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Do all the Most Massive Neutron Stars Exhibit the Extreme Precisions? A Possibility

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Abstract: ***In atomic clock the precision shows the ellipticity ranging $10^{-18} – 10^{-19}$ for its stability. In the case of double neutron star systems (DNSs) all of them have masses > 2.5 $M_\odot$ and exhibit equatorial deformation ranging from $10^{-18} – 10^{-2}$, i.e. subdividing ($10^{-18} – 10^{-19}$) supports the ellipticity which is for the Atomic Clock Precision values while another subdividing ($10^{-20} – 10^{-21}$) favours the possible further extreme precision values beyond that of Atomic Clock. Till date the detected heaviest neutron star is pulsar PSR J1748–2021B with mass $2.5^{+0.047}_{-0.078}$ $M_\odot$. Since all the detected 21 double neutron star systems have masses > 2.5 $M_\odot$ it is suggested that "all the massive neutron stars exhibit extreme precision" which may be within that of the Atomic Clock case (i.e. $10^{-18} – 10^{-19}$) or beyond the Atomic Clock case (i.e. $10^{-20} – 10^{-21}$). This implies that beyond the Atomic clock case, further extreme precision is possible and double neutron star systems are the potential sources for measuring the extreme precision parameters. This author encourages the Pulsar communities to observe extreme precision during their observations.***

Keywords: neutron star, millisecond pulsars, atomic clock, extreme precision

1. Introduction

Neutron stars are the densest manifestations of massive compact objects in the observable universe. So, the knowledge of neutron star properties is essential as probe of understanding the high density behavior of equation of state (EOS) for the baryonic matter in the region of beta equilibrium. Not only that, the precise gravitational mass and radius measurements of neutron star offer the scientists / astronomers to find an effective ways to constrain the EOS of high density matter in its interior. Again, theoretical studies of dense matter (a) have considerable large uncertainty in the high- density behavior of EOSs, and (b) is essential in order to check new observational masses and radii leading to the effective constraints on the EOSs of dense matter. For example, recent astrophysical observations like GW170817, GW190814, PSR J0740 + 6620, etc.

Compact stellar objects such as white dwarfs, neutron stars, pulsars, magnetars provide a unique natural laboratory for studying matter under extreme conditions (Archibald et al. 2009; Linares et al. 2014) as the microscopic description of dense nuclear matter inside the neutron star remains still an open challenging problems.

Millisecond pulsars (MSPs) in compact binaries provide a good opportunity to the scientists / astronomers to search for the most massive neutron stars (Linares etal. 2018). In particular, the companion stars are often strongly irradiated by the pulsar, even accretion displacing the effective center of light from their barycenter resulting which mass measurements become uncertain. Recent discovery of Red back (RB) and Black widow (BW) binary MSPs, through a series of optical spectroscopic and photometric observations, suggest that

a) their companion or secondary stars are light (~ 0.1 $M_\odot$ in the so- called red backs) or ultra-light (~ 0.01 $M_\odot$ in the black widows);
b) They are strongly irradiated by the pulsar wind; and high energy radiation powered by the neutron star's rotational energy loss.
c) 3 out of the nearly 20 known red back MSPs have shown transitions between the radio pulsars and accretion disk states ; and
d) Provide a long sought link between MSPs and low mass X-ray binaries (de Martino etal. 2013).

Till today, more than 550 millisecond pulsars (MSPs) with spin period $\lesssim$ 30 ms have been detected. But not all binary companions to these MSPs are white dwarfs, but some of them may have a more exotic matter. These binary MSPs are divided into two sub-classes depending on the mass of their companions- black widows (BWs) with non-degenerate companions (< 0.1 $M_\odot$) and red backs (RBs) with semi-degenerate companions (0.3 – 0.7) $M_\odot$.

In this study we investigate the significant roles of double neutron stars (DNSs), BWs and RBs towards understanding the extreme precision using their equatorial deformations (i.e. ellipticity) due to their high speed rotation.

