Whistler Mode Radiation induced by Kinetic Alfven Waves in burning plasma driven by Nonlinear Force

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Abstract
The emission of electromagnetic radiation in the whistler mode within burning plasma has been studied in the presence of low-frequency kinetic Alfvén wave (KAW) turbulence. This process occurs through wave–particle interactions driven by a nonlinear force. The nonlinear force arises from the resonant interaction between ions and the modulated field, which leads to instabilities. Using a Maxwellian model for the particle distribution function, the fluctuating parts of the ion distribution were obtained. These fluctuations generate a nonlinear force. By applying the ion momentum equation and the ion continuity equation, a nonlinear dispersion relation for the whistler mode was derived that incorporates this force. The growth rate of the whistler mode was then estimated using experimental data from fusion devices. The results suggest that whistler mode instabilities can develop in burning plasma when kinetic Alfvén wave turbulence is present.

Key words: Burning plasma, Kinetic Alfven wave turbulence, ion-acoustic wave, wave-particle interaction, Nonlinear force

1 Introduction

Radiation phenomena associated with plasma inhomogeneity are a common feature in burning plasma. Burning plasma is a self-organised complex system consisting of energetic particles—these include electrically charged fusion products as well as superthermal ions and electrons, generated both by fusion reactions and by external power sources used for heating and current drive [1]. The study of plasma instabilities driven by large-amplitude electromagnetic waves, based on plasma maser theory, has long been an active field of research in burning plasma. The energy density of fast ions and charged fusion products forms a significant fraction of the total plasma energy density.

Whistler modes, meanwhile, are naturally found high above Earth in the ionosphere, a layer of the atmosphere located approximately 50 to 600 miles above the Earth's surface. Whistler waves in the 100–200 MHz range are produced when lightning bolts generate pulses of electromagnetic waves that travel between the Northern and Southern hemispheres. As these waves propagate, their frequency shifts, and when converted into audio signals, they produce a sound similar to whistles. Remarkably, similar whistler waves have now been discovered in the hot plasma inside a tokamak [5]. Whistler modes are stabilised as the magnetic field

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increases, consistent with wave–particle resonance mechanisms.
Kinetic Alfvén wave (KAW) turbulence is supported by plasma inhomogeneity in burning plasma, which is associated with strong confining magnetic fields. Therefore, an inhomogeneous particle distribution model is considered in this investigation. Among the various collective electromagnetic instabilities in burning plasma, low-frequency kinetic Alfvén waves are regarded as the primary drivers of anomalous transport phenomena. In these inhomogeneous regions, KAW instabilities are excited by superthermal energetic particles [13,14,15]. This creates a strong possibility that resonant interactions allow KAWs to exchange energy with such particles.
Resonant interactions between kinetic Alfvén waves and whistler waves occur in burning plasma, as both may be slowed down by Coulomb collisions, which are naturally present in such systems. The important role of KAWs and their interaction with energetic particles in burning plasma was recognized in pioneering works by Belikov et al. [6], Rosenbluth and Rutherford [7], and Mikhailovskii [8].

Thus, in the inhomogeneous regions of burning plasma, kinetic Alfvén waves—excited by superthermal energetic particles [2,3,4]emerge as a low-frequency mode strongly coupled with plasma particles. Among the possible collective instabilities, KAWs are considered the most significant contributors.

In this investigation, we wish to analyse how kinetic Alfven wave turbulent energy along with the nonlinear force $\mathbf{E} \times \mathbf{B}$ which initiates kinetic drift motion may transfer its energy through acceleration of alpha particles to non-resonant whistler wave on the basis of nonlinear wave energy up-conversion process called plasma-maser effect. Plasma-maser effect, which is especially important in strongly magnetised plasma where $\Omega_i > \omega_{pi}$ was proposed by Nambu [18,19], where mode-mode coupling is considered among waves. Plasma-maser effect occurs when resonant as well as non-resonant waves are present in the system. The resonant waves are those for which the Cherenkov resonant condition $\omega - \mathbf{v}.\mathbf{k} = 0$ is satisfied. Whereas the nonresonant waves are those for which both the Cherenkov resonant condition and the nonlinear scattering conditions are not satisfied i.e $\Omega - (\mathbf{K}.\mathbf{v}) \neq 0$ and $\Omega - \omega - (\mathbf{K} - \mathbf{k}).\mathbf{v} \neq 0$. Here, ω and Ω are frequencies of the resonant and nonresonant waves respectively and $\mathbf{k}$ and $\mathbf{K}$ are the corresponding wave numbers.

