# An Inventory Model for Deteriorating Item Withfull Advance Payment and Price Discount Policy

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**Abstract:** ***This paper develops an inventory model for deteriorating items under a full advance payment policy with vendor-provided price discounts. The retailer covers the advance payment through a loan from a financial institution and repays the loan using sales revenue and earned interest. Two cases are considered depending on whether the accumulated revenue and earned interest is sufficient to meet the loan repayment obligation. The model incorporates price-dependent demand, constant deterioration, variable holding costs, discount benefits, loan interest, earned interest, and penalty charges. The objective is to determine the optimal selling price and replenishment cycle length that maximize the retailer's profit. Numerical examples and sensitivity analysis demonstrate the applicability of the model. The results indicate that demand parameters, purchasing cost, and financing-related factors significantly affect profitability, providing valuable managerial insights for inventory and financial decision-making.***

**Keywords:** Deteriorating items; Advance payment; Price discount; Price-dependent demand; Profit maximization.

## 1. Introduction and Literature Review

Inventory management plays a vital role in supply chain and operations management by ensuring the availability of products while minimizing ordering, holding, deterioration, and financing costs. In today's competitive business environment, firms must effectively coordinate both inventory and financial decisions to maximize profitability and maintain market competitiveness. Consequently, inventory models integrating operational and financial aspects have attracted significant attention from researchers and practitioners.

Many real-life products, including food items, pharmaceuticals, chemicals, electronic components, and fashionable goods, are subject to deterioration over time. Deterioration results in a reduction in quantity, quality, or economic value and therefore has a significant impact on inventory decisions. The foundation of inventory theory was established by Harris (1913) through the Economic Order Quantity (EOQ) model. Later, Ghare and Schrader (1963) introduced deterioration into inventory systems, while Covert and Philip (1973) extended the concept by incorporating Weibull deterioration. These pioneering studies laid the groundwork for modern deteriorating inventory models.

Demand behavior is another critical factor affecting inventory policies. Classical inventory models generally assume constant demand; however, in practice, demand is often influenced by selling price. Whitin (1955) was among the first researchers to establish a relationship between pricing and inventory control. Subsequently, Abad (1996, 2001) developed inventory models with price-dependent demand and demonstrated that joint optimization of pricing and replenishment decisions significantly improves profitability. Since then, price-dependent demand has become a widely accepted assumption in inventory modelling.

Financial considerations have also become increasingly important in inventory management. Goyal (1985) pioneered the study of trade credit by developing an EOQ model under permissible delay in payments. Later, Teng (2002), Chang et al. (2003), and Jaggi et al. (2008) extended trade-credit inventory models by incorporating deterioration, supplier credit, and demand-related factors. Their studies established that financing arrangements significantly affect inventory decisions and retailer profitability.

Significant contributions to deteriorating inventory systems have been made by Singh and his collaborators. An inventory model incorporating trade credit, linear demand, and variable deterioration was developed in Singh and Singh (2009). Subsequently, Singh and Malik (2011) proposed a two-storage inventory model for non-instantaneous deteriorating items under stock-dependent demand. The work of Singh and Vishnoi (2013) further extended the literature by integrating price-dependent demand, amelioration, deterioration, and multiple storage facilities within a supply chain framework. Later, Singh and Singh (2017) developed inventory and EPQ models that incorporated stock-level-dependent demand, selling-price-dependent demand, deterioration, partial backlogging, and permissible delay in payments. These studies significantly enhanced the understanding of deteriorating inventory systems by incorporating realistic demand patterns and financing policies.

With the growing importance of supply chain finance, considerable attention has been devoted to advance payment contracts, under which buyers are required to make partial or full payment before receiving goods. Although such arrangements improve suppliers' cash flow and reduce financial risk, they may impose financial burdens on retailers. In this direction, Singh and Kumar (2022) developed an EOQ model for non-instantaneous deteriorating items with imperfect quality under an advance-cash-credit payment policy and demonstrated the significant impact of financing arrangements on optimal inventory decisions. More recently, Singh and Singh (2023) investigated a sustainable deteriorating inventory system under an advance payment discount policy and showed that the integration of financial incentives and environmental considerations can improve overall system performance.

Recent studies have focused on developing more realistic inventory models by incorporating financing policies, sustainability issues, and advanced demand structures. Khan et al. (2023) investigated pricing and inventory decisions under installment advance payment contracts and discount policies. Das (2024) analyzed a production-inventory model with price-sensitive and environmentally conscious demand under a two-level trade-credit policy. Mondal et al. (2024) proposed a two-warehouse inventory model incorporating deterioration, trade credit, and partial advance payment. Mahato et al. (2024) examined sustainable inventory systems with controllable deterioration under carbon-emission regulations and trade-credit financing.

