Solving a Multi Objective Fuzzy Variable Linear Programming Problem using Ranking Functions

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Abstract: A new method for solving fuzzy variable linear programming problems directly using linear ranking functions is described and is applied for Simplex method in [3]. In this paper, we extend the concept for solving a Multi objective fuzzy variable linear programming problem using ranking functions. A numerical example is given to check the feasibility of the proposed method.

Keywords: Multi objective fuzzy variable linear programming, trapezoidal fuzzy number, ranking function.

1. Introduction

Many real world problems require handling and evaluation of fuzzy data for decision making. To evaluate and compare different alternatives among them, it is necessary to rank fuzzy numbers. With the development of fuzzy set theory, fuzzy ranking has become a topic that has been studied by many researchers. The concept of maximizing decision making was proposed by Bellman and Zadeh[1]. Zimmermann [2] presented a fuzzy approach to solve multi objective linear programming problems. Therefore we extend [3] to solve the Multi objective fuzzy variable linear programming problem by using ranking functions. This paper is organized as follows: In section 2, fuzzy numbers and their arithmetic operations are provided. Section 3 focuses on ranking functions given in [3].Section 4 deals with Multi objective fuzzy variable linear programming problem and Finally, a numerical example is presented in section 5 to illustrate the proposed method.

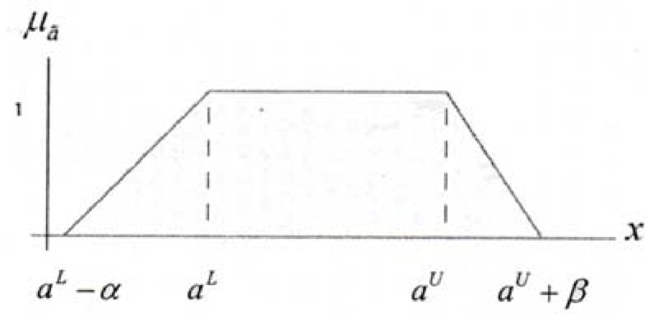
2. Fuzzy concepts

Definition 2.1: A fuzzy set $\tilde{a}$ in R is a set of ordered pairs:

$$\tilde{a} = \{(x, \mu_{\tilde{a}}(x)) \mid x \in R\}$$

$\mu_{\tilde{a}}(x)$ is called the membership function of x in $\tilde{a}$ which maps R to a subset of the non negative real numbers whose supremum is finite.

Definition 2.2: A fuzzy number $\tilde{a}$ is a trapezoidal fuzzy number if the membership function of it be in the form:



We may show any trapezoidal fuzzy number by $\tilde{a} = (a^L, a^U, \alpha, \beta)$, where the support of $\tilde{a}$ is (aL – α, aU + β), and the core of $\tilde{a}$ is $[a^L, a^U]$. Let F(R) is the set of trapezoidal fuzzy numbers.

Note that, we consider F(R) throughout this paper.

2.3 Arithmetic operations

Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers and $x \in R$.
Then, we define

$$x > 0,\ x\tilde{a} = (xa^L, x a^U, x\alpha, x\beta)$$
$$x < 0,\ x\tilde{a} = (xa^U, x a^L, -x\beta, -x\alpha)$$
$$\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta)$$
$$\tilde{a} - \tilde{b} = (a^L - b^U, a^U - b^L, \alpha - \theta, \beta - \gamma)$$

3. Ranking functions

We introduce the following linear ranking function which is similar to [3].

For $\widetilde{a} = (a^L, a^U, \alpha, \beta) \in F(R)$, we define a ranking function $\Re : F(R) \to R$ by

$$\Re(\widetilde{a}) = aL + a^U + \frac{1}{2}(\beta - \alpha)$$

4. Multi Objective fuzzy variable linear programming

A fuzzy variable linear programming (FVLP) problem is defined as follows:

$$\text{Max } \widetilde{z} = c\,\widetilde{x}$$

$$\text{Subject to } \quad A\widetilde{x} = \widetilde{b}$$

$$\widetilde{x} \geq 0$$

where $\widetilde{b} \in (F(R))^m$, $\widetilde{x} \in (F(R))^n$, $A \in R^{m\times n}$, $c \in R^n$

Let us consider the following Multi objective fuzzy variable linear programming (MOFVLP) problem having fuzzy variables in objective function and the constraints.

$$\text{(MOFVLPP): Max } f_i(\widetilde{x}) \qquad i=1,2\ldots\ldots k$$

$$\text{Subject to } \quad A_r\widetilde{x} \leq \widetilde{b}_r \qquad r = 1,2\ldots..m$$

$$\widetilde{x} \geq 0$$

5. Numerical example

We consider a Multi objective fuzzy variable linear programming problem

$$\text{Max } Z_1 = 5\widetilde{x}_1 + \widetilde{x}_2$$

$$\text{Max } Z_2 = \widetilde{x}_1 + \widetilde{x}_2$$

$$\text{Subject to } \quad \widetilde{x}_1 + \widetilde{x}_2 \leq (5, 7, 2, 2)$$

$$\widetilde{x}_1 \leq (4, 6, 2, 2)$$

$$\widetilde{x}_1, \widetilde{x}_2 \geq 0$$

First we solve, $\quad \text{Max } Z_1 = 5\widetilde{x}_1 + \widetilde{x}_2$

$$\text{Subject to } \quad \widetilde{x}_1 + \widetilde{x}_2 \leq (5, 7, 2, 2)$$

$$\widetilde{x}_1 \leq (4, 6, 2, 2)$$

$$\widetilde{x}_1, \widetilde{x}_2 \geq 0$$

Now, we rewrite the above problem in the standard form

$$\text{Max } Z_1 = 5\widetilde{x}_1 + \widetilde{x}_2 + 0\widetilde{x}_3 + 0\widetilde{x}_4$$

$$\text{Subject to } \quad \widetilde{x}_1 + \widetilde{x}_2 + \widetilde{x}_3 = (5, 7, 2, 2)$$

$$\widetilde{x}_1 + \widetilde{x}_4 = (4, 6, 2, 2); \quad \widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{x}_4 \geq 0$$

