Finding Increment Statistics on various types of Wavelets under 1-D Fractional Brownian Motion Synthesis

Abhishek Sharma[1], Bhubneshwar Sharma[2]

[1]M.Tech student, Department of ECE
Eternal University, H.P (India)

[2]Assistant Professor, Department of ECE
Eternal University, H.P (India)
bhubnesh86@gmail.com

Abstract: ***This research paper demonstrates a method to analyze effect of refinement, fractal index and item state on statistical behavior of various wavelets and finally we conclude that Haar wavelet type has highest standard deviation , median absolute deviation and mean absolute deviation values in all of the wavelets that we discussed in this paper and dmey wavelet has the lowest standard deviation, mean absolute deviation and median absolute deviation values. We use value of refinement = 10, value of fractal index = 0.1, length =100 and item state =1 in each wavelet type to analyze effect on first order increment and at the same time we analyze statistical behavior of Histogram, cumulative histogram, autocorrelation and FFT (Fast Fourier Transform) energy spectrum.***

Keywords: *Fractional Brownian motion, Fast Fourier Transform*

1. Introduction - Fractional Brownian motion

Wavelet analysis is carried out in order to know about decomposition from which perfect reconstruction is achieved. The methods used to recreate Brownian motion are vital tools in learning how certain processes behave in the hope that predicting behavior may be possible to serve current scientific needs. One main use of Brownian motion is in the entertainment industry where the resulting Brownian curves; created in 2 dimensions, model very realistic landscapes which have fractal dimension between 2 and 3. Fractal geometry has given rise to an area of mathematics that gives further understanding of how dynamic systems work and as the research continues in this area, more practical uses will be discovered. Normalized fractional Brownian motion is also called a Fractal Brownian motion. It is a Brownian motion without independent increments. It is a Continuous - time Gaussian process BH(t) on [0, T], which starts at zero, has expectation zero for all t in [0, T], and has the following covariance function:

$$E[B_H(t)B_H(s)] = \tfrac{1}{2}(|t|^{2H} + |s|^{2H} - |t-s|^{2H}),$$

Where H is a real number in (0, 1), called the Hurst index or Hurst parameter associated with the fractional Brownian motion. The value of H determines what kind of process the fBm is:

If H = 1/2 then the process is Wiener process.

If H > 1/2 then the increments of the process are positively correlated.

If H < 1/2 then the increments of the process are negatively correlated.

2. Properties

Modern applications of wavelet theory are diverse as wave propagation, data compression, signal processing, image processing, pattern recognition, computer graphics, the detection of aircrafts and submarines and some other medical image technology. The technique entails distorting datasets extracted from patterns to generate multi fractal spectra that illustrate how scaling varies over the dataset. The techniques of multifractal analysis have been applied in a variety of practical situations such as predicting earthquakes and interpreting medical images.

a. Regularity

Sample-paths are almost nowhere differentiable. However, almost-all trajectories are Holder continuous of any order strictly less than H: for each such trajectory, there exists a constant c such that

$$|B_H(t) - B_H(s)| \le c|t-s|^{H-\varepsilon}$$

for every ε > 0.

b. Self-similarity

Probability distribution in case of self similarity has been shown below:

$$B_H(at) \sim |a|^H B_H(t).$$

This property is due to the fact that the covariance function is homogeneous of order 2H and can be considered as a fractal property.

c. Stationary increments

The probability distributions in case of stationary increments is shown below

$$B_H(t) - B_H(s) \sim B_H(t-s).$$

d. Long-range dependence

For H > ½ the process exhibits long-range dependence,

$$\sum_{n=1}^{\infty} E[B_H(1)(B_H(n+1) - B_H(n))] = \infty.$$

3. Random cut method

Brownian motion is the erratic movement of microscopic particles. This property was first observed by botanist Robert Brown in 1827, when Brown conducted experiments regarding the suspension of microscopic pollen samples in liquid solution. The experiments showed that the motion of the particles, in a given time interval, was related to heat, the viscosity of the liquid, and the particle size. Temperature is the only factor that has a direct relationship to particle motion whereas viscosity and size have indirect relationships. Particles move more rapidly as the temperature increases, as the viscosity decreases, or when the average particle size is lowered. In this method in order to produce the graphs above for H = 1/2 a series of random cuts are made. These cuts take the form of step functions whose heights area random Gaussian values and whose jump in height occurs at random times. This can be stated mathematically as follows

