Non Linear Modeling of Split Phase Type Single Phase Induction Motor

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Abstract: *Single-phase induction machines are found in various appliances such as refrigerators, washing machines, driers, air conditioners and fans. This paper presents a mathematical model and simulation results of dynamic characteristics of the split phase induction motor for different values of the inductor and moment of inertia at no-load conditions. Based on the state vector analysis of the system, the d–q axis model of the split phase induction motor is deduced. It reveals the periodicity and chaos for various system parameters. Accordingly, a chaotic-speed fan is derived varying the motors' internal parameters or the operations.*

**Keywords:** split phase induction motor, bifurcation, chaos, non-linear, periodicity

1. Introduction

In home appliances, single-phase induction motors are commonly used.The split phase induction motor has two windings, the main winding and the auxiliary winding. These windings are displaced in space by 90 electrical. The auxiliary winding has higher current ratio between resistance and reactance, by designing it at a higher current density. It is assumed that the system has magnetic linearity and negligible slotting of the stator and rotor. Also the parameters here are considered, for simplicity, to be lumped rather than distributed.

Chaos is a phenomenon that occurs in nature, from as large as the universe to as tiny as a particle. Chaos can be defined as the qualitative study of the unstable periodic long term behaviour in a deterministic system that exhibits sensitive dependence on initial condition. A chaos theory is a field of study in mathematics, with applications in several disciplines including physics, engineering, economics, biology, and philosophy.

2. D-Q axis modeling of the motor

The D-Q model a single phase induction machine can be considered to be an unsymmetrical two phase induction machine. To build a Motor model, we need mainly The Stator and Rotor Voltage equations. Let us take the following motor
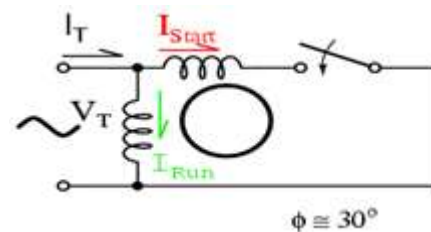
(a)
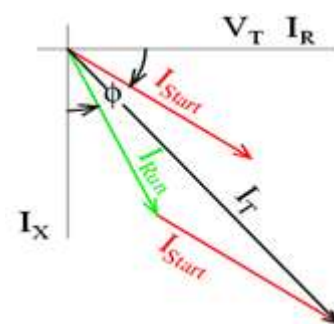
(b)

Figure 1: (a) Motor Model (b) Phasor diagram

The voltage equation can be written as

$$v_{qs} = r_m i_{qs} + \frac{p}{\omega_b} \lambda_{qs}$$

$$v_{ds} = r_a i_{ds} + \frac{p}{\omega_b} \lambda_{ds}$$

$$0 = r'_r i'_{qr} + \frac{p}{\omega_b} \lambda'_{qr} - \frac{1}{k}\frac{\omega_r}{\omega_b} \lambda'_{qr}$$

$$0 = k^2 r'_r i'_{dr} + \frac{p}{\omega_b} \lambda'_{dr} + k \frac{\omega_r}{\omega_b} \lambda'_{dr}$$

The Flux equations

$$\lambda_{qs} = x_{lm} i_{qs} + x_m (i_{qs} + i'_{qr})$$

$$\lambda_{ds} = x_{la} i_{ds} + k^2 x_m (i_{ds} + i'_{dr})$$

$$\lambda'_{qr} = x'_{sr} i'_{qs} + x_m (i_{qs} + i'_{qr})$$

$$\lambda'_{dr} = k^2 x'_{sr} i'_{dr} + k^2 x_m (i_{ds} + i'_{dr})$$

Where K= D axis to Q axis turns ratio and K=1 also.
Here we used Stationary Reference frame.

$$v^s_{qs} = v^s_{ds} = v_s$$

The Torque equation is given by

$$T_e = \frac{p}{2}\frac{X_{mq}}{\omega_b}\left(i^s_{qs} i'^s_{dr} - i^s_{ds} i'^s_{qr} \right)$$