The plasma maser process is best understood in terms of a high-frequency nonlinear force [18,19]. This force arises from the resonant interaction between energetic ions and modulated fields, which are produced by the coupling of high-frequency whistler waves with low-frequency kinetic Alfvén waves in burning plasma. The nonlinear force can either accelerate or decelerate energetic ions, and the accelerated ions may then emit electrostatic or electromagnetic radiation.
In this work, using a local approximation, we apply a modified Maxwell distribution function

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for inhomogeneous plasma [20] and derive the nonlinear force responsible for the emission of electromagnetic radiation in the whistler mode in the presence of kinetic Alfvén waves in burning plasma. Furthermore, we estimate the growth rate of the whistler wave by employing the ion momentum equation and the ion continuity equation in a tokamak environment.

2 Formulation of the Problem

We assume tokamak plasma in presence of kinetic Alfven wave turbulence with whistler mode as super imposing perturbation field in the system.The confining magnetic field with small gradient is taken along the z-direction $B_0 = B_0(y)$ and the system has spatial gradient in y-direction.

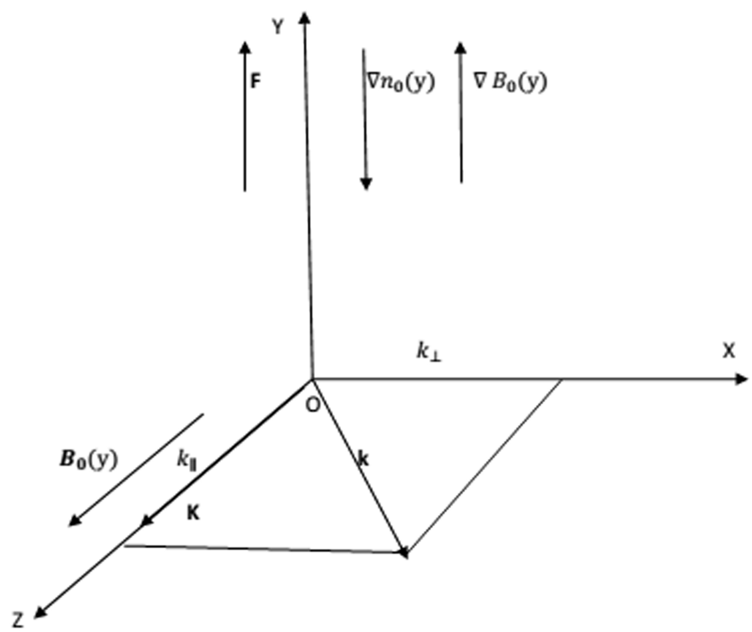

Fig.1:Geometry of the model: $\mathbf{k} = (k_\perp, 0, k_\parallel)$ is the propagating vector of kinetic Alfven wave $\mathbf{K} = (0, 0, K_\parallel)$ is the propagating vector of Whistler wave, $\nabla n_0(y)$ is the density gradient along the negative Y-direction, $\nabla B_0(y)$ is the magnetic gradient along the positive y-direction.

For such an inhomogeneous plasma [24] the particle distribution function [25] is considered as

$$f_{0i}(y,\mathbf{v}) = \left(\frac{m_i}{2T_i\pi}\right)^{\frac{3}{2}} \left[1+\lambda\left(y+\frac{v_x}{\Omega_i}\right)\right] exp\left[-(\frac{m_i\mathbf{v^2}}{2T_i}-\frac{Fy}{T_i})\right] \tag{1}$$

where $f_{0i}(y,\mathbf{v})$ is the distribution function for the guiding center: i refers to the ion (i), $v_x = v_\perp cos\theta, v_\parallel$ and $v_\perp$ are the components of velocity along and perpendicular to the direction of the external magnetic field, θ is the phase angle of the particle in the orbit, $\Omega_i = \frac{eB_0}{mc}$ is the ion cyclotron frequency, $\lambda = \left[-\frac{1}{f_{0i}}\frac{df_{0i}}{dy}\right]_{y=0}$ is the density gradient and $\mathbf{F} = F\hat{y}$ is the external force applied in the y-direction which is responsible for drifting motion of alpha particles in equation of motion as follows

$$\frac{d\mathbf{v}}{dt} = \Omega_i(\mathbf{v}\times\mathbf{\hat{z}}) + \frac{\mathbf{F}}{\mathbf{m}}\mathbf{\hat{y}}$$

Due to this external force applied perpendicular to magnetic field would result drifting velocity

$$V_F = \frac{\mathbf{F}\times\mathbf{B_0}}{eB_0^2}$$

at the zeroth order [22].

In present case we have

$$V_F = \frac{F}{m_i\Omega_i}\hat{x}$$

The interaction of high frequency whistler mode with low frequency kinetic Alfven wave turbulence is governed by the Vlasov-Maxwell equations.