2. Ellipticity, Neutron Star's Maximum Mass and Extreme Precision

Dense matter situated in the interior of a neutron star is difficult to understand from first principle. Because the strong coupling nature of neutron star matter is so dense but not asymptotically dense (Kosinski etal 2007). This means that when a young, isolated neutron star rotates tens of times each seconds or a few thousand revolution per minute then it generates beams of radio, visible light, X-rays, and gamma rays that can be observed as pulsed emission when the beams sweep the earth. In other words, they also generate "powerful outflows" or "winds". In binary system, in particular recycled MSPs this "wind" impinges on the atmosphere of its companion resulting which this could turn out into a unique laboratory for analyzing the nature and energy of the outflow.

The pulsar binary system, comprises a millisecond period pulsar and a low mass companion star (with a mass less than 1 $M_\odot$ (i.e. < 1 $M_\odot$) evolve from low mass X-ray binaries {LMXBs) in which pulsar accretes mass and angular momentum from the companion over Gyrs (Bhattacharya, van den Heuvel 1991). Under the situation when the accretion is quenched, it turns into pulsar binaries. The significant consequence is that the pulsar (i) spins rapidly with a millisecond period and (ii) having a high mass (exceeding 2 $M_\odot$) allows for constraints on the equation of state of dense matter (Linares 2021). In fact, in these pulsar binaries the companion's spin is tidally locked, leading to only one side i.e. facing the pulsar and receives the pulsar's output energy. This means that a strong companion's winds interact with the pulsar wind (Sanchez, Romani 2017).

Follow up of the optical observations of gamma ray pulsars (i.e. pulsar-like sources), in particular eclipsing binary systems, suggests two types of populations which are differentiated by the companion mass- (a) blackwidow (BW) and redback (RB) systems. In the case of BW pulsars they have very low mass degenerate companions (0.02 $M_\odot$ < M < 0.05 $M_\odot$) and the RB pulsars have slightly more massive companions (0.2 $M_\odot \le M \le 0.4 M_\odot$). Another important fact is that the companions of these types of pulsars are "ablated" by the pulses that heats and strips material off of the companion's surface into the surrounding medium. Therefore, the companions in BW systems are degenerate stars which are highly "ablated", while the RBs tend to be non-degenerate and are less "ablated" companions (Dilullo et al 2019).

2.1 Searching the Neutron Star's Maximum Mass

From Double Neutron Star System

Double neutron stars (DNSs) are a unique and rare population of neutron stars. In this DNS the neutron stars are less heavy than those in the other NS binaries, i.e. WD-NS systems. Not only that, pulsars in these DNS systems are rotating faster than regular pulsars (i.e. with P ≤ 200 ms) and due to emission of gravitational radiation (GW) their orbit, as a consequence, decays and they merge. This makes the DNS systems as prime candidates for

(a) detection of gravitational waves as well as for (b) finding the neutron star with maximum masses, (c) sources for heavy r-process nucleosynthesis (Lattimer, Schramm 1974), and candidates as progenitors of short gamma ray bursts (GRBs) (Levinson, Eichler 1989).

From Black Widow (BW) pulsars

Black widows (BWs) are the systems with a millisecond pulsar and a low mass companion. According to our present understanding, a millisecond pulsars are spun up to their fast rotation rates by accreting materials from their main sequence companion. This eventually leads to their full destruction, leaving behind massive isolated millisecond pulsars (i.e. massive neutron stars) (Manchester et al. 2017; Ginsburg, Quataert 2020). In some studies (Romani et al 2016) it is suggested that over time, the companion of this system is reduced to a few percent of the solar mass such that the gravitational waves are too weak and under such situation the "ablation or evaporation of the companion follow the pulsar's spin down energy (Ruderman et al. 1989; Levinson, Eichler 1991).

