International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2025: 7.089

Wiener Indices of Interval-Valued Fuzzy Graphs

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Abstract: ***The Wiener index of a graph G is defined as the sum of the distances between all unordered pairs of vertices. In this paper the Wiener index and hyper-Wiener index of interval-valued Pythagorean fuzzy graph, interval-valued complex Pythagorean fuzzy graph, interval-valued picture fuzzy graph, interval-valued intuitionistic fuzzy graph, and interval-valued Fermatean neutrosophic graph are investigated.***

Keywords: Distance, edge length, interval-valued fuzzy graph, membership value, non-membership value, Pythagorean fuzzy graph, Wiener indices

1. Introduction

Fuzzy graph theory is a combination of graph theory and fuzzy set theory which have been applied in various fields of science and engineering [1].An interval-valued fuzzy graph [IVFG] generalizes standard fuzzy graphs by assigning membership degrees as subintervals [0,1].An uncertainties are well expressed using picture fuzzy set, picture fuzzy graph would be prominent research direction for modeling the uncertain optimization problems [2].The concept of antipodal interval-valued fuzzy graphs and self-median interval fuzzy graphs were introduced by H. Rashmanlou and M. Pal [3].The concept of m-PIVFGs and complement of m-PIVFGs were introduced with illustrative examples in [4].An interval-valued picture fuzzy set (IVPFS) is one of the generalizations of an interval-valued fuzzy set to handle the uncertainties in the data during analysis [5].The concept of interval-valued weak fuzzy graphs and Fermat's weak fuzzy graphs were introduced in [6].The definition of a self-centered IVIFG and the necessary and sufficient conditions for an IVIFG to self-centered was given in [7].An overview on the fuzzy graph and its various kinds appear in [8].Interval-valued intuitionistic fuzzy graphs has various uses in modern science and technology, especially in the fields of neutral networks, computer science, operation research, and decision-making [9].The properties of various types of degrees, order and size of intuitionistic fuzzy graphs and a new definition for complete intuitionistic fuzzy graph and intuitionistic regular fuzzy graph have been studied in [10].The development of interval-valued Pythagorean neutrosophic set provides a more comprehensive framework for handling uncertainty, ambiguity, and incomplete information in various fields, leading to more robust decision-making processes, improved problem-solving capabilities, and better management of complex systems [11].Pythagorean fuzzy graph theory plays significant role in modeling many real-world problems containing uncertainties in different fields such as decision-making theory, computer science, optimization problems, data analysis, networking etc.[12]. V. N. S. Rao Repalle introduced the concept of interval-valued intuitionistic fuzzy line graphs (IVFLG) and explored the results related to IVFLG [13]. Some Kulli-Basava indices and Kulli-Basava polynomials were studied for spherical fuzzy graph by considering membership degree (α), neutral membership degree (β) and non-membership degree (γ) in the interval [0, 1] in [14].

The Wiener index was introduced by Harold Wiener in 1947, to study the boiling points of paraffin [15]. The Wiener index of connected graph G is denoted by W(G) and is defined as the sum of distances between all unordered pairs of vertices in G i.e.

$$\text{Wiener index} = W(G) = \frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v), \quad (1)$$

where $d_G(u,v)$ is the distance between two vertices.

The hyper-Wiener index was introduced by M. Randic [16][17][18] as:

Hyper-Wiener index =

$$WW(G) = \frac{1}{2}\sum_{u,v\in V(G)}[d_G(u,v) + d_G(u,v)^2]. \quad (2)$$

Below are five types of interval-valued fuzzy graphs defined in terms of conditions for vertices and edges of the graph G (V, E).

Interval-valued Pythagorean fuzzy graph:
Let G^*= (V, E) be a crisp graph. An Interval-Valued Pythagorean Fuzzy Graph (IVPFG) [19-20] on G^* is an ordered pair G(A,B) where:

A is an interval-valued Pythagorean fuzzy set on the vertex set V, and B is an interval-valued Pythagorean fuzzy relation on the edge set E.