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial\ \mathbf{r}} - \left\{\frac{e}{m_i}\left(\mathbf{E}+\frac{\mathbf{v}\times\ \mathbf{B}_0}{c}\right)+\frac{F}{m_i}\right\}.\frac{\partial}{\partial\mathbf{v}}\right]F_{0i}(\mathbf{r},\mathbf{v},t) = 0 \tag{2}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \tag{3}$$

$$\nabla\times\mathbf{B} = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi}{c}J \tag{4}$$

$$J = -en_i\int\mathbf{v}f_{0i}(\mathbf{r},\mathbf{v},t)d\mathbf{v} \tag{5}$$

$$\nabla.\mathbf{E} = 4\pi en_i\int f_{0i}(\mathbf{r},\mathbf{v},t)d\mathbf{v} \tag{6}$$

On the basis of linear response theory [23] the unperturbed ion distribution of a turbulent plasma can be expressed as

$$F_{0i} = f_{0i} + \epsilon f_{1i} + \epsilon^2 f_{2i} \tag{7}$$

$$\mathbf{E}_{0i} = \epsilon \mathbf{E}_l + \epsilon^2 \mathbf{E}_2 \tag{8}$$

and

$$\mathbf{B}_{0i} = \mathbf{B}_0 + \epsilon \mathbf{B}_l \tag{9}$$

where ϵ is a small parameter associated with the kinetic Alfven wave turbulent field $\mathbf{E}_l=(E_{l\perp}, 0, E_{l\|})$ and $\mathbf{B}_l=(0, B_{ly}, 0)$ with propagating vector $\mathbf{k}=(k_\perp, 0, k_\|)$, f_{0i} is space and time average part, f_{1i}, f_{2i} are fluctuating part of the distribution function. E_2 is the second order electric field.
Putting these in Eq.(2), we have to the order of ϵ

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial\ \mathbf{r}} - \left\{\frac{e}{m_i}\left(\frac{\mathbf{v}\times\ \mathbf{B}_0}{c}\right) + \frac{F}{m_i}\right\}.\frac{\partial}{\partial\ \mathbf{v}}\right] f_{1i} = \frac{e}{m_i}\left(\mathbf{E}_l + \frac{\mathbf{v}\times\mathbf{B}_l}{c}\right).\frac{\partial}{\partial\mathbf{v}} f_{0i} \tag{10}$$

Using Fourier transforms according to

$$A(\mathbf{r}, \mathbf{v}, t) = \sum_{\mathbf{k},\omega} A(\mathbf{k}, \omega) exp[i'(\mathbf{k.r} - \omega t)] \tag{11}$$

and integrating along unperturb orbit, the fluctuating part f_{1i} of the low-frequency turbulent field is obtained from Eq.(6) as

$$\begin{aligned}
f_{1i}(\mathbf{k},\omega) &= \frac{e}{m_i}\int_{-\infty}^{0}\left(\mathbf{E}_l + \frac{\mathbf{v}\times\mathbf{B}_l}{c}\right).\frac{\partial f_{0j}}{\partial v} exp[i'\{\mathbf{k}.(\mathbf{r}' - \mathbf{r}) - \omega\tau\}]\mathbf{d}\tau \\
&= \frac{e}{m_i}\int_{-\infty}^{0}\left(E_{l\perp}\frac{\partial f_{0i}}{\partial v_x} + E_{l\|}\frac{\partial f_{0i}}{\partial v_\|}\right) exp[i'\{\mathbf{k}.(\mathbf{r}' - \mathbf{r}) - \omega\tau\}]\mathbf{d}\tau \\
&= i'\sum\frac{e}{m_i}\left[\frac{m_i}{T_i k_\perp}E_{l\perp}\{1 - \left(k_\| v_\| - \omega - \frac{\lambda T_i k_\perp}{m_i\Omega_i}\right)M_{ab}\}f_{0i} - E_{l\|}\frac{\partial f_{0i}}{\partial v_\|}M_{ab}\right]
\end{aligned} \tag{12}$$

where

$$M_{ab} = \sum_{ab}\frac{J_a(\alpha_i)J_b(\alpha_i)exp[i'(b-a)]}{a\Omega_i + k_\| v_\| + \omega - k_\perp V_F + i'^{0+}}$$

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$$\alpha_i = \frac{k_\perp v_\perp}{\Omega_i}$$

$$V_F = \frac{F}{m_i \Omega_i}$$

which represent the $F \times B$ drift, ω and $\mathbf{k}$ are frequency and wave number of kinetic Alfven wave fields and i'^{0+} represents small imaginary part of ω.

