A Study on Achromatic Coloring Graphs and its Applications

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Abstract: ***The achromatic number χa (G) of a graph is the greatest number of color in a vertex coloring such that each pair of colors appears on at least one edge. In this paper we give some properties of achromatic coloring for complete graphs and its applications.***

Keywords: Achromatic coloring, achromatic number, Lower bound, Upper bound

1. Introduction

1.1 Graph Coloring

It is more than 200 years old. It has a many practical applications like computer science, telecommunications, operation research, designs of experiments etc. It is a special case of graph labelling.

1.2 Achromatic Coloring

It is a proper vertex coloring such that each pair of color classes is adjacent by at least one edge. The largest possible number of colors in an achromatic coloring is called the achromatic number and is denoted by χa(G), where G is a finite un directed graph with no loops and multiple edges.

The achromatic number was defined and studied by Harary, Hedetniemi and prins [2]. Yannakakis and Gavril [5] Proved that determining this value for general graphs is NP – complete. The NP-completeness of the achromatic number for trees was established by Cairnie and Edwards [6].

Further it is solvable for paths, cycles, complete bipartite graphs. Roichman [9] gives the achromatic number for Hypercubes.

2. Definitions

2.1 Chromatic Coloring and Chromatic Number

Let G (V, E) be a graph and $\phi : V \rightarrow N$ a labelling of the vertices. ϕ is a proper coloring if for every u,v ε V with (u,v) ε E then ϕ (u) $\neq$ ϕ (v), in other words neighbouring vertices should not be assigned the same color. The chromatic number denoted by χ (G).

2.2. Achromatic Coloring and Achromatic Number

The achromatic coloring of a graph is a proper vertex coloring such that each pair of color classes is adjacent by at least one edge.

The largest possible number of colors in an achromatic coloring of a graph G is called the and it is denoted by χ_a(G)

2.3 Pseudo Achromatic Number

The pseudo achromatic number α (G) is the maximum k for which there exists a complete coloring of G . If the coloring is required also to be proper, then such a maximum is known as the achromatic number and it will be denoted here by χ_a(G).

2.4 Line Distinguishing Coloring

Let G (V, E) be a graph. A coloring $\phi : V \rightarrow N$ of the vertices is a line distinguishing coloring iff for every edge (u, v) ε E the edge color (ϕ (u), ϕ (v)) is unique, (i.e). It appears at most once.

2.5 Central graph

Let G be a finite undirected graph with no loops and multiple edges. The central graph C(G) of a graph is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G.

Table 1: Properties of Achromatic Coloring

Properties	*Achromatic Coloring*
Lower Bound	In general graphs it is difficult to find lower bound. For particular graphs with large girth (at least 5) admit algorithms with relatively low approximation ratio for the achromatic number. This result gives on the observation that $\chi_a(G) \leq$ m/n for graphs G with n vertices, m edges and girth at least 5.
Upper Bound	In general graphs it is difficult to find upper bound. For a particular case approximating the achromatic number for general or bipartite graphs, the approximation ratio guarantees are just barely sub-linear in the number of vertices.
NP-complete	The problem of achromatic coloring is NP-complete for general graphs.
NP hard	The problem of achromatic coloring is NP-hard for trees, bipartite graphs, interval graphs, bipartite permutation and quasi-threshold graphs

Some of the most important results on achromatic coloring graphs, that appeared in the literature survey .For studying these particular kinds of coloring by presenting several potential applications [8].

3. Observation

Theorem 3.1:

The pseudo achromatic number α (G) is the maximum k for which there exists a complete coloring of G. If the coloring is required also to be proper, then such a maximum is known as the achromatic number and it will be denoted here by $\chi_a(G)$. Clearly $\chi(G) \leq \chi_a(G) \leq \alpha(G)$,where $\chi(G)$ denotes, the chromatic number of G.

Theorem 3.2:

Exhibiting an explicit coloring and showing a general upper bound is follows that, if $q = 2^\beta$, for some $\beta \in$ IN, and $n = q^2 + q + 1$, then $q^3 + q \leq \alpha(n) \leq q^3 + \frac{1}{2} q^2 + 1$.Besides those implied by Bouchet's theorem, very few exact values for $\chi_a(n)$ are known.

Corollary 3.1.1:

We were able to calculate exactly $\alpha(n)$ in the following family, if $q = 2^\beta$, for some $\beta \in$ IN, and $n = q^2+2q+2$, then $\alpha(n) = q(n+1) = q^3+2q^2+3q$.

Theorem 3.3:

Let G be a graph and H be a sub graph of G. Then $\chi_a(G) \geq \chi_a(H)$.

The following observation is from [4] for the complete graphs Cn, K_n etc.

Theorem 3.4

For any complete graph K_n , the number of edges in $L[C(K_n)]=n(n-1)^2/2$

Theorem 3.5

For any cycle K_n , $\chi_a\{ L[C(K_n)]\}=2n-3$.