Despite substantial progress in deteriorating inventory systems, trade-credit financing, and advance payment policies, limited attention has been given to inventory models that simultaneously consider full advance payment, supplier-provided discounts, external financing through financial institutions, earned interest on sales revenue, and penalty interest on overdue balances. In practice, retailers often borrow funds to make advance payments, earn interest on accumulated sales revenue, and may incur penalty charges if loan repayment obligations are not fully satisfied. These financial interactions significantly affect pricing and replenishment decisions and therefore require comprehensive investigation.

Motivated by this research gap, the present study develops an inventory model for deteriorating items under a full advance payment policy with vendor-provided price discounts. The retailer finances the advance payment through a loan obtained from a financial institution and repays the loan using accumulated sales revenue together with earned interest. The proposed model incorporates price-dependent demand, constant deterioration, time-dependent holding costs, purchasing discounts, loan interest charges, earned interest on sales revenue, and penalty interest on overdue balances. Two financial scenarios are examined depending on whether the accumulated sales revenue and earned interest are sufficient to satisfy the loan repayment obligation. The objective is to determine the optimal selling price and replenishment cycle length that maximize the retailer's profit. Numerical examples and sensitivity analyses are provided to illustrate the applicability of the proposed model and to derive meaningful managerial insights.

## 2. Assumptions & Notations

### 2.1 Assumptions

For the development and analysis of the proposed inventory system, the following assumptions are adopted:

1) The inventory system considers a single deteriorating item.
2) The replenishment process is instantaneous, and the entire order quantity is received in a single lot at the beginning of each replenishment cycle. The lead time for replenishment is assumed to be a constant L.
3) The item deteriorates continuously over time at a constant deterioration rate $\theta$, where $0<\theta<1$.
4) The market demand for the item is assumed to depend on the selling price and is given by $D(p)= a- bp$, where $a>0$ is the base demand parameter and $b>0$ is the price sensitivity coefficient.
5) The inventory holding cost per unit per unit time is taken to be proportional to the storage time and is expressed as $h(t)= h_0 + h_1t$, where $h_0 >0$ denotes the initial holding cost per unit per unit time and $h_1 >0$ represents the rate at which the holding cost increases with storage duration.
6) The vendor offers a discount rate of $\delta$ ($0<\delta<1$) to the retailer when the total purchasing cost is prepaid in a single installment at the time of placing the order.
7) The retailer obtains a loan from a third-party financial institution to cover the advance payment of the entire purchasing cost to the vendor at the time of placing the order. When the entire purchasing cost is prepaid through a single advance payment, the retailer borrows the entire purchasing cost at the beginning of the advance payment period. Consequently, the retailer incurs interest charges on the borrowed amount at a constant interest rate $I_c$ from the time the loan is obtained until the predetermined repayment deadline M.
8) The retailer repays the loan principal together withthe accrued interest at the predetermined repayment deadline M using sales revenue generated from customers and the interest earned on the deposited sales revenue from the time it is received until the repayment deadline M. If the accumulated sales revenue together with the interest earned thereon is insufficient to settle the total loan repayment amount by the repayment deadline, the financial institution charges a penalty interest at a higher rate $I_p$ ($I_c< I_p$) on the unpaid balance for the overdue period.
9) Shortages are not permitted during the planning horizon.

### 2.2 Mathematical Notation

The following table outlines the notations adopted for the mathematical formulation of the problem.

| Notations | Description |
|---|---|
| a, b | The demand parameters. |
| $\theta$ | The deterioration parameter. |
| L | The fixed lead time between the order placement date and the delivery of the ordered items by the vendor. |
| Q | The order quantity per replenishment cycle. |
| A | The order cost per replenishment cycle. |
| $C_p$ | The purchasing cost of the item per unit. |
| P | The selling price of the item per unit. |
| $C_d$ | The deterioration cost per unit per unit time. |
| $h_0$, $h_1$ | The inventory holding cost parameters. |
| M | The predetermined repayment deadline with $M \leq T$ |
| $I_C$ | The interest rate charged on the borrowed amount. |
| $I_P$ | The penalty interest rate charged on the unpaid balance ($I_c < I_p$) |
| $I_e$ | The interest rate earned on the sale revenue ($I_e \leq I_c$) |
| I (t) | The inventory level during the planning horizon [0, T] |
| T | The length of the inventory replenishment cycle time. |
| Z (p, T) | The retailer's total profitper replenishment cycle per unit time |

## 3. Mathematical Formulation of the Inventory Model

Based on the stated assumptions, an advance-payment scenario is considered in which the retailer covers the entire purchasing cost through a loan obtained from a third-party financial institution. The retailer prepays the entire purchasing cost in a single installment at the time of placing the order and, consequently, receives a discount at the rate $\delta$ $(0<\delta<1)$ offered by the vendor.