2.1.2 Ellipticity of MSPs (i. e. heaviest Neutron Stars)

In general, the ellipticity of a star means the ratio between the size of deformation (or mountain) Δr compared to the star's radius "r". While in practical this is a ratio of the difference between two perpendicular moments of inertia and the third perpendicular moment of inertia.

Idealizing a neutron star with the shape of a slightly deformed, homogeneous ellipsoid and having a small ellipticity (ϵ) (Krastev et. al 2018; Talukder et. al 2014; Shapiro, Teukolosky 1983) which can be expressed as

$$\epsilon = (I_{xx} - I_{yy}) / I_{zz} \quad (1)$$

where I_{xx} , I_{yy} , and I_{zz} are the principal moments of inertia of the neutron star w.r.t. rotation axis such that I_{zz} is assumed to be aligned with the spin axis.

Regarding the ellipticity of magnetic origin it was shown by different authors as

(A) According to Moriya and Tauris (2016)

$$\epsilon < (5/3\ G)^{1/2} \{C\ R^3\ P_o\ B_{dipole}/2^4\ \pi\ I\}$$

$$\cong 3x\ 10^{-4} (B_{dipole}/\ 10^4\ G)\ (P_o/1ms) \quad (2)$$

with the fiducial standard neutron star properties of I = 10^{45} g cm^2, R = 10Km, P_o = initial spin period, and the angle between the spin axis and the principal axis of the neutron star distortion = π/ 2 (Cutler and Jones 2001).

(B) According to Bonazzola and Gourgoulhon 1996; Konno etal 2000; Regimbou and de Freitas Pacheco 2006

$$\epsilon = \kappa.\ (B_o^2\ R^4\ /\ GM^2).\ \sin^2\phi \quad (3)$$

where B_o = the dipole magnetic field, M and R are the mass and radius of the star, respectively. Φ = the angle between the rotation and the magnetic dipole axes, κ = the distortion parameter that depends on both the star equation of state and the magnetic field configuration.

By considering the spin period (P) and period derivative ($\dot{P}$) (i.e. observable quantities, moment of inertia (I), and η (as the fraction of rotational power ($\dot{E}_{rot}$) emitted in the form of gravitational waves (i.e. $\dot{E}_{GW}$), i.e. the efficiency for gravitational wave generation (de Araujo 2016c) then ellipticity in eqn. (3) can be rewritten as

$$\epsilon = [3\ I\ c^3\ /\ 4\pi^2\ GM^2\ R^2\ ].\ P\ \dot{P}\ (1 - \eta)\ \kappa \quad (6)$$

and with η relating to pulsar dipole magnetic field which can be expressed as

$$\eta = \left(\frac{288}{5}\right) .\ \{ I^3\ c\ \dot{P}\ /\ GM^4\ R^4\ P \}\ .\ (1 - \eta\ )^2\ \kappa^2 \quad (7)$$

Since ϵ and η << 1, then one can obtain the expression for ellipticity as

$$\epsilon\ \cong\ (3\ I\ c^3\ /\ 4\pi^2\ GM^2\ R^2).\ P\ \dot{P}\ \kappa \quad (8)$$

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Regarding the distortion parameter ‘κ’ which depends on the EOS and on the magnetic field configuration. For realistic case κ = 10 but values as high as κ = 1000 can be considered although they are probably unrealistic.

Therefore, using the fiducial values for neutron star namely M = 1.4 M$_\odot$, R = 10km, I = 10^{38} Kg m^2, κ = 10 in eqn. (8) the ellipticity can be expressed as

$$\epsilon = 1.72814 \times 10^4 \, P \, \dot{P} \tag{9}$$

In Table 1 the estimated ellipticity of double neutron star systems (DNSs), Black Widow (BW) pulsars and Red back (RB) are shown.