Theorem 3.6

For any complete graph Kn , χ_a { $L[C(K_n)]\}=2n-3$, Here $L[C(K_n)]$

Satisfies the following properties

(i) Maximum degree of vertices = Minimum degree of the vertices = n-1.

(ii)L[C (K_n)] contain n copies of vertex disjoint K_{n-1}

(iii) There is a cycle 'c' of length 2n with alternate edges from each of the complete graph K_{n-1}

In this paper, we improve the following corollary by the observation of theorem 3.6 with the example. Here the restriction is $n \geq 3$

Corollary 3.1.2

For any complete graph K_n , $\chi_a\{ L[C(K_n)]\}=2n-3, n \geq 3$

Proof:

In theorem 3.6, Substitute

(i) n=1 then $\chi_a\{ L[C(K_n)]\}= -1$ (- ve)

(ii) (ii) Subtitute n=2 then $\chi_a\{L [C (K_n)]\}=1$(+ ve) .

Here the graph is the complete graph, so that in this corollary our restriction is $n \geq 3$.

Following example is given for line graph of central graph of K_5 is equal to 9.

Example 3.2.1

Consider $L(C(K_5))$ with 6 copies of vertex disjoint K_5 complete graph which satisfies maximum and minimum degree of the vertices are 5 then $\chi_a\{ L[C(K_5)]\}=2(6) -3= 9$

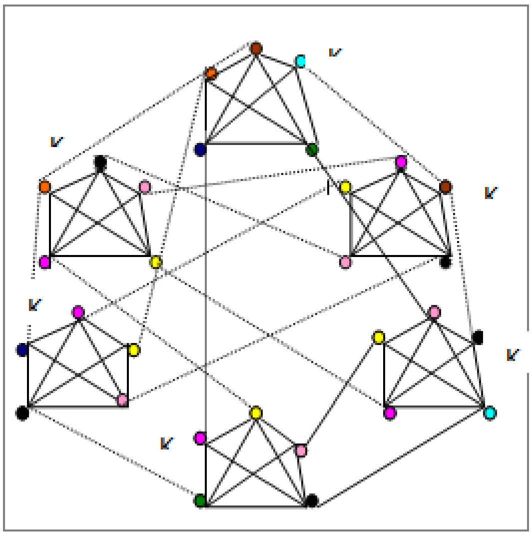

Figure 1: $\chi_a\{ L[C(K_5)]\} = 9$

4. Applications

4.1 Graph theory

It has many applications like, radio navigation, and image compression. In computer science, graphs are used to represent networks of communication, data organization, computational devices, the flow of computation, etc.

4.2 Graph coloring

The problem of coloring a graph has number of applications. Some of them are scheduling, bandwidth allocation, pattern matching and puzzle Sudoku. Few examples are given as follows:

(List coloring, Multi coloring, Minimum sums coloring, Harmonious Coloring etc).

4.3 Achromatic Coloring:

Greedy approach for finding an achromatic coloring with a large number of colors However, using a semi-greedy approach to extracting small independent sets, Chaudhary and Vishwanathan [1] gave the first sub linear approximation algorithm for the achromatic number problem with an approximation ratio of $O(n/\sqrt{\log n})$ for any graph with n vertices. They conjectured that the achromatic number can be approximated with in a ratio of $O(p\chi_a(G))$ for any graph G. In support of their conjecture, they gave an algorithm that returns a $O(p\ \chi_a(G)) = O(n7/20)$ ratio approximation for graphs G with girth (i.e. length of the shortest simple cycle) at least 7.

5. Conclusion

In this paper we discussed about the achromatic number of central graphs and its properties. This type of coloring gives a several applications.

References

[1] A.Chaudhary and Vishwanathan, "Approximation Algorithms for the Achromatic Number, Journal of Algorithms 41, 404–416 (2001)

[2] F.Harary, S. Hedetniemi, and G. Prins, "An interpolation theorem for graphical Homomorphism ",Portugal,Math, Vol.26, pp453-462,1967.

[3] F.Hughes and G.MacGillivray, "The achromatic number of graphs: a survey and some new results, Bill. Inst. Combin. Appl. 19 (1997), pp27-56.

[4] K.Thilagavathi And N. Roopesh, "Achromatic coloring of line graph of central graph", Proceedings of the international conference on mathematics and computer science-2009.

[5] M.Yanakakis and F.Gavril, "Edge dominating sets in graphs", SIAM J.Appl.maths. Vol 38, pp364-372, 1980.

[6] N.Cairnie and K.J. Edwards, 'Some results on the achromatic number', Journal of Graph Theory, Vol.26, pp129-136, 1997.

[7] O.Frank,F.Harary and M.Plantholt, "The line-distinguishing chromatic number of a graph, Ars combin. 14(1982)241-267.

[8] Vasiliki Mitsou, Book on "Vertex Labelings and Colorings of Graphs" The Graduate Center, The City University of New Yark.

[9] Y.Roichman, 'On the Achromatic number of Hypercubes', Journal of Combinatorial Theory, Series B, Vol.79, PP177-182, 2000.

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