$$V_B(t) = \sum_{j=-\infty}^{\infty} A_j P(t - t_j)$$

Where

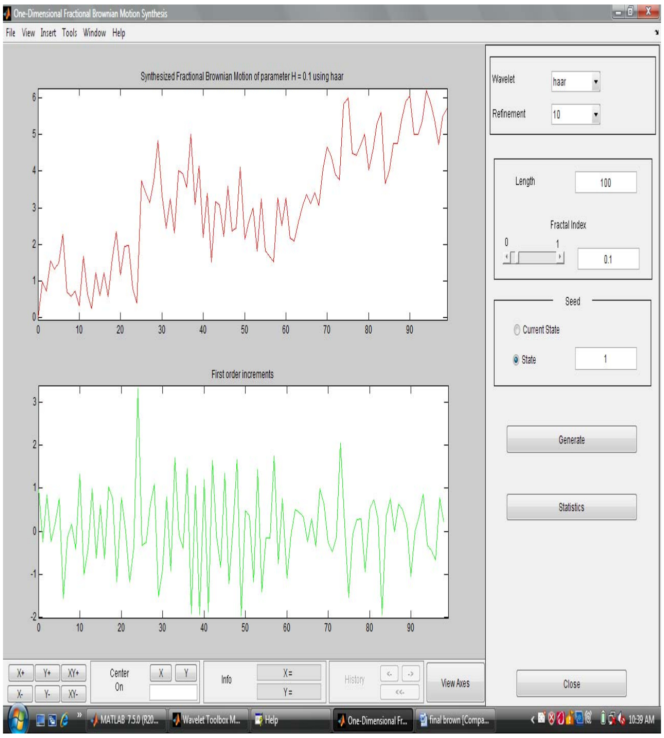
$$P(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

4. Results and Discussion

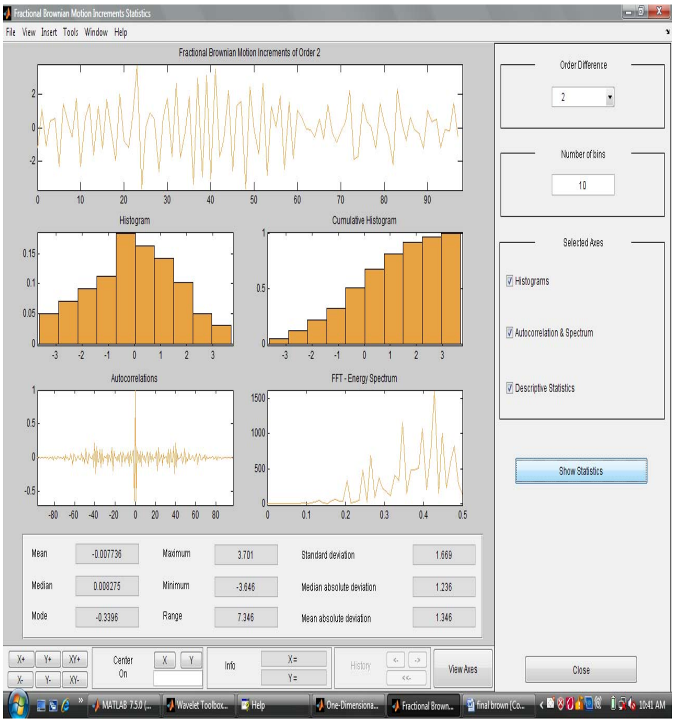
Here we are going to find how the deviation values get changed when we change the type of wavelets and analyze effect on statistical behavior.

Case 1

(a) Using Haar Wavelet put value of refinement=10, length=100, fractal index=0.1, item state=1, Then 1-D fractional Brownian motion synthesis H=0.1 is shown in diagram below.