Table 1: Various parameters period (P), period derivative ($\dot{P}$), ellipticity (ϵ), and Mass of the pulsar (m_{system}) for double neutron star systems (DNSs), Black Widow pulsars (BWs) and Redback pulsars (RBs).

Name	Period (P) s	Period Derivative ($\dot{P}$) s s^{-1} x 10^{-20}	Ellipticity (ϵ)	Mass of primary (m_p) M$_\odot$	Mass of companion (m_c) M$_\odot$	Mass of the pulsar (m_{system}) M$_\odot$	Reference
Double Neutron star system							
J0737-3039	0.022699	0.0176	6.90396 x 10^{-20} = 6.90396(-20)	1.3381	1.2489	2.58708	Burgay 2003
J1518-4904	0.04935	0.00027	2.30266 (-22)	1.41	1.31	2.7183	Lazarus 2016
B1534+12	0.037904	0.0242	2.9433 (-19)	1.3700	1.3454	2.67846	Janssen 2008
J1756-2251	0.028462	0.0102	5.017 (-20)	1.341	1.230	2.56999	Stairs2002
J1811-1736	0.1041	0.00901	1.62084 (-19)	1.64	> 0.93	2.57	Zhao 2004
J1829+2456	0.041009	0.00053	3.7566 (-21)	1.306	1.299	2.59	Martinez2017
J1906+0746	0.144073	203.00	5.05425(-15)	1.291	1.322	2.6134	Faulkner 2005
J1913 +16	**0.059031**	**0.0063**	**8.80379 (-19)**	**1.4398**	**1.3886**	**2.8284**	Hulse 1995
J1930-1852	0.185520	0.1800	5.77088 (-18)	< 1.32	>1.30	2.59	Sengar2022
J0453+1559	0.045782	0.00186	1.47158 (-20)	1.559	1.174	2.734	Swiggum 2015
J1913+1102	**0.0273**	**0.0016**	**7.5485 (-21)**	**1.62**	**1.27**	**2.889**	Lyne2000
J1946+2052	0.0170	0.009	2.6440 (-20)	< 1.31	>1.18	2.50	Champion2004
J1208-5936	0.0287	< 0.0004	1.9839 (-21)	1.26	1.32	2.586	Lorimer2006
J1846-0513	0.0234	0.0101	4.08428 (-20)	< 1.345	>1.2845	2.629	Kramer1998
J1411+2551	0.0624	0.00096	1.0352 (-20)	<1.62	>0.92	2.538	Martinez2015
J1325-6253	0.0289	0.00048	2.39727 (-21)	< 1.59	> 0.98	2.57	Agazie2021
J1759+5036	0.176	0.00243	7.3909 (-20)	<1.919	>0.7006	2.67	Swiggum2023
J1018-1523	0.0832	0.00109	1.56721 (-20)	<1.1	>1.16	2.3	Han2021
J2150+3427	0.654	0.0360	4.0687 (-18)	<1.67	>0.98	2.59	Su2024
J1901+0658	0.0757	0.00217	2.8387 (-20)	<1.68	>1.11	2.79	Su2024
J0509+3801	**0.0765**	**0.0793**	**1.04836 (-18)**	**1.46**	**1.34**	**2.805**	Ding2024
J1757-1854	0.0215	0.0263	9.7717 (-20)	1.3412	1.3917	2.733	
Double Neutron Star system in Globular Cluster							
J1807-2500B	0.004186			1.3655	1.2064	2.57190	
B2127 + 11C	0.030529			1.358	1.354	2.71279	
J2215 +5135				2.27	0.33	2.60	Breton2013
Black-Widow (BW) Pulsars							
J0023+0923	0.00305	1.142345	6.0211 (-19)		0.018		Arzoumanian2018, Lee 2018
J0251-2606	0.00254	0.7572	5.05099 (-19)		0.032		Cromartie2016
J0312-0921	**0.003704**	**1.9725**	**1.262577(-18)**		**0.009**	**2.6**	Samuel
J0610-2100	0.00386	1.235	8.24035 (-19)		0.025		Pallanca2012
J0952-0607	0.00141	0.4773	1.16302 (-19)		0.026		Bassa2017, Ho2019
J1311-3430	0.00256	2.0999	9.27455 (-19)		0.0104		Romani2012
J1446-4701	0.002194	0.98075	3.7177 (-19)		0.022		Ng2014
J1513-2500	0.00219	0.2161	8.17857 (-20)		0.020	1.7	Sanpa-arsa2016
J1544+4937	2.15928	0.2933	1.09482(-19)		0.025		Tang2014
J1544-2555	0.002390	1.1645	4.81009 (-19)				Li2014
J1555-2908	0.00178	4.4550	1.37617 (-18)		0.060		Lorimer2019
J1627+3219	**0.00218**	**5.478**	**2.06375 (-19)**		**0.022**	**2.7**	
J1641+8049	0.002117	0.0895	3.12612 (-19)		0.047		Lynch2018
J1731-1847	0.002344	2.5401	1.0294 (-19)		0.039		Ng2014
J1745+1017	0.002652	0.2729	1.2508 (-19)		0.016		Barr 2013
J1805+0615	0.002128	2.2758	8.37275 (-19)		0.027		Cromartie2016
J1928+1245	0.00321	1.680	8.77253 (-19)		0.010		Parent2019
J1959+2048	0.001607	1.68515	4.68183 (-19)		0.036		Huang2012,Wu2012
J2017-1614	0.002314	0.245	9.79855 (-20)		0.030	2.4	Sampa-arsa2016
J2051-0827	0.00451	1.2733	9.9211 (-19)		0.039		Shaifullah2016
J2052+1218	0.001995	0.67037	2.2996 ((-19)		0.042		Draghis2019
J2055+3829	0.002089	0.09996	3.60914 (-20)		0.027		Giillemot2019
J2115+5448	0.002603	7.49	3.36909 (-18)		0.025		Sampa-arsa2016