After applying perturbation to the quasisteady state by a high frequency test whistler wave field $\mu\delta\mathbf{E}$ with a propagating vector $\mathbf{K} = (0, 0, K_\parallel)$, electric field $\delta\mathbf{E} = (E_h, 0, 0)$, magnetic field $\delta\mathbf{B} = (0, B_h, 0)$ and a frequency Ω, the total perturb electric field, magnetic field and the particle distribution function becomes

$$\delta\mathbf{E} = \mu\delta\mathbf{E}_h + \mu\epsilon\delta\mathbf{E}_{lh} + \mu\epsilon^2\Delta\mathbf{E} \tag{13}$$

$$\delta\mathbf{B} = \mu\delta\mathbf{B}_h + \mu\epsilon\delta\mathbf{B}_{lh} + \mu\epsilon^2\Delta\mathbf{B} \tag{14}$$

$$\delta f_i = \mu\delta f_{hi} + \mu\epsilon\delta f_{lhi} + \mu\epsilon^2\Delta f_i \tag{15}$$

Where $\delta\mathbf{E}_{lh}, \Delta\mathbf{E}$, $\delta\mathbf{B}_{lh}, \Delta\mathbf{B}$ are the modulation fields, δf_{lhi}, Δf_i are particle distribution function corresponding to modulating fields and δf_{hi} is the fluctuating part of particle distribution function due to a high frequency whistler wave. Linearising Vlasov equation Eq.(2) we obtain

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial \mathbf{r}} - \left\{\frac{e}{m_i}\left(\quad (\mathbf{E}_{0i} + \delta\mathbf{E}_h) + \frac{\mathbf{v} \times (\ \mathbf{B}_0 + \delta\mathbf{B})}{c}\right) + \frac{F}{m_i}\right\}.\frac{\partial}{\partial \mathbf{v}}\right](F_{0i} + \partial f) = 0$$

$$\Rightarrow \left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial \mathbf{r}} + \frac{e}{m_i}\left\{(\epsilon\mathbf{E}_l + \epsilon^2\mathbf{E}_2 + \mu\delta E_h + \mu\delta\mathbf{E}_{lh} + \mu\epsilon^2\Delta\mathbf{E})\right.\right.$$

$$\begin{aligned} + \quad & \left.\left.\frac{\mathbf{v} \times (\mathbf{B}_0 + \mu\delta\mathbf{B}_h + \mu\epsilon\delta\mathbf{B}_{lh} + \mu\epsilon^2\Delta\mathbf{B})}{c} + \frac{F}{m_i}\right\}.\frac{\partial}{\partial \mathbf{v}}\right] \times (f_{0i} + \epsilon f_{1i} + \epsilon^2 f_{2i} \\ + \quad & \mu\delta f_h + \mu\epsilon\partial f_{lh} + \mu\epsilon^2\Delta f) = 0 \end{aligned} \tag{16}$$

To the order of μ, $\mu\epsilon$, $\mu\epsilon^2$, after omitting the second order field quantities we have

$$P\delta f_{hi} = \frac{e}{m_i}\left(\delta\mathbf{E}_h + \frac{\mathbf{v} \times \delta\mathbf{B}_h}{c}\right).\frac{\partial f_{0i}}{\partial \mathbf{v}} \tag{17}$$

$$P\delta f_{lhi}=\frac{e}{m_i}\left(\mathbf{E}_l+\frac{\mathbf{v}\times\mathbf{B}_l}{c}\right).\frac{\partial}{\partial\mathbf{v}}\delta f_{hi}+\frac{e}{m_i}\left(\delta\mathbf{E}_h+\frac{\mathbf{v}\times\delta\mathbf{B}_h}{c}\right).\frac{\partial}{\partial\mathbf{v}}f_{li}+\frac{e}{m_i}\left(\delta\mathbf{E}_{lh}+\frac{\mathbf{v}\times\delta\mathbf{B}_{lh}}{c}\right).\frac{\partial}{\partial\mathbf{v}}f_{0i} \tag{18}$$

$$P\Delta f_i=\frac{e}{m_i}\left(\mathbf{E}_l+\frac{\mathbf{v}\times\mathbf{B}_l}{c}\right).\frac{\partial}{\partial\mathbf{v}}\delta f_{lhi}+\frac{e}{m_i}\left(\delta\mathbf{E}_{lh}+\frac{\mathbf{v}\times\delta\mathbf{B}_{lh}}{c}\right).\frac{\partial}{\partial\mathbf{v}}f_{li} \tag{19}$$

where the operator P is given by

$$P=\frac{\partial}{\partial t}+\mathbf{v}.\frac{\partial}{\partial\mathbf{r}}-\left\{\frac{e}{m_i}\left(\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}_0}{c}\right)+\frac{F}{m_i}\right\}.\frac{\partial}{\partial\mathbf{v}}$$