For every vertex v∈ V,
$A(v) = ([\mu_A^L(v), \mu_A^U(v)], [\nu_A^L(v), \nu_A^U(v)])$,

where: $0 \le \mu_A^L(v) \le \mu_A^U(v) \le 1$, $0 \le \nu_A^L(v) \le \nu_A^U(v) \le 1$,

and the Pythagorean conditions are

$(\mu_A^L(v))^2 + (\nu_A^L(v))^2 \le 1$, $(\mu_A^U(v))^2 + (\nu_A^U(v))^2 \le 1$.

For every edge uv∈ E,

$B(u,v) = ([\mu_B^L(u,v), \mu_B^U(u,v)], [\nu_B^L(u,v), \nu_B^U(u,v)])$, such that: $(\mu_B^L(u,v))^2 + (\nu_B^L(u,v))^2 \le 1$, $(\mu_B^U(u,v))^2 + (\nu_B^U(u,v))^2 \le 1$,

and
$\mu_B^L(u,v) \le \min(\mu_A^L(u), \mu_A^L(v)), \mu_B^U(u,v) \le \min(\mu_A^U(u), \mu_A^U(v))$,

$$\nu_B^L(u,v) \ge \max(\nu_A^L(u), \nu_A^L(v)), \nu_B^U(u,v) \ge \max(\nu_A^U(u), \nu_A^U(v)).$$

Interval-valued complex Pythagorean fuzzy graph:

An Interval-Valued Complex Pythagorean Fuzzy Graph (IVCPFG) [21] combines fuzzy sets with interval-valued Pythagorean fuzzy sets. An interval-valued complex Pythagorean fuzzy graph is structured as G (V, A, B), where V is the underlying non-empty set of vertices,

A is an interval-valued complex Pythagorean fuzzy set on V, B is an interval-valued complex Pythagorean fuzzy relation on V×V.

Vertex set (A):

For each vertex set v∈ V,
$A(v) = \{[\mu_A^L(v)e^{2\pi i\alpha_A^L(v)}, \mu_A^U(v)e^{2\pi i\alpha_A^U(v)}], [\nu_A^L(v)e^{2\pi i\beta_A^L(v)}, \nu_A^U(v)e^{2\pi i\beta_A^U(v)}]\}$,

$$0 \le \mu_A^L(v) \le \mu_A^U(v) \le 1, 0 \le \nu_A^L(v) \le \nu_A^U(v) \le 1,$$

$$0 \le \alpha_A^L(v) \le \alpha_A^U(v) \le 1, 0 \le \beta_A^L(v) \le \beta_A^U(v) \le 1.$$

Condition:
1) Amplitude: $(\mu_A^L(v))^2 + (\nu_A^L(v))^2 \le 1, (\mu_A^U(v))^2 + (\nu_A^U(v))^2 \le 1$.
2) Phase: $(\alpha_A^L(v))^2 + (\beta_A^L(v))^2 \le 1$, $(\alpha_A^U(v))^2 + (\beta_A^U(v))^2 \le 1$.

Edge set (B):

For each edge set: (u,v) ∈ V×V,
$B = \{[\mu_B^L(u,v)e^{2\pi i\alpha_B^L(u,v)}, \mu_B^U(u,v)e^{2\pi i\alpha_B^U(u,v)}], [\nu_B^L(u,v)e^{2\pi i\beta_B^L(u,v)}, \nu_B^U(u,v)e^{2\pi i\beta_B^U(u,v)}]\}$.

Upper and lower bounds:
$\mu_B^L(u,v) \le \min[\mu_A^L(u), \mu_A^L(v)], \mu_B^U(u,v) \le \min[\mu_A^U(u), \mu_A^U(v)]$,

$$\nu_B^L(u,v) \ge \max[\nu_A^L(u), \nu_A^L(v)], \nu_B^U(u,v) \ge \max[\nu_A^U(u), \nu_A^U(v)].$$

Phase term bounds:

$$\alpha_B^L(u,v) \le \min(\alpha_A^L(u), \alpha_A^L(v)), \alpha_B^U(u,v) \le \min(\alpha_A^U(u), \alpha_A^U(v)),$$

$$\beta_B^L(u,v) \ge \max(\beta_A^L(u), \beta_A^L(v)), \beta_B^U(u,v) \ge \max(\beta_A^U(u), \beta_A^U(v)),$$

and $(\mu_B^L)^2 + (\nu_B^L)^2 \le 1, (\mu_B^U)^2 + (\nu_B^U)^2 \le 1$,

$(\alpha_B^L)^2 + (\beta_B^L)^2 \le 1, (\alpha_B^U)^2 + (\beta_B^U)^2 \le 1$.