J2214+3000	0.003119	1.47285	7.93876 (-19)		0.015		Schroeder2014
J2234+0944	0.003627	2.00998	1.25984(-18)		0.018		Arzoumanian2018
J2241-5236	0.00218	0.68965	2.60612(-19)		0.016		Keith2011, An2018
J2256-1024	0.002294	1.13535	4.50150(-19)		0.032		Hessels2011,
RedBack (RB) pulsars							
J0212+5321	0,00211	10-22-10-17			0.34		Perez2023
J0838-2827	0.00362	10-21-10-18			0.17	1.4- 2.1	Thongmeearkom2024
J0955-3947	0.00202	10-20		1.71	0.254	1.964	Thongmeearkom2024
J1023+0038	0.00169	1.2 x10-20	8.9125 (-19)	1.71	0.13	1.91	Archibold 2009
J1036-4353	0.002489	3.53x10-21			0.23	~1.4	Burgay2015
J1048+2339	0.00467	3 x 10-21			0.30	2.1	Deneva 2021
J1227-4853	0.00169	1.11x10-20	8.23379 (-19)		0.21		Roy1015
J1242-4712	0.00531	2.14x10-20		~1.4	0.09	>1.5	Ghosh2024
J1302-3258	0.00377	6.55x10-21			0.15	1.4-2.0	Bangale2024
J1306-4035	0.00276	10-20-10-19		1.77-2.3	0.44		Kaeme2018
J1417-4402	0.00266	10-20-10-19		1.7-2.13	0.33	2.1-2.5	Camilo2016
J1431-4715	0.00201	1.40910-20	1.24824(-18)	1.4-2.0	0.20	1.6-2.2	Bates2015
J1622-0315	0.00385	0.78x10-15			0.15	2.3	Smith2023
J1628-3205	0.00321	1.48x10-20			0.16	1.5-1.7	Smith2023
J1723-2837	0.00186	7.5x10-21		1.4	0.24	1.8-2.7	Crawford2013
J1803-6707	0.00213	10-20		1.4	0.16		Burgay2024
J1816+4510	0.00319	4.31x10-20	6.0548(-18)		0.28		Fiore2023
J1859+0313	0.03376	8.6x10-20			0.29		Wang2025
J1908+1105	0.00256	10-20-10-17			0.05		Daneva2021
J1910-5320	0.00233	3.7x 10-20		1.4	0.28	1.8-2.1	Dodge2024
J1931+1428	0.00261				0.25		Wang2025
J1932+2121	0.01424	3.53x10-19			0.11	1.58	Wang2025
J1947-1120	0.00224	10-22-10-17			0.20		Strader2015
J2039-5617	0.00265	>10-20			0.17	1.6	Clerk2021
J2055+1545	0.00216	9.77x10-21		1.7	0.24	2.0	Lewis2023
J2129-0429	0.00761	-----	-------	1.74	0.37	2.18	Bangale2024
J2215+5135	0.00761	3.34x10-20	3.83353(-18)	----	0.21	2.18	Bangale2024
J2333-5526	0.00210	--------	------	------	0.29	2.06	Thongmeearkom2024
J2339-0533	0.00288	1.41x10-20	1.7893(-18)	1.7	0.26	1.96	Pletsch2015