Using the Fourier transformation and method of characteristics[27] We have evaluated the fluctuating parts δf_{hi}, δf_{lhi}, Δf_i of the particle distribution function over the particle trajectories
We have from Eq.(13)

$$\begin{aligned}
\delta f_{hi}(\mathbf{K},\Omega) &= \frac{e}{m_i}\int_{-\infty}^{0}\left(\delta\mathbf{E}_h+\frac{\mathbf{v}\times\delta\mathbf{B}_h}{c}\right).\frac{\partial f_{0i}}{\partial\mathbf{v}}exp[i'\{\mathbf{K}.(\mathbf{r'}-\mathbf{r})-\boldsymbol{\Omega}\tau\}]\mathbf{d}\tau\\
&= \frac{e}{m_i}\int_{-\infty}^{0}\delta E_h\left[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)\frac{\partial f_{0i}}{\partial v_x}+\frac{K_\parallel v_\perp cos\theta}{\Omega}\frac{\partial f_{0i}}{\partial v_\parallel}\right]exp[i'\{\mathbf{K}.(\mathbf{r'}-\mathbf{r})-\boldsymbol{\Omega}\tau\}]\mathbf{d}\tau\\
&= \frac{i'e}{m_i}\delta E_h\left[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)f_{0i}\left[\frac{m_i v_\perp}{2T_i}\left\{\frac{exp(i'\theta)}{K_\parallel v_\parallel-\Omega-\Omega_i}+\frac{exp(-i'\theta)}{K_\parallel v_\parallel-\Omega+\Omega_i}\right\}\right.\right.\\
&+ \left.\left.\frac{\lambda}{\Omega_i(K_\parallel v_\parallel-\Omega)}\right]-\frac{K_\parallel v_\perp cos\theta}{\Omega}\frac{\partial f_{0i}}{\partial v_\parallel}\frac{1}{K_\parallel v_\parallel-\Omega}\right]\\
&= \frac{i'e}{m_i}\delta E_h\left[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)f_{0i}\left[\frac{m_i}{T_i}\frac{v_\perp cos\theta(K_\parallel v_\parallel-\Omega)}{\{(K_\parallel v_\parallel-\Omega)^2-{\Omega_i}^2\}}+\frac{\lambda}{\Omega_i(K_\parallel v_\parallel-\Omega)}\right]\right.\\
&- \left.\frac{K_\parallel v_\perp cos\theta}{\Omega}\frac{\partial f_{0i}}{\partial v_\parallel}\frac{1}{K_\parallel v_\parallel-\Omega}\right]
\end{aligned} \tag{20}$$

From Eq. (14)

$$\begin{aligned}
\delta f_{lhi}(\mathbf{K}-\mathbf{k}),\Omega-\omega) &= \frac{e}{m_i}\int_{-\infty}^{0}\left[\left(\mathbf{E}_l+\frac{\mathbf{v}\times\mathbf{B}_l}{c}\right).\frac{\partial\delta f_{hi}}{\partial\mathbf{v}}+\left(\delta\mathbf{E}_h+\frac{\mathbf{v}\times\delta\mathbf{B}_h}{c}\right).\frac{\partial f_{li}}{\partial\mathbf{v}}\right.\\
&\left.+\left(\delta\mathbf{E}_{lh}+\frac{\mathbf{v}\times\delta\mathbf{B}_{lh}}{c}\right).\frac{\partial f_{0i}}{\partial\mathbf{v}}\right]\times exp[i'\{(\mathbf{K}-\mathbf{k}).(\mathbf{r'}-\mathbf{r})-(\boldsymbol{\Omega}-\omega)\tau\}]\mathbf{d}\tau\\
&= I_{lh}^1+I_{lh}^2+I_{lh}^3(say)
\end{aligned} \tag{21}$$