$$T_e = T_L + J \frac{d\omega_r}{dt} + B_m \omega_r$$

Now the equations can be written as

$$
\begin{bmatrix} v^s_{qs} & v^s_{ds} & 0 & 0 \end{bmatrix} =
\begin{bmatrix}
R_{qs} + \frac{p}{\omega_b} X_{qs} & 0 & \frac{p}{\omega_b} X_{mq} & 0 \\
0 & R_{ds} + \frac{p}{\omega_b} X_{ds} & 0 & \frac{p}{\omega_b} X_{md} \\
\frac{p}{\omega_b} X_{mq} & -\frac{\omega_r}{\omega_b} X_{md} & R'_{qr} \frac{p}{\omega_b} X'_{qr} & -\frac{\omega_r}{\omega_b} X'_{dr} \\
\frac{\omega_r}{\omega_b} X_{mq} & \frac{p}{\omega_b} X_{md} & \frac{\omega_r}{\omega_b} X'_{qr} & R'_{dr} + \frac{p}{\omega_b} X'_{dr}
\end{bmatrix}
\cdot \begin{bmatrix} i^s_{qs} & i^s_{ds} & i'^s_{qr} & i'^s_{dr} \end{bmatrix}^T
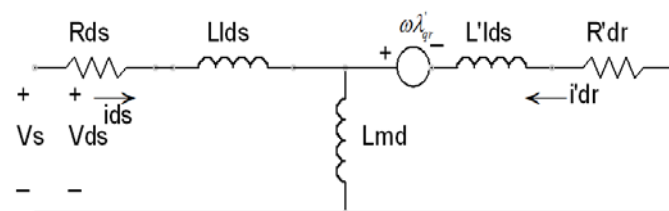$$

Where,

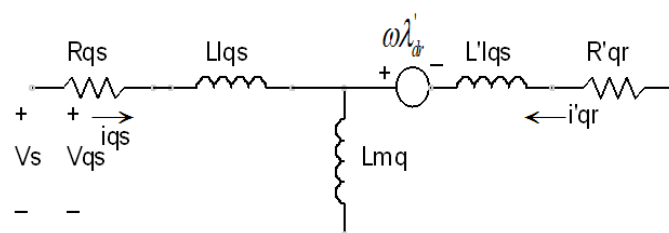**Figure 2:** Equivalent D-axis model of the induction motor

Figure 3: Equivalent Q-axis model of the induction motor

Rdr = Direct axis rotor resistance,
Rqr= Q-axis rotor resistance,
Llds= Direct axis stator leakage inductance,
Llqs= Q-axis stator leakage inductance,
Lmd =Direct axis mutual inductance,
Lldr = Direct axis rotor leakage inductance,
Xmd = Direct axis magnetizing reactance,
Llqr = Q-axis rotor leakage inductance,
Lmq = Q-axis mutual inductance,
ωr = Rotor angular speed and
ω= Speed of the reference frame,
Xmq = Q-axis magnetizing reactance.
Rqs = Q-axis stator resistance

3. MATLAB / SIMULINK Modeling

The variation of system settling points with the variation of system parameters is defined as bifurcation of the system. Here we vary the (Bm/J) ratio keeping other variables constant. The Two (main and auxiliary) stator windings are displaced 90 degree in space. The modeling of the single phase induction motor projects the non-linear model of the system. It can be found that the chaotic speed waveforms offer the well-known chaotic properties, namely random-like but bounded oscillations. Also, these waveforms are a-periodic and very sensitive to the initial condition. Physically, these chaotic motions reveal the unbalanced status of the interaction between the magnetic fields by the main winding and the auxiliary winding.
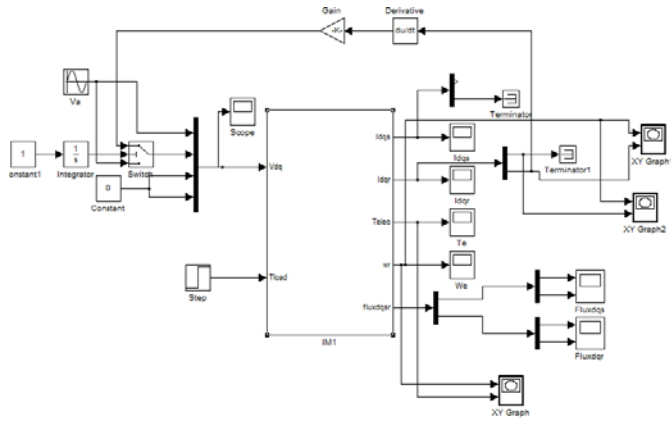
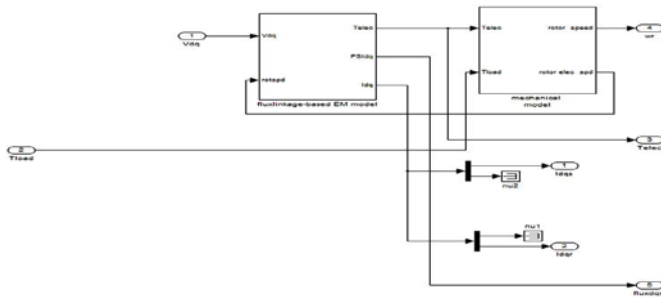**Figure 4:** Dynamic model of a split phase induction motor.