The retailer places an order for Q units of the item and prepays the entire purchasing cost in a single installment at the time of placing the order. As a result, the retailer receives a discount at the rate $\delta$ $(0 < \delta < 1)$ offered by the vendor. The vendor delivers the ordered quantity after a constant lead time $L$. Upon receipt of the order, the retailer begins the replenishment cycle with an initial inventory level of Q units. As customer demand and item deterioration occur continuouslyover time, the inventory level gradually decreases and eventually reaches zero at time t= T, at which point the replenishment cycle ends. At this instant, a new replenishment cycle begins with the arrival of the next order, ensuring the continuous operation of the inventory system without shortages. The inventory system under this scenario is illustrated in Figure 1.

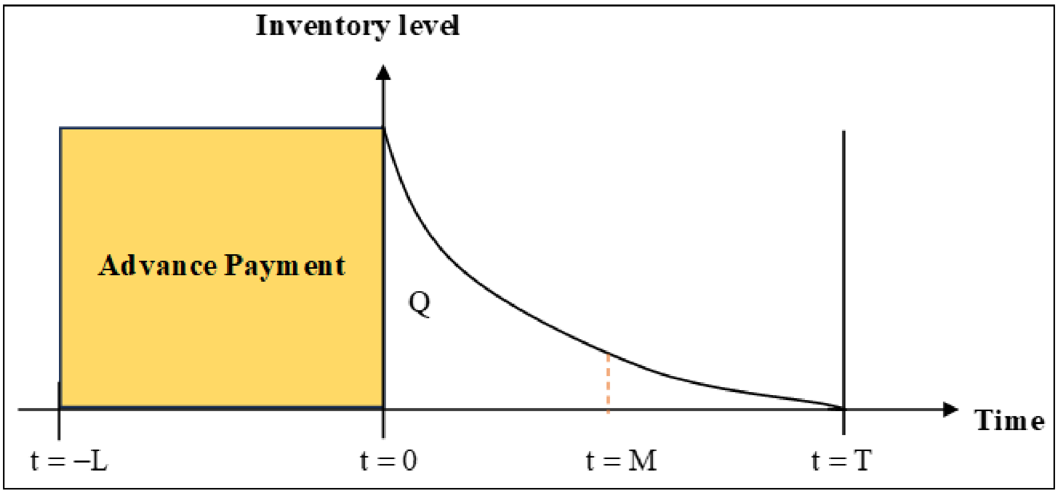



**Figure 1:** Graphical presentation of the inventory system with Advance Payment

The inventory system is described by the following differential equation:

$$\frac{dI(t)}{dt}+\theta\cdot I(t)=-D(p), \qquad 0\leq t\leq T \qquad (1)$$

with the boundary conditions $I(0)=Q$ and $I(T)=0$.

Solving the differential equation with the help of boundary condition $I(T)=0$, we get

$$I(t)=\frac{(a-bp)}{\theta}\left\{e^{\theta(T-t)}-1\right\}, \qquad 0\leq t\leq T \qquad (2)$$

Now, with the help of the boundary condition $I(0)=Q$, the order quantity per replenishment cycle is given by

$$Q=\frac{(a-bp)}{\theta}\left\{e^{\theta T}-1\right\} \qquad (3)$$

The retailer's total profit per replenishment cycle consists of the following components, which are calculated as follows:

**Ordering Cost:** The total ordering cost per replenishmentcycle is given by

$$OC=A \qquad (4)$$

**Holding Cost:** The holding cost incurred during a replenishment cycle is given by

$$HC=\int_0^T (h_0+h_1 t) I(t)\,dt=\frac{(a-bp)}{\theta}\left[h_0\left\{\frac{(e^{\theta T}-1)}{\theta}-T\right\}+h_1\left\{\frac{(e^{\theta T}-1)}{\theta^2}-\frac{T}{\theta}-\frac{T^2}{2}\right\}\right] \quad (5)$$

**Deterioration Cost:** The deterioration cost per replenishment cycle is given by

$$DC=C_d\left(Q-\int_0^T D(p)\,dt\right)=C_d\,(a-bp)\left\{\frac{(e^{\theta T}-1)}{\theta}-T\right\} \quad (6)$$

**Purchasing Cost:** The total purchasing cost for the retailer is calculated as

$$PC=C_p\cdot Q=C_p(a-bp)\left\{\frac{e^{\theta T}-1}{\theta}\right\} \quad (7)$$

**Discounted Purchasing Cost:** Since the retailer prepays the entire purchasing cost in a single installment at the time of placing the order, the retailer receives the maximum discount $\delta\,\%$ offered by the vendor. Consequently, the effective purchasing cost is reduced, and the discounted purchasing cost is given by

$$DPC=(1-\delta)C_p\cdot Q=(1-\delta)C_P(a-bp)\left\{\frac{e^{\theta T}-1}{\theta}\right\} \quad (8)$$