2.2.3 Ellipticity, Atomic clock Stability, and Extreme Precision

According to the present knowledge of deformation magnitude the theoretical models indicate that high speed neutron stars with a spin frequency (υ) ~ 700 – 1000 Hz can develop; equatorial deformation in the range of $\epsilon \sim 10^{-5}$ to 10^{-6} for fluid-like stars or smaller in solid crust models. Therefore, ellipticity ~ 10^{-20} to 10^{-21} means it is not a standard value for equatorial oblateness $\epsilon = (I_{xx} - I_{yy})/ I_{zz}$, rather this value may refer to a "*highly specific minimal residual deviation*", potentially applicable for a "*slower rotator*" or a "*different parameter in a specific model*".

For example,

1) In the case of the high spin in combination with a potentially high mass and a solid crust it means the deformation, under such situation, can be smaller than that of a pure fluid, but still large enough to cause detectable gravitational wave emissions or affect pulse timing, though 10^{-20} is exceptionally low for typical structural oblateness.
2) In the case of black widow (BW) pulsar, in particular PSR B1957+20, which is an eclipsing binary millisecond pulsar, having spin period 1.67074 millisecond or approximately 622Hz. Such high spin black widow pulsars are thought to have extremely low ellipticity $\epsilon \sim 10^{-9}$ to 10^{-10} in order to maintain their stability although this theoretical limits sometimes suggest- even smaller values are suitable for specific emission mechanisms i.e. the value 10^{-20} likely refers to a different such as an upper limit on the intrinsic spin down rate (i.e. period derivative $\dot{P}$ measured in s s^{-1}) due to low energy loss or a very small derivative of the spin frequency (i.e. $\upsilon \sim 10^{-20}$ Hz s^{-2}). Not only that, this extremely low spin down rates ($\dot{P} \sim 10^{-20}$- 10^{-21}) are very crucial for confirming their nature as "recycled" or "reborn" neutron star that have gained angular momentum from a companion.
3) In the case of double neutron star systems, particularly in globular clusters, they are very crucial for testing stellar evolution theories because their formation follows a separate- pathways which are different from those in the galactic disk.

Regarding the atomic clock stability, ellipticity in the probe field of an atomic clock means the laser or microwave radiation used to interrogate the atoms is not perfectly linearly or circularly polarized and as a result it can cause systematic frequency shifts that degrade the ultraprecise atomic clocks.