Figure 5: Sub-systems modeling for the induction motor

4. Motor Parameters

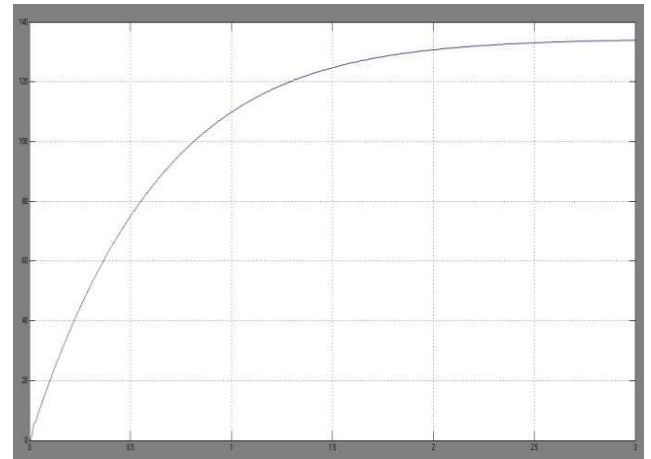| Parameters	Value	Parameters	Value
Pair Of Poles	1	Llqr	0.3105 H
Rdr	20 Ω	Lmq	0.3528 H
Rqr	20 Ω	J	2.310e-4Kg-m^2
Llds	0.111 H	Bm	1.470e-4N.m.s
Llqs	0.4111 H	Lmd	0.2448 H
Rqs	4.5Ω	Rsd	9.5 Ω
Lldr	0.2767 H	Frequency (f)	40Hz

5. Results and Simulations

Figure 6: Speed-Torque Curve
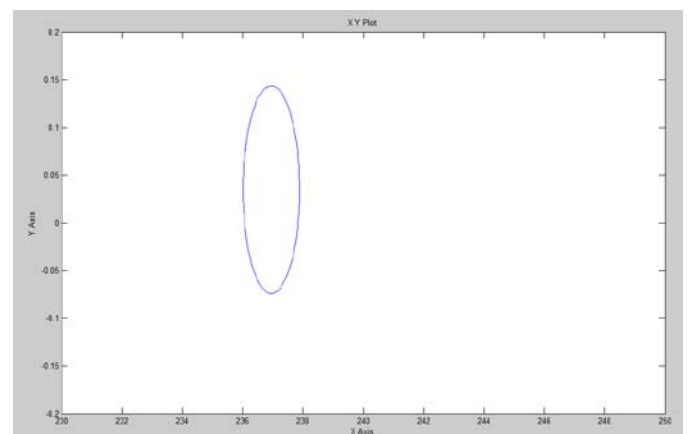

Figure 7: Speed Waveform
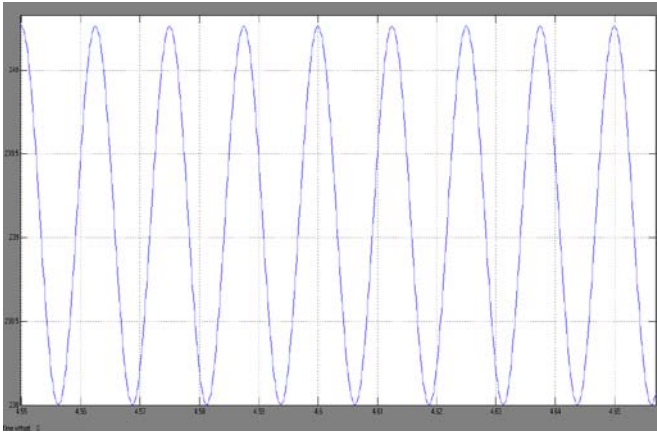

Figure 8: ids Vs Speed
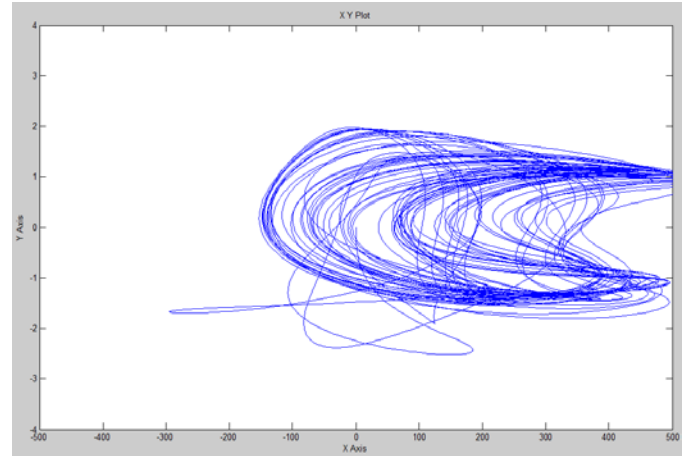
Figure 9: Speed waveform