**Interest Charged on Borrowed Amount:** Since the retailer obtains a loan from a third-party financial institution to cover the advance payment of the entire purchasing cost at the time of placing the order, the retailer incurs interest charges on the borrowed amount $\left[(1-\delta)C_P(a-bp)\left\{\frac{e^{\theta T}-1}{\theta}\right\}\right]$. The retailer repays the loan principal together with the accrued interest at the predetermined repayment deadline $t=M$, then the interest charged to the retailer during the time interval $[-L,M]$ is calculated as

$$ICB=ICB[-L,M]=I_c(L+M)\cdot(DPC)=I_c(L+M)\cdot(1-\delta)C_P(a-bp)\left\{\frac{e^{\theta T}-1}{\theta}\right\} \quad (9)$$

**Loan Repayment Amount:** The total loan repayment amount at time $t=M$ is given by

$$LRA[-L,M]=DPC+ICB=\{1+I_c(L+M)\}\cdot(1-\delta)C_P(a-bp)\left\{\frac{e^{\theta T}-1}{\theta}\right\} \quad (10)$$

**Sale Revenue:** The retailer begins selling the products after receiving them from the vendor, and deposits the resulting sales revenue into an interest-bearing account. Over the period $[0,M]$, the retailer accumulates a total sales revenue given by

$$SR[0,M]=p\int_0^M D(p)\,dt=p\,(a-bp)M \quad (11)$$

**Interest Earned:** Over the period $[0,M]$, the interest earned on the accumulated sales revenue is calculated as follows:

$$IE[0,M]=pI_e\int_0^M D(p)\,t\,dt=\frac{1}{2}p\,I_e\,(a-bp)M^2 \quad (12)$$

Comparing the accumulated sales revenue together with the interest earned thereon $(SR[0,M]+IE[0,M])$ with the total loan repayment amount $LRA[-L,M]$ at the repayment deadline $t=M$, two cases may arise:

**Case 1: When the accumulated sales revenue together with the interest earned thereon is greater than or equal to the total loan repayment amount at the repayment deadline** $t=M$, i.e.,

$$(SR[0,M]+IE[0,M])\geq LRA[-L,M],$$

then the retailer is able to repay the entire loan repayment amount at the repayment deadline $t=M$. After repaying the loan principal together with the accrued interest, no penalty interest is charged by the financial institution. Consequently, a surplus amount remains in the interest-bearing account.

The surplus amount remaining in theinterest-bearing account at the repayment deadline $t=M$ is given by

$$SA=SR[0,M]+IE[0,M]-LRA[-L,M]$$

$$SA=(a-b\,p)\left[pM+\frac{1}{2}pI_eM^2-(1-\delta)C_p\,\{1+I_c(L+M)\}\left(\frac{e^{\theta T}-1}{\theta}\right)\right] \quad (13)$$

Since this surplus amount remains deposited in the interest-bearing account during the interval $[M,T]$, it continues to earn interest. Therefore, the interest earned on the surplus amount during the interval $[M,T]$ is given by

$$IES=I_e(T-M)\cdot(SA)=I_e(T-M)(SR[0,M]+IE[0,M]-LRA[-L,M])$$

$$IES = I_e(T-M)(a-b\,p)\left[pM + \frac{1}{2}pI_eM^2 - (1-\delta)C_p\,\{1+I_c(L+M)\}\left(\frac{e^{\theta T}-1}{\theta}\right)\right] \quad (14)$$

Furthermore, during the period $[M,T]$, the retailer continues to sell the items and deposits the resulting sales revenue into the interest-bearing account. The accumulated sales revenue generated during this period $[M,T]$ is given by

$$SR[M,T] = p\int_M^T D(p)\,dt = p\,(a-bp)(T-M) \quad (15)$$

Consequently, the retailer also earns interest on the accumulated sales revenue generated during this period. Therefore, the total interest earned on the accumulated sales revenue over the interval $[M,T]$ is calculated as follows:

$$IE[M,T] = pI_e\int_M^T D(p)\,t\,dt = \frac{1}{2}p\,I_e\,(a-bp)(T^2-M^2) \quad (16)$$

Hence, in this case, the retailer's total profitper replenishment cycle per unit time is given by

$$Z(p,T) = \frac{1}{T}(SA + IES + SR[M,T] + IE[M,T] - OC - HC - DC)$$

$$Z(p,T) = \frac{(a-bp)}{T}\left[\begin{array}{l}\{1+I_e(T-M)\}\left[pM + \frac{1}{2}pI_eM^2 - (1-\delta)C_p\,\{1+I_c(L+M)\}\left(\frac{e^{\theta T}-1}{\theta}\right)\right] \\ +p\left\{(T-M)+\frac{1}{2}I_e(T^2-M^2)\right\} - \frac{A}{(a-bp)} - \frac{h_0}{\theta}\left\{\frac{(e^{\theta T}-1)}{\theta} - T\right\} \\ -\frac{h_1}{\theta}\left\{\frac{(e^{\theta T}-1)}{\theta^2} - \frac{T}{\theta} - \frac{T^2}{2}\right\} - C_d\left\{\frac{(e^{\theta T}-1)}{\theta} - T\right\}\end{array}\right] \quad (17)$$