Recent studies (Yudin etal. 2023) on the "probe-field-ellipticity-induced shift" shows a significant concern for the next generation of atomic-lattice and ion-trap clocks because of the limiting in their performance due to unwanted ac Stark shifts (i.e. the standard ac Stark shift at the level of 4 x 10^{-18},. As an impacts of ellipticity on the atomic clocks it is suggested:

a) Systematic shift- due to uncontrolled ellipticity that causes a shift in the atomic transition frequency leading to inaccuracies, i.e. if uncontrolled, then these shifts can reach fractional levels on the order of 10^{-18} to 10^{-19}.

b) Hyper-Ramsey shift- The use of advanced interrogation techniques such as hyper-Ramsey spectroscopy, can suppress the total ac Stark shift (including ellipticity-induced shifts) to a level lower than 10^{-19}.

c) Observation of a probe stark shift of $(-3.2 \pm 1.7) \times 10^{-20}$.

d) First order Zeeman shift of $(-1.6 \pm 2.0) \times 10^{-19}$.

During the study of the clock stability, after careful control of systematic effects Nicholson et al (2015) did not found any residual drifts that affect the clock stability at 2×10^{-18} after thousands of seconds of averaging time. This means a long- term drift under control of the low 10^{-18} (Nicholson et al 2015) is capable to offer a complete characterization of the clock stability with the short- term stability measurements i.e. the atomic clock achieves fractional stability of 2.2×10^{-18} at 1s. In other words, the total uncertainty of the atomic clock has reduced to 2.1×10^{-18} in fractional frequency units.

3. Conclusion

Exhibition of Extreme Precision by MSPs (i.e. Massive Neutron Stars)

At present the most massive neutron star that has been discovered is PSR J1748 – 2021B which as a mass of $2.5^{+0.047}_{-0.078}$ M⊙ (Freire et al 2021). In this study we consider MSPs i.e. m_{system} with mass > 2.5 as massive neutron star. In Table 1 21 double neutron star (DNS) systems, 3 DNSs located in globular clusters, 27 black widow (BW) pulsars, and 29 red back (RBs) and the ellipticities in the shape of their equatorial deformation due to high- speed rotation. For example, the masses of double neutron stars J1913 + 16, J1913+1102, and J0509+3801 are 2.8284 M⊙, 2.889 M⊙, and 2.805 M⊙, respectively with their corresponding ellipticities are 8.80379×10^{-19}, 7.5484×10^{-21}, and 1.04836×10^{-18}, respectively. While for double neutron star systems in globular cluster systems only PSR B2127-11C possesses heaviest mass m_{system} = 2.71279 M⊙ but ellipticity is not yet confirmed.

For BWs, the highest mass m_{system} observed in PSR J0312-0921, J1627+3219 with mass 2.6 M⊙, ellipticity $\epsilon = 1.262577 \times 10^{-18}$, and mass 2.7 M⊙, ellipticity $\epsilon = 2.06375 \times 10^{-19}$, respectively. For all red back (RBs) pulsars uncertainties both in ellipticities and final masses (i.e. m_{system}) constraint to obtain confirmed values. Since the DNSs almost all of them have masses > 2.5 M⊙ and ellipticities ranging 10^{-18} to 10^{-21}, i.e. (10^{-18} to 10^{-19}) covering for precisely comparable to Atomic Clock stability and (10^{-20} to 10^{-21}) for favouring the further extreme precision overwhelming the that values above the Atomic Clock.

Since the equatorial deformation in the case of Atomic Clock i.e. the value of $\epsilon \sim 10^{-18}$ - 10^{-19} are considered as stable, then one can say that double neutron stars (DNS) and black widows (BW) millisecond pulsars are more stable and are capable to exhibit extreme precision. Not only that it can be said that the detected double neutron star systems (DNS) almost all of them exhibit extreme precision through equatorial deformation due to their high speed rotation.

The author, therefore, encourages the scientists/ astronomers associated in the Pulsar Communities to observe the extreme precisions during their observations.

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