Figure 12: ids Vs Speed (chaos)

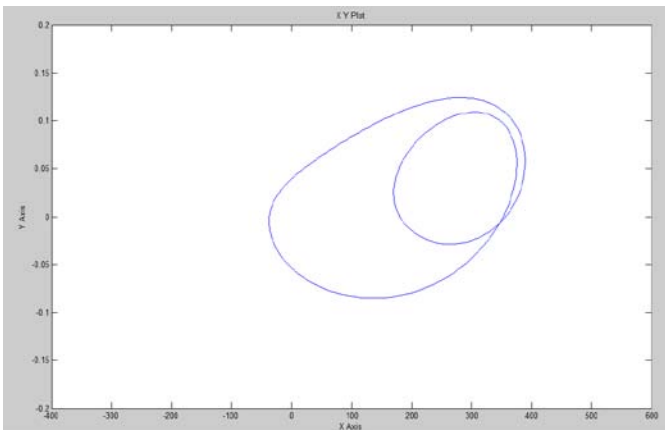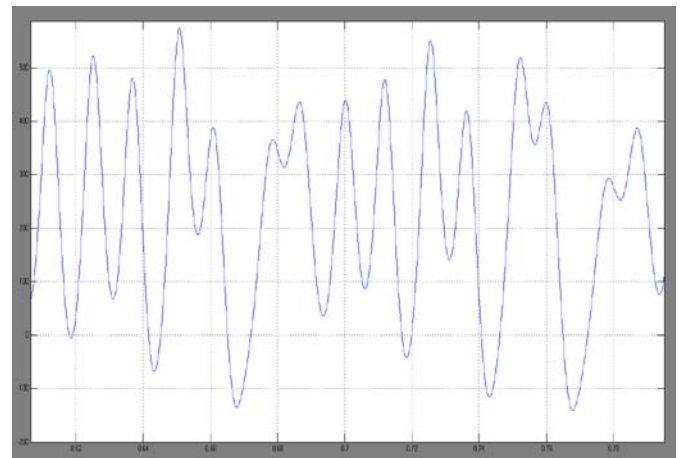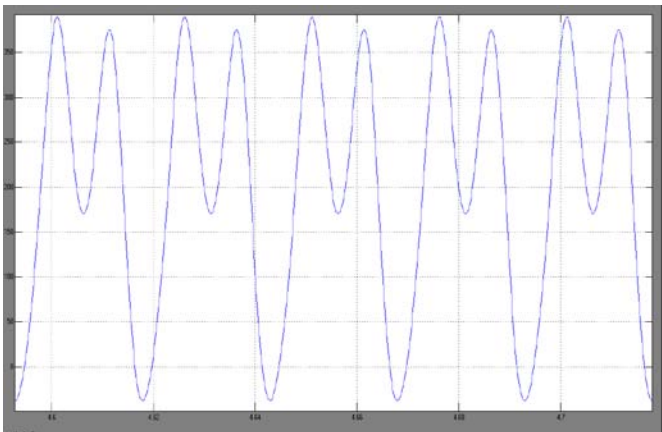**Figure 10:** ids Vs Speed

Figure 13: Speed waveform

6. Conclusions

The non-linear phenomenon in split-phase induction motors are observed as the (Bm/J) ratio is varied. Figures have shown above represents the speed waveforms and the corresponding phase-portraits, at various periodic-speed operations, i.e. the period-1, period-2 and chaotic operations. As the moment of inertia decreases the angular speed becomes chaotic. We found lots of application of chaotic speed of induction motor such as cooling system, mixture grander; air compressor etc. These waveforms are consistent with the well-known phenomenon of inevitable torque pulsation. It should be noted that once the operating condition are known, the motor parameter whose variation bringing chaos may be more than one possible. We also find out the chaotic speed by varying frequency, input voltage etc.

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Figure 11: Speed waveform

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