**Case 2: When the accumulated sales revenue together with the interest earned thereon is less than the total loan repayment amount at the repayment deadline $t = M$,** i.e.,

$$(SR[0,M] + IE[0,M]) < LRA[-L,M],$$

thenthe retailer is unable to repay the entire loan repayment amount at the repayment deadline $t = M$. Consequently, an unpaid balance remains outstanding, and the financial institution charges penalty interest on the unpaid amount at the rate $I_p$ for the overdue period $[M,T]$. The outstanding balance at the repayment deadline $t = M$ is given by

$$UB = LRA[-L,M] - (SR[0,M] + IE[0,M])$$

$$UB = (a-b\,p)\left[(1-\delta)C_p\,\{1+I_c(L+M)\}\left(\frac{e^{\theta T}-1}{\theta}\right) - pM - \frac{1}{2}pI_eM^2\right] \quad (18)$$

Since the unpaid balance remains outstanding during the interval $[M,T]$, the penalty interest charged on the unpaid balance over the period $[M,T]$ is given by

$$PI[M,T] = I_p(T-M)\cdot UB = I_p(T-M)\{LRA[-L,M] - (SR[0,M] + IE[0,M])\}$$

$$PI[M,T] = I_p(T-M)(a-b\,p)\left[(1-\delta)C_p\,\{1+I_c(L+M)\}\left(\frac{e^{\theta T}-1}{\theta}\right) - pM - \frac{1}{2}pI_eM^2\right] \quad (19)$$

Furthermore, during the period $[M,T]$, the retailer continues to sell the items and deposits the resulting sales revenue into the interest-bearing account. The accumulated sales revenue generated during this period $[M,T]$ is given by

$$SR[M,T] = p\int_M^T D(p)\,dt = p\,(a-bp)(T-M) \quad (20)$$

Consequently, the retailer also earns interest on the accumulated sales revenue generated during this period. Therefore, the total interest earned on the accumulated sales revenue over the interval $[M,T]$ is calculated as follows:

$$IE[M,T] = pI_e\int_M^T D(p)\,t\,dt = \frac{1}{2}p\,I_e\,(a-bp)(T^2-M^2) \quad (21)$$

At the end of the replenishment cycle, i.e., at $t = T$, the retailer settles the outstanding balance together with the accrued penalty interest. Hence, in this case, the retailer's total profitper replenishment cycle per unit time is given by

$$Z(p,T) = \frac{1}{T}(SR[M,T] + IE[M,T] - UB - PI[M,T] - OC - HC - DC)$$

$$Z(p,T) = \frac{(a-bp)}{T}\left[\begin{array}{l}p\left\{(T-M)+\frac{1}{2}I_e(T^2-M^2)\right\} - \{1+I_p(T-M)\} \\ \left[(1-\delta)C_p\,\{1+I_c(L+M)\}\left(\frac{e^{\theta T}-1}{\theta}\right) - pM - \frac{1}{2}pI_eM^2\right] \\ -\frac{A}{(a-bp)} - \frac{h_0}{\theta}\left\{\frac{(e^{\theta T}-1)}{\theta} - T\right\} \\ -\frac{h_1}{\theta}\left\{\frac{(e^{\theta T}-1)}{\theta^2} - \frac{T}{\theta} - \frac{T^2}{2}\right\} - C_d\left\{\frac{(e^{\theta T}-1)}{\theta} - T\right\}\end{array}\right] \quad (22)$$

## 4. Numerical Examples and Optimization

The objective is to determine the optimal selling price p* and replenishment cycle length T* that maximize the retailer's profit function Z (p, T). Thus, the optimization problem can be stated as

$$\underset{p,T}{\text{Max}}\left\{ Z(p,T)\right\}$$

The optimal solution is obtained by solving the first-order necessary conditions

$$\frac{\partial Z(p,T)}{\partial p}=0 \quad \text{and} \quad \frac{\partial Z(p,T)}{\partial T}=0.$$

Since the resulting equations are highly nonlinear and analytically intractable, closed-form solutions are difficult to obtain. Therefore, numerical optimization techniques are employed using software such as Mathematica Software 11.2 to determine the optimal solution.