Interval-valued picture fuzzy graph:

Let G* = (V,E) be a underlying graph. An Interval-Valued Picture Fuzzy Graph (IVPFG) [22-24] on G* is an ordered pair G = (A,B), where

A is the interval-valued fuzzy vertex set, B is the interval-valued picture fuzzy edge set.

For every vertex v∈ V,

$A(v) = = \{[\mu_v^L(u), \mu_v^U(u)], [\eta_v^L(u), \eta_v^U(u)], [\nu_v^L(u), \nu_v^U(u)]\}$,

satisfies, $0 \le \mu_v^U(u)+\eta_v^U(u)+\nu_v^U(u) \le 1$, where μ is membership interval, η − neutral interval and ν is non − mebership interval.

For every edge (u,v)∈ E,

$B(u,v) = \{[\mu_v^L(u,v), \mu_v^U(u,v)], [\eta_v^L(u,v), \eta_v^U(u,v)], [\nu_v^L(u,v), \nu_v^U(u,v)]\}$.

Conditions: $\mu_E^L(u,v) \le \min(\mu_v^L(u), (\mu_v^L(v)), \mu_E^U(u,v) \le \min(\mu_v^U(u), \mu_v^U(v))$,

$\eta_E^L(u,v) \le \min(\eta_v^L(u), \eta_v^L(v)), \eta_E^U(u,v) \le \min(\eta_v^U(u), \eta_v^U(v))$,

$\nu_E^L(u,v) \ge \max(\nu_v^L(u), \nu_v^L(v)), \nu_E^U(u,v) \ge \max(\nu_v^U(u), \nu_v^U(v))$,

and $0 \le \mu_E^U(u,v)+\eta_E^U(u,v)+\nu_E^U(u,v) \le 1$.

Interval-valued intuitionistic fuzzy graph:
An Interval-Valued Intuitionistic Fuzzy Graph (IVIFG) [25] is denoted by G (V, E), where

Vertex set (V):
For a vertex $u \in V, \mu_1(u) = [\mu_1^L(u), \mu_1^U(u)], \nu_1(u) = [\nu_1^L(u), \nu_1^U(u)]$, where μ and ν denote the degree of membership and non-membership,
satisfies the condition: $0 \le \mu_1^U(u) + \nu_1^U(u) \le 1$.

Edge set (E):
For any edge e = (u,v) ∈ E, $\mu_2(u,v) = [\mu_2^L(u,v), \mu_2^U(u,v)], \nu_2(u,v) = [\nu_2^L(u,v), \nu_2^U(u,v)]$.

Lower and upper bounds:

$$\mu_2^L(u,v) \le \min[\mu_1^L(u), \mu_1^L(v)], \mu_2^U(u,v) \le \min[\mu_1^U(u), \mu_1^U(v)],$$

$$\nu_2^L(u,v) \ge \max[\nu_1^L(u), \nu_1^L(v)], \nu_2^U(u,v) \ge \max[\nu_1^U(u), \nu_1^U(v)],$$

and $0 \le \mu_2^U(u,v) + \nu_2^U(u,v) \le 1$.

Interval-valued Fermatean neutrosophic graph:
Let G^*= (V, E) be a crisp graph. An Interval-Valued Fermatean Neutrosophic Graph (IVFNG) [26] on G^* is an ordered pair G = (A, B), where

A is the interval-valued Fermatean neutrosophic vertex set,

B is the interval-valued Fermatean neutrosophic edge set.

For every vertex set v ∈ V,
$A(v) = ([T_A^L(v), T_A^U(v)],[I_A^L(v), I_A^U(v)],[F_A^L(v), F_A^U(v)])$,

such that: $(T_A^U(v))^3 + (F_A^U(v))^3 \leq 1$,and $0\leq T_A^L\leq T_A^U\leq 1$, $0\leq I_A^L\leq I_A^U\leq 1$, $0\leq F_A^L\leq F_A^U\leq 1$, where T_A- truth membership interval, I_A- indeterminacy membership interval, and

F_A- falsity membership interval.