where

$$
\begin{aligned}
I^1_{lh} &= \frac{e}{m_i}\int_{-\infty}^{0}\left(\mathbf{E}_l+\frac{\mathbf{v}\times\mathbf{B}_l}{c}\right).\frac{\partial\delta f_{hi}}{\partial\mathbf{v}}exp[i'\{(\mathbf{K}-\mathbf{k}).(\mathbf{r}'-\mathbf{r})-(\boldsymbol{\Omega}-\omega)\tau\}]\mathbf{d}\tau \\
&= \left(\frac{e}{m_i}\right)\left(E_{l\perp}\frac{\partial f_{hj}}{\partial v_x}+E_{l\parallel}\frac{\partial f_{hj}}{\partial v_\parallel}\right)exp[i'\{(\mathbf{K}-\mathbf{k}).(\mathbf{r}'-\mathbf{r})-(\boldsymbol{\Omega}-\omega)\tau\}]\mathbf{d}\tau \\
&= \left(\frac{e}{m_i}\right)^2\delta E_h\Bigg[E_l\Bigg[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)f_{0i}\frac{m_i}{T_i}\frac{(K_\parallel v_\parallel-\Omega)}{\{(K_\parallel v_\parallel-\Omega)^2-{\Omega_i}^2\}}-\frac{K_\parallel}{\Omega}\frac{\partial f_{0i}}{\partial v_\parallel}\frac{1}{K_\parallel v_\parallel-\Omega}\Bigg]N_{st} \\
&\quad +E_{l\perp}\Bigg[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)f_{0i}\left\{\frac{m_i}{T_i}\frac{v_\perp cos\theta(K_\parallel v_\parallel-\Omega)}{\{(K_\parallel v_\parallel-\Omega)^2-{\Omega_i}^2\}}+\frac{\lambda}{\Omega_i(K_\parallel v_\parallel-\Omega)}\right\} \\
&- \frac{K_\parallel v_\perp cos\theta}{\Omega}\frac{\partial f_{0i}}{\partial v_\parallel}\frac{1}{K_\parallel v_\parallel-\Omega}\Bigg]\times\frac{m_i}{T_i k_\perp}\left[1+\left\{(\Omega-\omega)-(K_\parallel-k_\parallel)v_\parallel-\frac{\lambda T_i k_\perp}{m_i\Omega_i}\right\}N_{st}\right] \\
&+ E_{l\parallel}\frac{\partial}{\partial v_\parallel}\Bigg[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)f_{0i}\left\{\frac{m_i}{T_i}\frac{v_\perp cos\theta(K_\parallel v_\parallel-\Omega)}{\{(K_\parallel v_\parallel-\Omega)^2-{\Omega_i}^2\}}+\frac{\lambda}{\Omega_i(K_\parallel v_\parallel-\Omega)}\right\} \\
&- \frac{K_\parallel v_\perp cos\theta}{\Omega}\frac{\partial f_{0i}}{\partial v_\parallel}\frac{1}{K_\parallel v_\parallel-\Omega}\Bigg]N_{st}\Bigg]
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
I^2_{lh} &= \frac{e}{m_i}\int_{-\infty}^{0}\left(\delta\mathbf{E}_h+\frac{\mathbf{v}\times\mathbf{B}_h}{c}\right).\frac{\partial f_{li}}{\partial\mathbf{v}}exp[i'\{(\mathbf{K}-\mathbf{k}).(\mathbf{r}'-\mathbf{r})-(\boldsymbol{\Omega}-\omega)\tau\}]\mathbf{d}\tau \\
&= \left(\frac{e}{m_i}\right)\delta E_h\left[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)\frac{\partial f_{1i}}{\partial v_x}+\frac{K_\parallel v_\perp cos\theta}{\Omega}\frac{\partial f_{1i}}{\partial v_\parallel}\right]exp[i'\{(\mathbf{K}-\mathbf{k}).(\mathbf{r}'-\mathbf{r})-(\boldsymbol{\Omega}-\omega)\tau\}]\mathbf{d}\tau \\
&= \left(\frac{e}{m_i}\right)^2\delta E_h\Bigg[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)\frac{m_i}{T_i k_\perp}\left[1+\left\{(\Omega-\omega)-(K_\parallel-k_\parallel)v_\parallel-\frac{\lambda T_i k_\perp}{m_i\Omega_i}\right\}N_{st}\right] \\
&\quad\times\frac{m_i}{T_i k_\perp}\left[E_{l\perp}\left\{1-(k_\parallel v_\parallel-\omega-\frac{\lambda T_i k_\perp}{m_i\Omega_i})M_{ab}\right\}f_{0i}-E_{l\parallel}\frac{\partial f_{0i}}{\partial v_\parallel}M_{ab}\right] \\
&\quad+\frac{K_\parallel v_\perp cos\theta}{\Omega}\frac{\partial}{\partial v_\parallel}\left[\frac{m_i}{T_i k_\perp}E_{l\perp}\left\{1-(k_\parallel v_\parallel-\omega-\frac{\lambda T_i k_\perp}{m_i\Omega_i})M_{ab}\right\}f_{0i}-E_{l\parallel}\frac{\partial f_{0i}}{\partial v_\parallel}M_{ab}\right]N_{st}\Bigg]
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
I^3_{lh} &= \frac{e}{m_i}\int_{-\infty}^{0}\left(\delta\mathbf{E}_{lh}+\frac{\mathbf{v}\times\mathbf{B}_{lh}}{c}\right).\frac{\partial f_{0i}}{\partial\mathbf{v}}exp[i'\{(\mathbf{K}-\mathbf{k}).(\mathbf{r}'-\mathbf{r})-(\boldsymbol{\Omega}-\omega)\tau\}]\mathbf{d}\tau \\
&= i'\frac{e}{m_i}\delta E_{lh}\int_{-\infty}^{0}\left[\left(\frac{-k_\perp}{|\mathbf{K}\text{-}\mathbf{k}|}-\frac{|K-k|}{\Omega-\omega}v_\parallel\right)\frac{\partial f_{0i}}{\partial v_x}+\left(\frac{K_\parallel-k_\parallel}{|\mathbf{K}\text{-}\mathbf{k}|}-\frac{|\mathbf{K}\text{-}\mathbf{k}|}{\Omega-\omega}v_\perp cos\theta\right)\frac{\partial f_{0i}}{\partial v_\parallel}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \times exp[i'\{(\mathbf{K}-\mathbf{k}).(\mathbf{r}'-\mathbf{r})-(\mathbf{\Omega}-\omega)\tau\}]\mathbf{d}\tau \\
= \;& -i'\frac{e}{m_i}\delta E_{lh}\Bigg[\left(\frac{k_\perp}{|\mathbf{K\text{-}k}|}+\frac{|\mathbf{K\text{-}k}|}{\Omega-\omega}v_\parallel\right)f_{0i}\frac{m_i}{T_i k_\perp}\left[1+\left\{(\Omega-\omega)-(K_\parallel-k_\parallel)v_\parallel-\frac{\lambda T_i k_\perp}{m_i\Omega_i}\right\}N_{st}\right] \\
& +\left(\frac{K_\parallel-k_\parallel}{|\mathbf{K\text{-}k}|}-\frac{|\mathbf{K\text{-}k}|}{\Omega-\omega}v_\perp cos\theta\right)\frac{\partial f_{0i}}{\partial v_\parallel}N_{st}\Bigg]
\end{aligned}
\tag{24}
$$