To verify that the obtained solution corresponds to a maximum profit, the Hessian matrix

$$H=\begin{bmatrix} \frac{\partial^2 Z}{\partial p^2} & \frac{\partial^2 Z}{\partial p \partial T} \\ \frac{\partial^2 Z}{\partial T \partial p} & \frac{\partial^2 Z}{\partial T^2} \end{bmatrix}$$

is evaluated at the optimal point (p*, T*). If the Hessian matrix H is negative definite, then the profit function Z((p, T) is concave in the neighborhood of (p*, T*), and the obtained solution represents the global maximum.

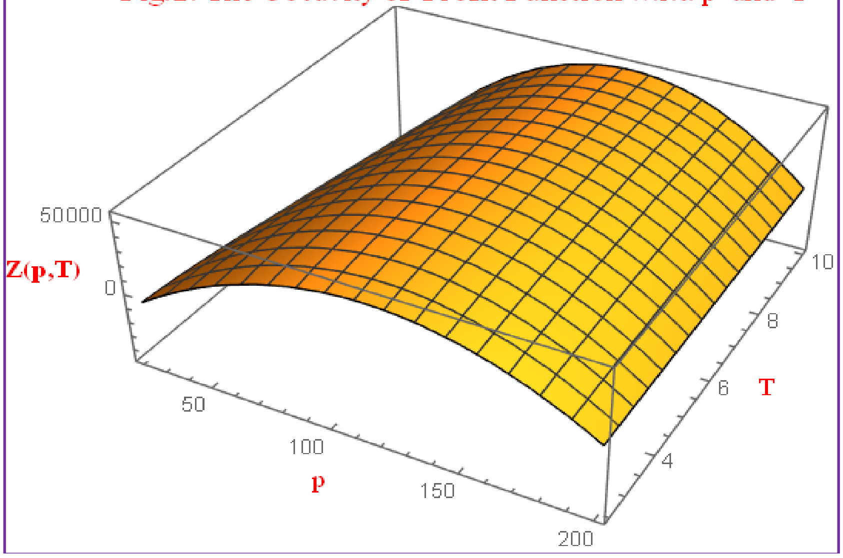
To illustrate the applicability of the proposed model, the following numerical examples are presented.

**Example 1:** The following parameter values are considered for the computational study:
a= 1000 units per month, b= 5 units per ₹ per month, A= ₹200 per order, $C_p$= ₹ 20 per unit, $C_d$= ₹ 2 per unit per month, $\theta$= 0.08 per month, $h_0$= ₹ 1.5 per unit per month, $h_1$= ₹ 0.20 per unit per month, $\delta = 10\%$, $I_e = 0.12$ per month, $I_c = 0.15$ per month, $I_p = 0.18$per month, $L = 0.25$ months, $M = 2.50$ months.

Using the above parameter values, the retailer's profit function $Z(p,T)$ is numerically optimized with respect to the decision variables, namely the selling price $p$ and the replenishment cycle length $T$. The resulting optimal values of the selling price $p^*$, replenishment cycle length $T^*$, order quantity $Q^*$ , and maximum profit $Z^*(p,T)$ are obtained as follows:
$p^*$ = ₹ 120.35 per unit, $T^*$ = 7.9908 months, $Q^*$ = 4455.85 units,

$SA = SR[0,M] + IE[0,M] - LRA[-L,M]$ = ₹ 24507.05, and $Z^*(p,T)$ = ₹ 54417.79.

Fig.2: The Cocavity of Profit Function w.r.t.'p' and 'T'