For every edge (u,v) ∈ E,
$B(u,v)=([T_B^L(u,v), T_B^U(u,v)],[I_B^L(u,v), I_B^U(u,v)],[F_B^L(u,v), F_B^U(u,v)])$,

Subject to:
$T_B^L(u,v) \leq \min[T_A^L(u), T_A^L(v)]$, $T_B^U(u,v) \leq \min[T_A^U(u), T_A^U(v)]$,

$I_B^L(u,v) \leq \min[I_A^L(u), I_A^L(v)]$, $I_B^U(u,v) \leq \min[I_A^U(u), I_A^U(v)]$, and

$F_B^L(u,v) \geq \max[F_A^L(u), F_A^L(v)]$, $F_B^U(u,v) \geq \max[F_A^U(u), F_A^U(v)]$,

with Fermatean condition: $(T_B^U(u,v))^3 + (F_B^U(u,v))^3 \leq 1$.

The symbols and notations follow standard conventions as established [27-29]. In this paper the Wiener index and hyper-Wiener index of interval-valued Pythagorean fuzzy graph, interval-valued complex Pythagorean fuzzy graph, interval-valued picture fuzzy graph, interval-valued intuitionistic fuzzy graph, and interval-valued Fermatean neutrosophic graph are obtained.

2. Materials and Methods

The graphs of interval-valued fuzzy graphs such as: Pythagorean, complex Pythagorean, picture, intuitionistic and Fermatean neutrosophic graphs are represented in figures (1-5). Membership, neutral and non-membership values in each case are determined from the respective graphs. The conditions for vertices and edges in all studied interval-valued graphs are satisfied. Wiener index and hyper-Wiener index are distance based topological indices. The shortest distances for all unordered pairs of vertices are obtained from the respective graphs.

3. Results and Discussion

Theorem 1: The Wiener index and hyper-Wiener index of interval-valued Pythagorean fuzzy graph respectively are 0.8449 and 1.5587.

Proof: By using figure (1), table (1) and equations (1-2), we obtain Wiener index and hyper-Wiener index of interval-valued Pythagorean fuzzy graph.

The edge weight is defined as:

$$w(e) = \frac{\mu_L + \mu_U + \sqrt{1-\nu_L^2} + \sqrt{1-\nu_U^2}}{4},$$

where μ_L, μ_U are the lower and upper membership values and ν_L, ν_U are lower and upper non-membership values. Then the edge length L(e) = 1- w(e).

Wiener index = $W(G)=\frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)$ = 0.8449.

Hyper-Wiener index = WW(G) = $\frac{1}{2}\sum_{u,v\in V(G)}[d_G(u,v) + d_G(u,v)^2]$ = 1.5587.

Table 1: Edge, membership value, non-membership value, edge weight, edge length and shortest distance of interval-valued Pythagorean fuzzy graph.

Edge	Membership value	Non-membership value	Edge weight w(e)	Edge length L(e)	Shortest distance d(u,v)
(x,z)	[0.4,0.6]	[0.2,0.6]	0.6947	0.3051	0.3051
(y,z)	[0.4,0.6]	[0.2,0.5]	0.7114	0.2885	0.2885
(x,y)	[0.5,0.7]	[0.1,0.6]	0.7487	0.2513	0.2513

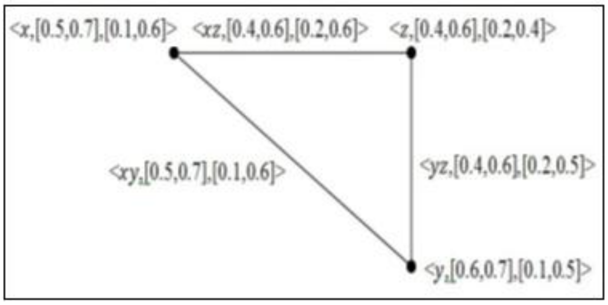

Figure 1: Interval-valued Pythagorean fuzzy graph.

Theorem 2: The Wiener index and hyper-Wiener index of interval-valued complex Pythagorean fuzzy graph respectively are 1.85 and 5.2725.