where

$$N_{st}=\sum_{st}\frac{J_s(\alpha_i)J_t(\alpha_i)exp[i'(s-t)\theta]}{(K_\parallel-k_\parallel)v_\parallel-s\Omega_e+\omega-\Omega-k_\perp V_F}$$

and

$$\alpha_i=\frac{k_\perp v_\perp}{\Omega_i}$$

3 Nonlinear Force

$$
\begin{aligned}
P\Delta f_i(K,\Omega) &= \frac{e}{m_i}\left(\mathbf{E}_l+\frac{\mathbf{v}\times\mathbf{B}_l}{c}\right).\frac{\partial\delta f_{lhi}}{\partial\mathbf{v}}+\frac{e}{m_i}\left(\mathbf{E}_{lh}+\frac{\mathbf{v}\times\mathbf{B}_{lh}}{c}\right).\frac{\partial f_{li}}{\partial\mathbf{v}} \\
&= G
\end{aligned}
\tag{25}
$$

Introducing Fourier transformation in space and time we obtain

$$\Delta f_i(K,\Omega)=\frac{F(K,\Omega)}{i'(K_\parallel v_\parallel-\Omega)} \tag{26}$$

The nonlinear force can be defined as the rate of change of momentum and is given by

$$
\begin{aligned}
G_{Nh} &= \frac{\partial}{\partial t}\int m_i\mathbf{v}\Delta f d\mathbf{v} \\
&= \frac{\partial}{\partial t}\int m_i\Delta f_i(\mathbf{K},\Omega)exp[i'(\mathbf{K}.\mathbf{r}-\Omega t)]\mathbf{v}d\mathbf{v} \\
&= m_i\int\frac{\Omega}{\Omega-K_\parallel v_\parallel}G(K,\Omega)\mathbf{v}d\mathbf{v}
\end{aligned}
\tag{27}
$$