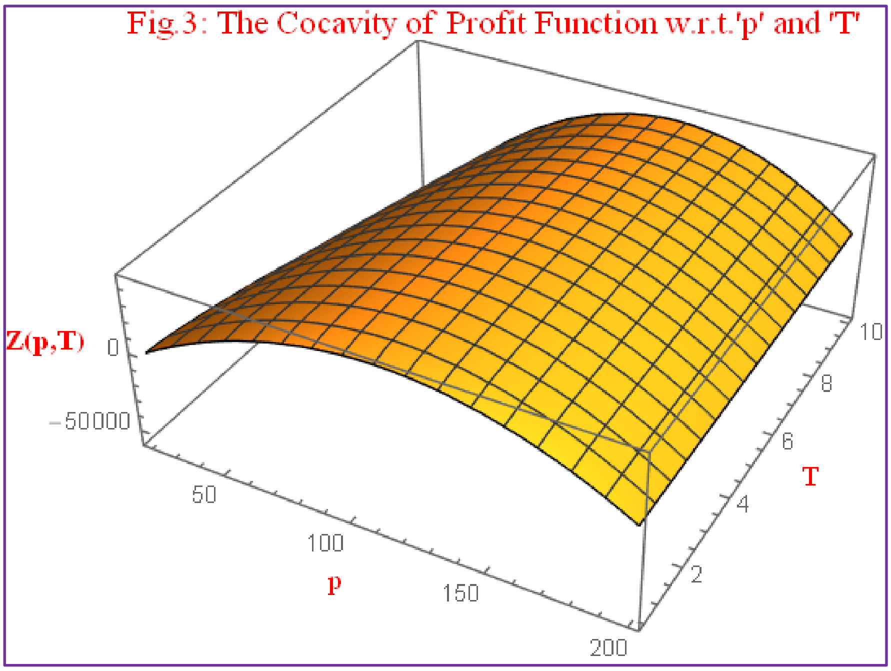



**Example 2:** The following parameter values are considered for the computational study:

a= 1000 units per month, b= 5 units per ₹ per month, A= ₹ 200 per order, $C_p$= ₹ 20 per unit, $C_d$= ₹ 2 per unit per month, $\theta = 0.08$ per month, $h_0$ = ₹ 1.5 per unit per month, $h_1$ = ₹ 0.20 per unit per month, $\delta = 10\%$, $I_e = 0.12$ per month, $I_c = 0.15$ per month, $I_p = 0.18$ per month, $L = 0.25$ months, $M = 0.50$ months.

Using the above parameter values, the retailer's profit function $Z(p,T)$ is numerically optimized with respect to the decision variables, namely the selling price $p$ and the replenishment cycle length $T$. The resulting optimal values of the selling price p*, replenishment cycle length T*, order quantity Q*, and maximum profit $Z^*(p,T)$ are obtained as follows:

P*= ₹ 116.59 per unit, T*= 4.2140 months, Q*= 2089.98 units, $UB = LRA[-L, M] - (SR[0, M] + IE[0, M]) =$ ₹ 16810.54, and $Z^*(p, T) =$ ₹ 46365.41.




## 5. Sensitivity Analysis

To investigate the effects of changes in the model parameters on the optimal selling price (p*) replenishment cycle length (T*), order quantity (Q*), and maximum profit (Z*(p, T)), a sensitivity analysis is performed. Each parameter is varied individually by $-20\%, -10\%, +10\%,$ and $+20\%$ from its base value while keeping all other parameters unchanged.

**Table 1:** Sensitivity Analysis of the Optimal Solution

| Parameter | % Change in parameter | p* | T* | Q* | Z*(p, T) | % change in Z*(p, T) |
|---|---|---|---|---|---|---|
| a | -20 | 97.96 | 6.5266 | 2658.34 | 30843.70 | -43.32% |
| | -10 | 109.12 | 7.2516 | 3483.17 | 41701.20 | -23.37% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 131.65 | 8.7343 | 5583.77 | 69067.80 | +26.92% |
| | +20 | 142.99 | 9.4728 | 6873.09 | 85721.40 | +57.52% |
| b | -20 | 148.68 | 9.8390 | 6064.35 | 75852.20 | +39.39% |
| | -10 | 132.91 | 8.8174 | 5147.56 | 63734.00 | +17.12% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 110.14 | 7.3174 | 3921.31 | 47059.90 | -13.52% |
| | +20 | 101.67 | 6.7640 | 3499.78 | 41119.10 | -24.44% |
| A | -20 | 120.35 | 7.9893 | 4454.60 | 54422.80 | +0.01% |
| | -10 | 120.35 | 7.9901 | 4455.12 | 54420.30 | 0.00% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.36 | 7.9917 | 4456.17 | 54415.30 | 0.00% |
| | +20 | 120.36 | 7.9925 | 4456.70 | 54412.80 | -0.01% |
| $C_p$ | -20 | 118.93 | 9.3278 | 5619.09 | 59527.40 | +9.39% |
| | -10 | 119.62 | 8.6064 | 4976.97 | 56864.40 | +4.50% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 121.12 | 7.4638 | 4026.91 | 52149.90 | -4.17% |
| | +20 | 121.93 | 7.0104 | 3670.00 | 50031.60 | -8.06% |
| $C_d$ | -20 | 120.34 | 8.0125 | 4472.55 | 54481.70 | +0.12% |
| | -10 | 120.35 | 8.0017 | 4464.08 | 54449.70 | +0.06% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.36 | 7.9801 | 4447.24 | 54386.00 | -0.06% |
| | +20 | 120.36 | 7.9693 | 4438.85 | 54354.20 | -0.12% |
| θ | -20 | 121.09 | 9.4060 | 5090.29 | 56563.70 | +3.94% |
| | -10 | 120.67 | 8.6236 | 4740.92 | 55428.80 | +1.86% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.11 | 7.4678 | 4218.19 | 53503.00 | -1.68% |