Proof. Consider figure (2) for IVCPFG with three vertices (V_1, V_2, V_3) and edges (V_1, V_2), (V_2, V_3) and (V_1, V_3). For an edge e = (u,v), the membership amplitude is (μ_L,μ_U), non-membership amplitude is (ν_L, ν_U), membership phase ($\propto_L$,$\propto_U$) and non-membership phase is

(β_L,β_U). The term ($\propto_L$,$\propto_U$) is lower and upper membership phase values and (β_L,β_U) is lower and upper non-membership phase values. Edge length is defined as:
$L(e) = \frac{1}{4}[(1-\mu_L) + (1-\mu_U) + (\nu_L + \nu_U)] + \frac{1}{4\pi}[(\propto_L + \propto_U) + (\beta_L + \beta_U)]$.

The shortest-path distance between vertices u and v is d(u,v) = min $\sum L(e)$.By using figure (2), table (2) and equations (1-2), we obtain Wiener index and hyper-Wiener index of interval-valued complex Pythagorean fuzzy graph as:
Wiener index = $W(G)=\frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)$=1.85.
Hyper-Wiener index = WW(G)
$=\frac{1}{2}\sum_{u,v\in V(G)}[d_G(u,v) + d_G(u,v)^2]$= 5.2725.

Table 2: Edge, membership value, non-membership value, edge length and shortest distance of interval-valued complex Pythagorean fuzzy graph

Edge	Membership amplitude (μ_L, μ_U)	Non-membership amplitude (ν_L, ν_U)	Membership phase $(\propto_L, \propto_U)$	Non-membership phase (β_L, β_U)	Edge length L(e)	Shortest distance d(u,v)
(V_1,V_2)	[0.7,0.8]	[0.2,0.3]	[0.30π,0.40π]	[0.10π,0.2π]	0.5	0.5
(V_2,V_3)	[0.5,0.6]	[0.4,0.5]	[0.2π,0.3π]	[0.3π,0.4π]	0.75	0.75
(V_1,V_3)	[0.65,0.75]	[0.25,0.35]	[0.35π,0.45π]	[0.15π,0.25π]	0.6	0.6

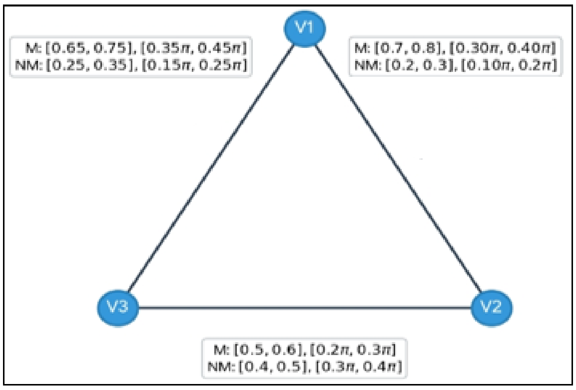

Figure 2. Interval-valued complex Pythagorean fuzzy graph

Theorem 3: The Wiener index and hyper-Wiener index of interval-valued picture fuzzy graph respectively are 0.7666 and 1.3542.

Proof: Consider figure for interval-valued picture fuzzy graph with three vertices V_1 = ([0.60,0.80], [0.10,0.30], [0.05,0.20]), V_2 = ([0.50,0.70], [0.20,0.40], [0.05,0.15]), V_3 = ([0.65,0.85], [0.10,0.25], [0.05,0.15]) and edges:(V_1, V_2), (V_2, V_3) and (V_1, V_3),the values of edges are represented in table (3). The interval-valued picture fuzzy graph is shown in figure (3). The membership interval $(\mu) = [\mu^L, \mu^U]$, neutral interval $(\eta) = [\eta^L, \eta^U]$ and non − mebership interval $(\nu) = [\nu^L, \nu^U]$, where μ^L, μ^U denote the lower and upper membership values and η^L, η^U the lower and upper neutral values and ν^L, ν^U lower and upper non − membership values are used to define edge length. Edge length is defined as:

$$L(e) = \frac{(1-\mu^L)+(1-\mu^U)+(\eta^L+\eta^U)+(\nu^L+\nu^U)}{6}.$$

By using figure (3), table (3) and equations (1-2), we get the Wiener index and hyper-Wiener index of interval-valued picture fuzzy graph as:

Wiener index = $W(G)=\frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)$=0.7666.

