0.7325	0.6555	0.6106	0.5944	0.6106	0.6555	0.7325	0.8473	1.0000
M=0.3	1.0000	0.7757	0.6300	0.5298	0.4729	0.4526	0.4729	0.5298	0.6300	0.7757	1.0000
M=0.4	1.0000	0.7265	0.5484	0.4383	0.3776	0.3576	0.3776	0.4383	0.5484	0.7265	1.0000

Table 1 gives different values of velocity at varying values of M at a distance y as the fluid flows. The table asserts that as M increases the velocity of the fluid decrease. For instance at y = 0.8 the values of the velocity decrease from 0.6106 to 0.3776 as the values of Hartman number changes from M = 0.2 to M = 0.4 respectively. It's noted that velocity changes in the same way for other values of y. Similarly as the distance y increase, the velocity values decrease at any given values of Hartman number. This can be attributed due to the presence of the transverse magnetic field normal to the plates which gives rise to Lorentz force which is resistive to the flow hence decelerating the flow of the fluid after plotting the velocity of the fluid against distance y down the flow on different values of Hartman number M, at h = 0.01 and Gr = 0.5, a similar result as that revealed by the table are obtained i.e. the velocity of the fluid decrease as the distance y increases. The same results are obtained for different values of M as shown in figure 3 below.

6.1 Velocity U(y) profiles at varying values of M at h = 0.01 and Gr = 0.5

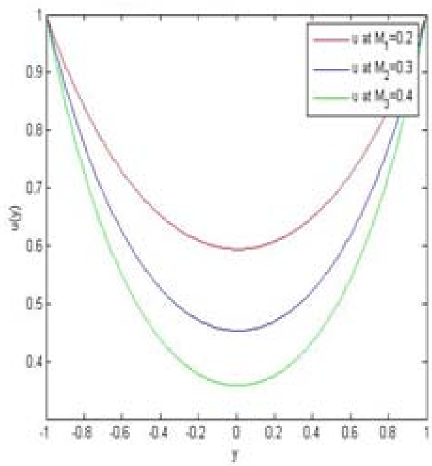


Figure 2: Effect of Hartman number on velocity at h = 0.01 & Gr = 0.5

Table 2: Effect of Grasshof number on velocity profile for various values of Gr at h = 0.1 and M = 0.2 at a distance y from the plates.

y	0	0.2	0.4	0.6	0.8	1.0	0.8	0.6	0.4	0.2	0
Gr = 0.4	1.0000	1.0335	1.0594	1.0777	1.0841	1.0923	1.0841	1.0777	1.0594	1.0335	1.0000
Gr = 0.6	1.0000	1.067	1.1188	1.1554	1.1773	1.1827	1.1773	1.1554	1.1188	1.067	1.0000
Gr = 0.8	1.0000	1.1007	1.1782	1.2331	1.2659	1.2768	1.2659	1.2331	1.1782	1.1007	1.0000

Table 2 shows the values of the velocity profile for various values of Gr as the distance x increases away from the plates at M = 0.2, h = 0.1. It's observed that an increase in Gr leads to an increase of the velocity. For instance taking a point y = 0.4 the velocity of the fluid changes from 1.0594 to 1.1782 at Gr = 0.4 and Gr = 0.8 respectively. Similarly the velocity of the fluid increases as one moves away from the plates at a given value of Grasshof number making the velocity of the fluid to be highest at the middle of the plates. This satisfies the natural situation since fluids flow in such a way that the velocity is highest at the middle position of the channel. This is attributed by the interaction of fluid molecules with plate molecules hence reducing its velocity near the plates. As Gr increases the velocity of the fluid also increases. When the values of velocity are plotted against their corresponding values of distance y away from the plates, Figure 3 below is obtained. Figure 4 shows that at each value of Grasshof number the velocity is higher at the middle of the plate.

6.2 Velocity profiles for U(y) at h = 0.1, M = 0.2 and Varying Gr

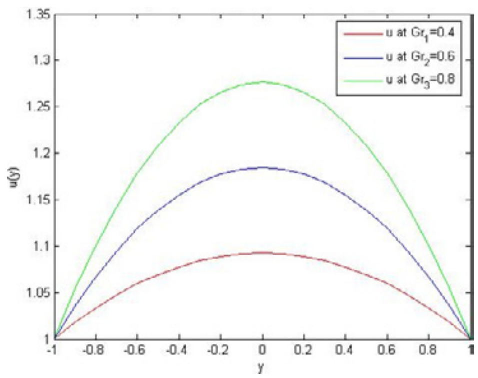


Figure 3: Effect of Grasshof number on velocity profile at h = 0.1 and M = 0.2

Table 3: Effect of Prandtl number on temperature distribution at h = 0.1 and V_o= 0.05 at varying values of Pr.

y	0	0.2	0.4	0.8	1.0	0.8	0.4	0.2	0
Pr = 0.40	1.0000	-4.4711	-8.7598	-13.7768	-14.9555	-13.7768	-8.7598	-4.4711	1.0000
Pr = 0.71	1.0000	-5.1619	-9.9993	-15.6734	-16.4912	-15.6734	-9.9993	-5.1619	1.0000
Pr = 1.0	1.0000	-5.8484	-11.2329	-17.5667	-18.4681	-17.5667	-11.2329	-5.8484	1.0000

Table 3 shows values of temperature at a distance y as the fluid flows at v_o = 0.5 and various values of Prandtl numbers. It is observed that the values of temperature decrease as the fluid flows (as the distance y from the leading edge increases). For instance at y = 0.4, the temperature decreases from -8.7598 to -11.2329 as Prandtl values change from 0.4 to 1.0 respectively. It's also noted that the temperature decrease with increase values of Pr. Figure 4 has been obtained by plotting temperature versus y at h=0.01, and V_o = 0.05 it is observed that the temperature decreases with increasing values of Prandtl number Pr i.e. the temperature is high near the plates than at the middle of the plates. This is justified due to the fact that the thermal conductivity of fluids decreases with increase of prandtl number Pr and hence decreases the thermal boundary layer thickness and the temperature profiles.

6.3 Te mperature d istribution at h = 0.1, v_o = 0.05 at varying Pr

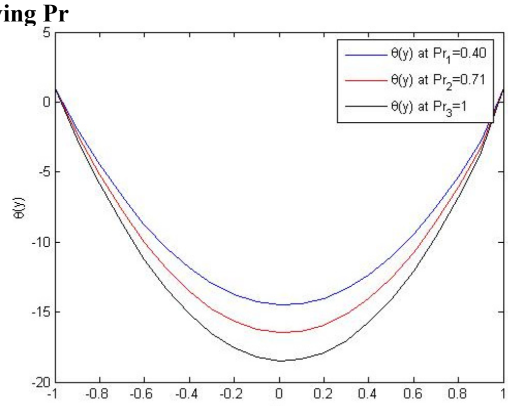


Figure 4: Effect of Prandtl number on temperature distribution at h = 0.1 and V_o= 0.05

7. Conclusions

The analysis of the effect Prandtl number on temperature distribution, Hartman number and Grashoff number on velocity profile on stokes free convection flow between parallel infinite porous plates in the presence of transverse magnetic field with constant heat flux has been carried out. The equations governing this flow were found to be non linear. In order to obtain their solution an efficient finite difference scheme has been developed. The results obtained for various values of these parameters were presented in tabular and graphical form. It can be concluded that an increase of Prandtl number decreases the temperature distribution as the fluid flows. Also an increase of Grasshof number increases the velocity profiles as the distance away from the plates from the plates increases while the increase of Hartman number causes a decrease of velocity profile as the distance from the leading edge increases.

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