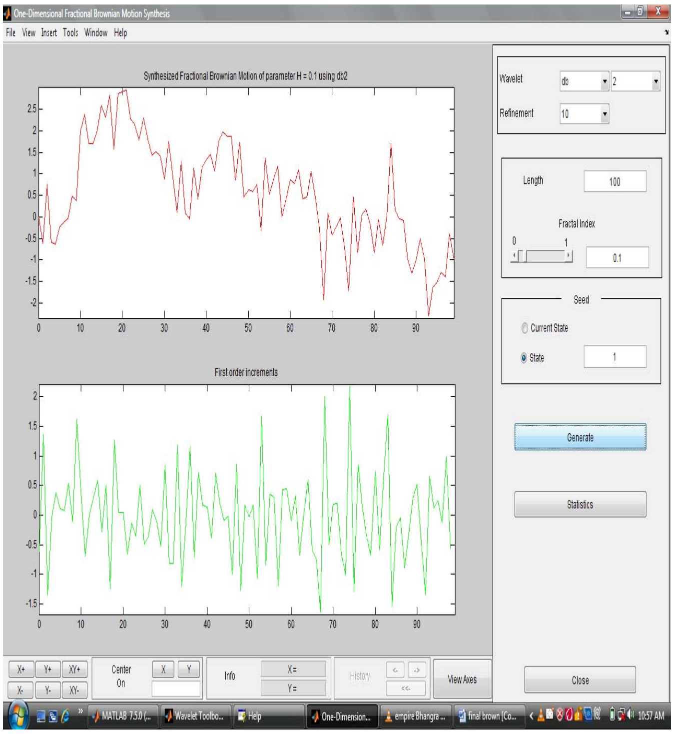


(b) 1-D fractional Brownian motion increments statistics using Haar wavelet with Order=2, no. of bins=10 is shown in diagram below.

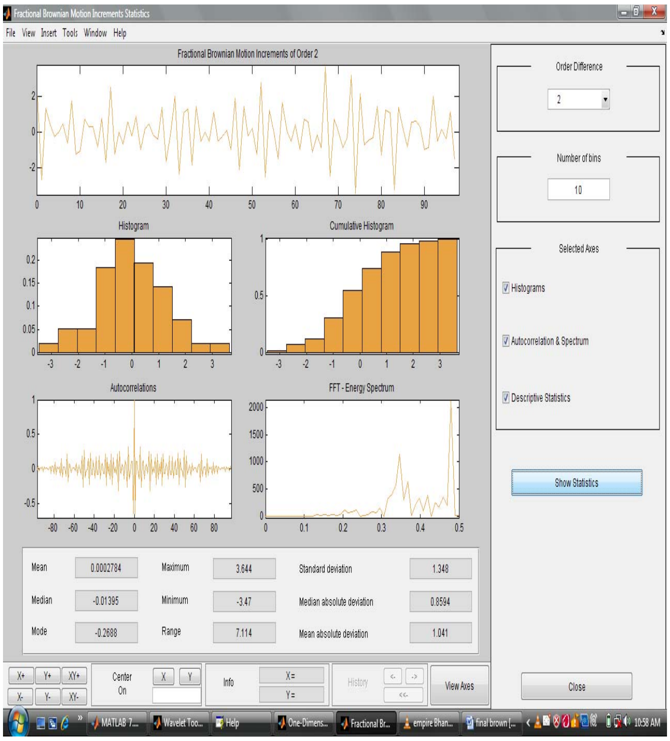


Case 2

(a) Using db2 wavelet, put value of refinement=10, length=100, fractal index=0.1, state=1, Then one dimensional fractional Brownian motion synthesis H=0.1 is shown in diagram below.