Hyper-Wiener index = WW(G)

$=\frac{1}{2}\sum_{u,v\in V(G)}[d_G(u,v)+d_G(u,v)^2]$= 1.3542.

Table 3: Edge, membership, neutral, non-membership values, edge length and shortest distance of interval-valued picture fuzzy graph

Edge	$[\mu^L, \mu^U]$	$[\eta^L, \eta^U]$	$[\nu^L, \nu^U]$	Edge length L(e)	Shortest distance d(u,v)
(V_1,V_2)	[0.50,0.70]	[0.15,0.35]	[0.05,0.20]	0.2583	0.2583
(V_2,V_3)	[0.60,0.80]	[0.10,0.30]	[0.05,0.15]	0.2	0.2
(V_1,V_3)	[0.40,0.60]	[0.20,0.40]	[0.05,0.20]	0.3083	0.3083

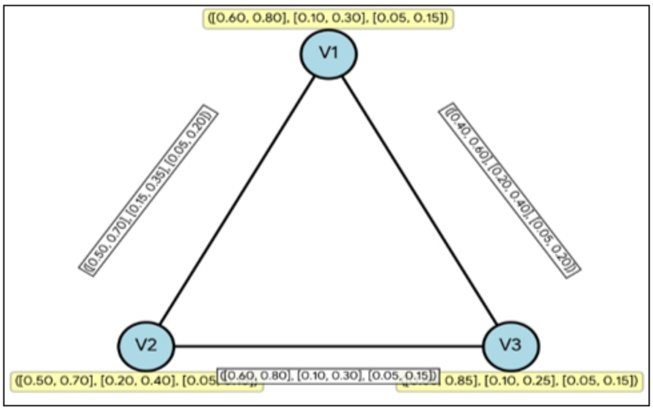

Figure 3: Interval-valued picture fuzzy graph

Theorem 4: The Wiener index and hyper-Wiener index of interval-valued intuitionistic fuzzy graph respectively are 2.49 and 8.6901.

Proof: The interval-valued intuitionistic fuzzy graph is shown in figure (4). The edge length is defined as: L(e) = - $\frac{1}{4}([\mu^L+\mu^U]+[1-\nu^L]+[1-\nu^U])$, where μ and ν denote the degree of membership and non membership, and $d(V_1,V_4)=d(V_1,V_2)+d(V_2,V_4)=0.7275.$

Using figure (4), table (4) and equations (1-2), we obtain the Wiener index and hyper-Wiener index of interval-valued intuitionistic fuzzy graph as:

$W(G)=\frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)$ =2.49.

Hyper-Wiener index = WW(G) = $\frac{1}{2}\sum_{u,v\in V(G)}[d_G(u,v)+d_G(u,v)^2]$= 8.6901.

Table 4: Edge, $[\mu^L, \mu^U]$, $[\nu^L, \nu^U]$, edge length and shortest distance of interval-valued intuitionistic fuzzy graph.

Edge	$[\mu^L, \mu^U]$	$[\nu^L, \nu^U]$	Edge length L(e)	Shortest distance d(u,v)
(V_1,V_2)	[0.1,0.2]	[0.3,0.4]	0.4	0.4
(V_2,V_3)	[0.07,0.1]	[0.4,0.5]	0.3175	0.3175
(V_2,V_4)	[0.01,0.1]	[0.4,0.4]	0.3275	0.3275
(V_1,V_3)	[0.15,0.2]	[0.2,0.3]	0.4625	0.4625
(V_3,V_4)	[0.02,0.1]	[0.5,0.6]	0.255	0.255

Paper ID: MR26815182008 DOI: https://dx.doi.org/10.21275/MR26815182008

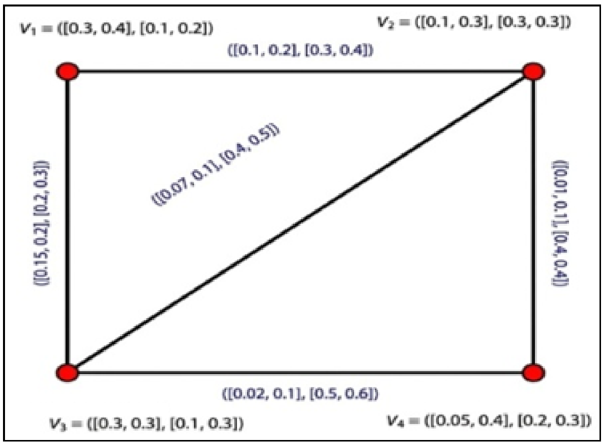

Figure 4: Interval-valued intuitionistic fuzzy graph

Theorem 5: The Wiener index and hyper-Wiener index of interval-valued Fermatean neutrosophic graph respectively are 1.8916 and 5.4697.