| | | | | | | |
|---|---|---|---|---|---|---|
| | +20 | 119.93 | 7.0275 | 4017.24 | 52664.90 | -3.22% |
| $h_0$ | -20 | 120.26 | 8.1977 | 4618.34 | 55025.30 | +1.12% |
| | -10 | 120.31 | 8.0932 | 4535.79 | 54719.10 | +0.55% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.40 | 7.8907 | 4377.83 | 54121.40 | -0.54% |
| | +20 | 120.45 | 7.7926 | 4302.27 | 53829.70 | -1.08% |
| $h_1$ | -20 | 120.43 | 8.1221 | 4551.01 | 54621.80 | +0.37% |
| | -10 | 120.39 | 8.0556 | 4502.59 | 54518.90 | +0.19% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.32 | 7.9278 | 4410.11 | 54318.40 | -0.18% |
| | +20 | 120.29 | 7.8663 | 4365.91 | 54220.60 | -0.36% |
| $\delta$ | -20 | 120.52 | 7.8667 | 4353.20 | 53899.50 | -0.95% |
| | -10 | 120.44 | 7.9282 | 4403.87 | 54157.60 | -0.48% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.27 | 8.0546 | 4508.56 | 54680.20 | +0.48% |
| | +20 | 120.19 | 8.1194 | 4562.64 | 54944.90 | +0.97% |
| $I_e$ | -20 | 119.42 | 7.1720 | 3902.78 | 49300.70 | -9.40% |
| | -10 | 119.91 | 7.6050 | 4192.07 | 51812.50 | -4.79% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.75 | 8.3356 | 4695.96 | 57105.90 | +4.94% |
| | +20 | 121.10 | 8.6446 | 4915.42 | 59868.30 | +10.02% |
| $I_c$ | -20 | 119.92 | 8.3386 | 4747.29 | 55822.90 | +2.58% |
| | -10 | 120.14 | 8.1608 | 4597.23 | 55112.30 | +1.28% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.57 | 7.8285 | 4321.91 | 53738.50 | -1.25% |
| | +20 | 120.80 | 7.6733 | 4195.46 | 53073.50 | -2.47% |
| L | -20 | 120.31 | 8.0212 | 4480.79 | 54542.90 | +0.23% |
| | -10 | 120.33 | 8.0060 | 4468.19 | 54480.30 | +0.11% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.37 | 7.9758 | 4443.17 | 54355.40 | -0.11% |
| | +20 | 120.39 | 7.9608 | 4430.77 | 54293.20 | -0.23% |
| M | -20 | 119.78 | 7.5940 | 4190.92 | 53184.00 | -2.27% |
| | -10 | 120.05 | 7.7858 | 4318.51 | 53825.70 | -1.09% |
| | 0 | 120.35 | 7.9908 | 4455.85 | 54417.79 | 0.00% |
| | +10 | 120.68 | 8.2048 | 4599.67 | 54962.50 | +1.00% |
| | +20 | 121.02 | 8.4246 | 4748.72 | 55462.10 | +1.92% |

The sensitivity analysis demonstrates that the market demand parameters $a$ and $b$ exert the most significant influence on the retailer's profitability. Variations in these parameters lead to substantial changes in the optimal selling price (p*) replenishment cycle length (T*), order quantity (Q*), and maximum profit (Z*(p, T)). Among the financial parameters, the interest earned rate $I_e$ and the purchasing cost $C_p$ exhibit considerable effects on profit, while the loan interest rate $I_c$ has a moderate impact. Conversely, the ordering cost $A$ , deterioration cost $C_d$, and lead time L have only marginal effects on the optimal solution. These findings indicate that the retailer's profitability is primarily driven by market demand conditions, financing policies, and procurement costs. Therefore, managerial efforts should focus on demand expansion, effective pricing strategies, favorable financing arrangements, and cost-efficient procurement practices to achieve higher profitability and improved inventory performance.

## 6. Conclusions

The model integrates several realistic features, including price-dependent demand, continuous deterioration, time-dependent holding costs, purchasing discounts, loan interest charges, earned interest on sales revenue, and penalty interest on overdue balances. Analytical expressions for inventory level, order quantity, and profit function are derived, and numerical optimization is employed to determine the optimal selling price and replenishment cycle length.

Numerical results demonstrate the effectiveness of the proposed approach in improving retailer profitability under advance payment arrangements. The sensitivity analysis reveals that the demand parameters and purchasing cost have the greatest impact on the optimal policy and profit. Among the financial parameters, the interest earned rate significantly affects profitability, while the loan interest rate has a moderate influence. In contrast, ordering cost, deterioration cost, deterioration rate, holding cost parameters, discount rate, lead time, and repayment deadline exhibit relatively smaller effects on the optimal solution.

The findings suggest that retailers should focus on demand enhancement, strategic pricing decisions, favourable financing arrangements, and efficient procurement practices to maximize profitability. The proposed model offers a useful decision-making framework for inventory systems involving deteriorating products and advance payment contracts. Future research may extend the model by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incorporating shortages, trade credit, inflation, stock-dependent demand, partial advance payment policies, uncertain deterioration rates, or fuzzy and stochastic environments to better reflect real-world supply chain conditions.

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