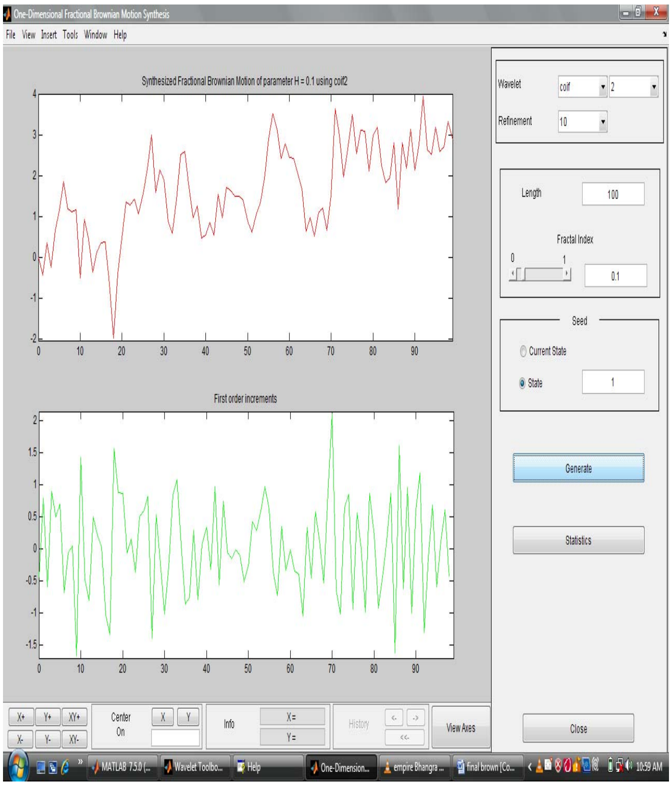


(b) 1-D fractional Brownian motion statistics using db2 wavelet with Order=2, no. of bins=10 is shown in diagram below.

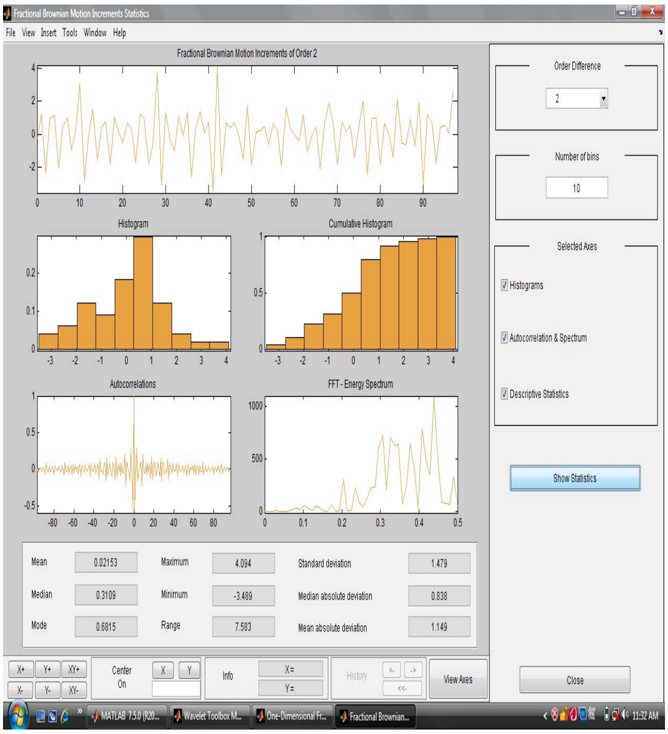


Case3

a) Using Coif Wavelet put the value of refinement=10, length=100, fractal index=0.1, state=1, then one dimensional fractional brownian motion synthesis h=0.1 is shown in diagram below.

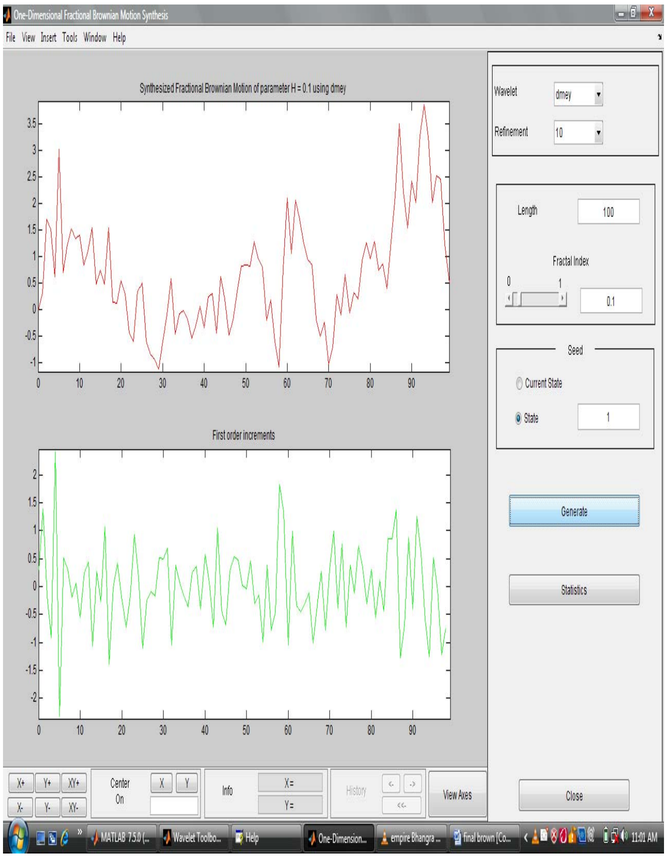


(a) 1-D fractional Brownian motion statistics of Coif Wavelet with Order=2, no. of bins=10 is shown in diagram below.