Proof. The interval-valued Fermatean neutrosophic graph is shown in figure (5), we take $x_1=V_1$, $x_2=V_2$ and $x_3=V_3$ for convenience. The edge length is defined as

$$\text{Edge length} = L(e) = \frac{(1-T^L)+(1-T^U)+I^L+I^U+F^L+F^U}{6}.$$

Using figure (5), table (5) and equations (1-2), we get the Wiener index and hyper-Wiener index of interval-valued Fermatean neutrosophic graph as:

$$\text{Wiener index} = W(G)=\frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)=1.8916.$$

$$\text{Hyper-Wiener index} = WW(G) = \frac{1}{2}\sum_{u,v\in V(G)}[d_G(u,v)+d_G(u,v)^2]=5.4697.$$

Table 5: Edge, truth, inderminacy, falsity, edge length and shortest distance of interval-valued Fermatean neutrosophic graph.

Edge	Truth $[T^L, T^U]$	Inderminacy $[I^L, I^U]$	Falsity $[F^L, F^U]$	Edge length L(e)	Shortest distance d(u,v)
(V_1,V_2)	[0.80,0.90]	[0.90,0.95]	[0.80,0.85]	0.6333	0.6333
(V_2,V_3)	[0.85,0.90]	[0.90,0.95]	[0.85,0.85]	0.6333	0.6333
(V_1,V_3)	[0.85,0.95]	[0.90,0.95]	[0.85,0.85]	0.625	0.625

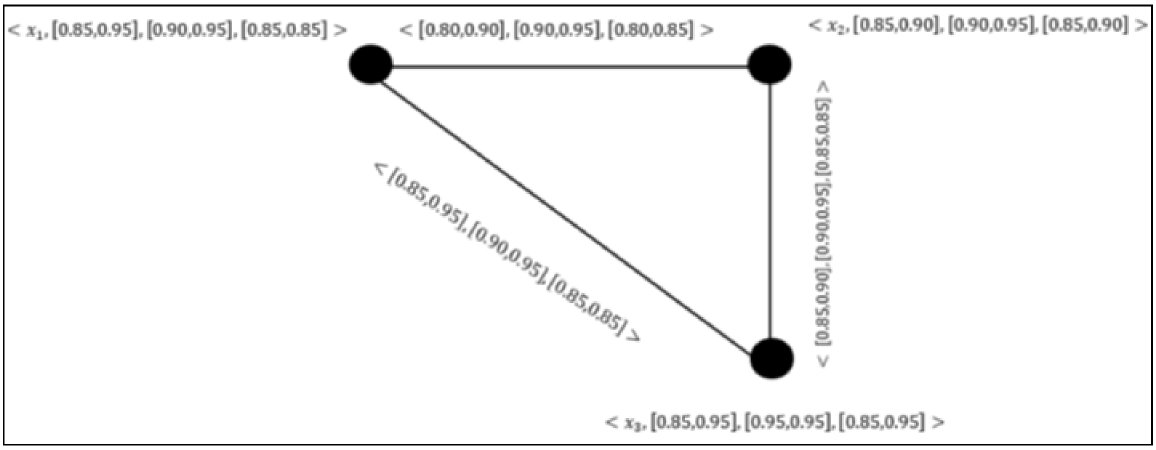

Figure 5: Interval-valued Fermatean neutrosophic graph

4. Conclusion

The distance-based Wiener index and hyper-Wiener index are obtained for interval-valued Pythagorean fuzzy graph, interval-valued complex Pythagorean fuzzy graph, interval-valued picture fuzzy graph, interval-valued intuitionistic fuzzy graph, and interval-valued Fermatean neutrosophic graph.

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