		5	1	0	0		
$\widetilde{C}_j$	Basis	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{x}_4$	RHS	$\Re$(RHS)
0	$\widetilde{x}_3$	1	1	1	0	(5, 7,2,2)	12
0	$\widetilde{x}_4$	1	0	0	1	(4,6,2,2)	10
	$\widetilde{Z}_j - \widetilde{C}_j$	-5	-1	0	0		
0	$\widetilde{x}_3$	0	1	1	-1	(-1,3,0,0)	2
5	$\widetilde{x}_1$	1	0	0	1	(4,6,2,2)	10
	$\widetilde{Z}_j - \widetilde{C}_j$	0	-1	0	0		
1	$\widetilde{x}_2$	0	1	1	-1	(-1,3,0,0)	
5	$\widetilde{x}_1$	1	0	0	1	(4,6,2,2)	
	$\widetilde{Z}_j - \widetilde{C}_j$	0	0	1	4		

Since all $\widetilde{Z}_j - \widetilde{C}_j \geq 0$, the optimal solution is obtained.

The values of the fuzzy decision variables are

$\widetilde{x}_1$=(4, 6, 2, 2) and $\widetilde{x}_2$ = (-1, 3, 0, 0) .

Hence, Max $\widetilde{Z}$ = (19, 33,10,10) $\quad \Re$ (Z) = 52.

Next, we solve Max $Z_2 = \widetilde{x}_1 + \widetilde{x}_2$

$$\text{Subject to } \quad \widetilde{x}_1 + \widetilde{x}_2 \leq (5, 7, 2, 2)$$

$$\widetilde{x}_1 \leq (4, 6, 2, 2)$$

$$5\widetilde{x}_1 + \widetilde{x}_2 \geq (19, 33, 10, 10)$$

$$\widetilde{x}_1, \widetilde{x}_2 \geq 0$$

Now, the above problem can be rewritten in the standard form

$$\text{Max } \quad Z_2 = \widetilde{x}_1 + \widetilde{x}_2 + 0\widetilde{x}_3 + 0\widetilde{x}_4 + 0\widetilde{x}_5$$

$$\text{Subject to } \quad \widetilde{x}_1 + \widetilde{x}_2 + \widetilde{x}_3 = (5, 7, 2, 2)$$

$$\widetilde{x}_1 + \widetilde{x}_4 = (4, 6, 2, 2)$$

$$5\widetilde{x}_1 + \widetilde{x}_2 + \widetilde{x}_5 = (19, 33, 10, 10)$$

$$\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{x}_4, \widetilde{x}_5 \geq 0.$$

		1	1	0	0	0		
$\widetilde{C}_j$	Basis	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{x}_4$	$\widetilde{x}_5$	RHS	$\Re$(RHS)
0	$\widetilde{x}_3$	1	1	1	0	0	(5, 7,2,2)	12
0	$\widetilde{x}_4$	1	0	0	1	0	(4,6,2,2)	10
0	$\widetilde{x}_5$	-5	-1	0	0	1	(-33,-19, 10 ,10)	…
	$\widetilde{Z}_j - \widetilde{C}_j$	-1	-1	0	0	0		
0	$\widetilde{x}_3$	0	1	1	-1	0	(-1,3,0,0)	2
1	$\widetilde{x}_1$	1	0	0	1	0	(4,6,2,2)	10
0	$\widetilde{x}_5$	0	-1	0	5	1	(-13, 11, 20 , 20)	…
	$\widetilde{Z}_j - \widetilde{C}_j$	0	-1	0	0	0		
1	$\widetilde{x}_2$	0	1	1	-1	0	(-1,3,0,0)	
1	$\widetilde{x}_1$	1	0	0	1	0	(4,6,2,2)	
0	$\widetilde{x}_5$	0	0	1	4	1	(-14, 14 20, 20)	
	$\widetilde{Z}_j - \widetilde{C}_j$	0	0	1	0	0		

Since all $\widetilde{Z}_j - \widetilde{C}_j \geq 0$, the optimal solution is obtained.

The values of the fuzzy decision variables are

$\widetilde{x}_1$=(4, 6, 2, 2) and $\widetilde{x}_2$ = (-1, 3, 0, 0) .

Hence, Max $\widetilde{Z}$ = (3, 9, 2, 2) $\Re$ (Z) = 12.

6. Conclusion

In this paper, we have considered a Multi objective fuzzy variable Linear programming problem with all variables of the objective function and constraints are considered as trapezoidal fuzzy numbers. For this case, the optimal solution is obtained by using the ranking functions as in [3]. A numerical example is solved by using the proposed method and that yields promising results.

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