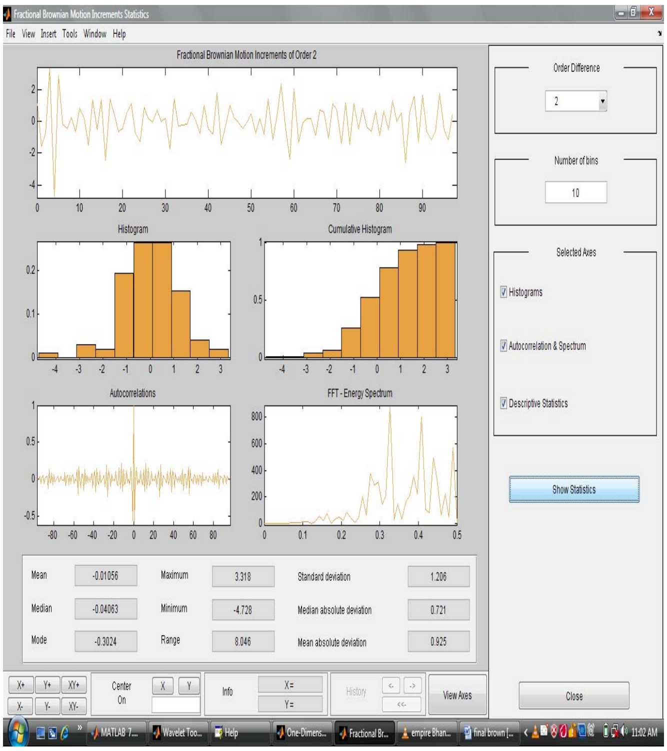


Case4

a) Using dmey Wavelet with refinement=10, length=100, fractal index=0.1, state=1, Then one dimensional fractional Brownian motion synthesis H=0.1 is shown in diagram below.



(a) 1-D fractional Brownian motion statistics of dmey Wavelet with Order=2, no. of bins=10 is shown in diagram below.



Experimental results show that this method works efficiently fast and offers acceptable results.

Results & Conclusion

1. The value standard deviation is more in haar wavelet and less in dmey.
2. The value of median absolute deviation is more in haar wavelet and less in dmey
3. The value of mean absolute deviation is more in haar wavelet and less in dmey wavelet.

Wavelet theory has already been shown to provide an appropriate tool both for analysis and synthesis of long range dependent processes in one dimension. Number of results has been provided for characterizing and analyzing fBm via wavelets. In this paper, we introduced method to find standard deviation, mean absolute deviation and median absolute deviation in different wavelets and we analyzed deviations in values of different wavelet so that we are able to find which type of wavelet has highest value and which type of wavelet has lowest value. Fractional Brownian motion (fBm) offers a convenient modeling for non-stationary stochastic processes with long-term dependencies and 1/f-type spectral behavior over a wide range of frequencies.

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Author Profile

Abhishek Sharma was born in 1990. He received Bachelor's Degree in Electronics and communication Department from Eternal University in 2012 and pursuing his Master's degree in Electronics and communication Department from Eternal University. His research areas of interest are Digital Signal Processing, Optical Fiber Communication, Microwave Engineering, Neural Networks and Control System. He has published research paper in reputed international journal entitled "Design and analysis of 2-D wavelet based GUI image fusion system approach for restoring image by performing wavelet decomposition" in international journal of advance research in computer science and software Engineering. He is working under supervision of his M.Tech Guide Er. Bhubneshwar Sharma.

Bhubneshwar Sharma was born in 1986. He received Bachelor's Degree in Electronics and Communication Engineering from Jammu University in 2007 and received Master's degree in Electronics and Communication Department from Punjab in 2009. He is currently working as Assistant Professor in the Department of Electronics and Communication Engineering in Eternal University, Himachal Pradesh, India. He is pursuing Ph.D. He has published research papers in International Journals and presented his work at conferences. He has delivered many lectures and chaired technical sessions at international & national platforms. He pursues a broad range of research interests that include Digital Signal Processing, Neural Networks, and Wireless Sensor Networks. His comprehensive academic background coupled with an excellent Knowledge and versatile experiences that vibrate with his confidence.