Therefore the Fourier component of the high frequency nonlinear force $G_{Nh}(\mathbf{K},\Omega)$ acting on unit volume of fast ion can be written as

$$
\begin{aligned}
G_{Nh}(\mathbf{K},\Omega) &= m_i n_b \int \frac{\Omega}{\Omega - K_{\|}v_{\|}} F\mathbf{v}d\mathbf{v} \\
&= e n_b \int \frac{\Omega}{\Omega - K_{\|}v_{\|}} \left[\left(\mathbf{E}_l + \frac{\mathbf{v}\times\mathbf{B}_l}{c} \right) . \frac{\partial \delta f_{lhi}}{\partial \mathbf{v}} + \left(\mathbf{E}_{lh} + \frac{\mathbf{v}\times\mathbf{B}_{lh}}{c} \right) . \frac{\partial f_{li}}{\partial \mathbf{v}} \right] \mathbf{v} d\mathbf{v}
\end{aligned} \tag{28}
$$

where n_b being the ion number density. The z-component of nonlinear force $G_{Nh}(\mathbf{K},\Omega)$ can be written as

$$
G_{Nhz}(\mathbf{K},\Omega) = G_{Nhz1}(\mathbf{K},\Omega) + G_{Nhz2}(\mathbf{K},\Omega) \tag{29}
$$

where

$$
G_{Nhz1}(\mathbf{K},\Omega) = e n_b \int \frac{\Omega}{\Omega - K_{\|}v_{\|}} \left(\mathbf{E}_l + \frac{\mathbf{v}\times\mathbf{B}_l}{c} \right) . \frac{\partial \delta f_{lhi}}{\partial \mathbf{v}} d\mathbf{v} \tag{30}
$$

and

$$
G_{Nhz2}(\mathbf{K},\Omega) = e n_b \int \frac{\Omega}{\Omega - K_{\|}v_{\|}} \left(\mathbf{E}_{lh} + \frac{\mathbf{v}\times\mathbf{B}_{lh}}{c} \right) . \frac{\partial f_{li}}{\partial \mathbf{v}} \Bigg) v_{\|} d\mathbf{v} \tag{31}
$$

$$
\begin{aligned}
G_{Nhz2}(\mathbf{K},\Omega) &= -e n_b \int \frac{\Omega}{\Omega - K_{\|}v_{\|}} \left\langle \delta E_{lh} \left(\frac{k_\perp}{|\mathbf{K}\text{-}\mathbf{k}|} + \frac{|\mathbf{K}\text{-}\mathbf{k}|}{\Omega - \omega} v_{\|} \right) \frac{\partial f_{1i}}{\partial v_x} \right\rangle v_{\|} d\mathbf{v} \\
&\quad + e n_b \int \frac{\Omega}{\Omega - K_{\|}v_{\|}} \left\langle \delta E_{lh} \left(\frac{K_{\|} - k_{\|}}{|\mathbf{K}\text{-}\mathbf{k}|} - \frac{|\mathbf{K}\text{-}\mathbf{k}|}{\Omega - \omega} v_\perp cos\theta \right) \frac{\partial f_{li}}{\partial v_{\|}} \right\rangle v_{\|} d\mathbf{v} \\
&= I_1 + I_2
\end{aligned} \tag{32}
$$

where

$$
I_1 = -e n_b \int \frac{\Omega}{\Omega - K_{\|}v_{\|}} \left\langle \delta E_{lh} \left(\frac{k_\perp}{|\mathbf{K}\text{-}\mathbf{k}|} + \frac{|\mathbf{K}\text{-}\mathbf{k}|}{\Omega - \omega} v_{\|} \right) \frac{\partial f_{1i}}{\partial v_x} \right\rangle v_{\|} d\mathbf{v}
$$

$$
I_2 = e n_b \int \frac{\Omega}{\Omega - K_{\|}v_{\|}} \left\langle \delta E_{lh} \left(\frac{K_{\|} - k_{\|}}{|\mathbf{K}\text{-}\mathbf{k}|} - \frac{|\mathbf{K}\text{-}\mathbf{k}|}{\Omega - \omega} v_\perp cos\theta \right) \frac{\partial f_{li}}{\partial v_{\|}} \right\rangle v_{\|} d\mathbf{v}
$$

We have considered the second part of the nonlinear force $G_{Nhz2}(\mathbf{K},\Omega)$ which is comes from the polarization coupling term. In the evaluation process taking only the imaginary part as they only contribute to growth or damping of plasma waves through the plasma maser interaction for which the condition is $\omega = \mathbf{k}.\mathbf{v}$ (other resonant conditions viz, $\Omega = \mathbf{K}.\mathbf{V}$ and $\Omega-\omega = (\mathbf{K\text{-}k}).\mathbf{V}$ are not satisfied)between the whistler wave and kinetic Alfven wave turbulence. Assuming $\Omega < Kv_{\parallel}$ and the phase angle θ to be vary vary small, we evaluate the nonlinear force considering the most dominant contribution to Bessel's term comes from the term a = b = s = t = 0.
Substituting the values of $\delta f_{li}(\mathbf{K\text{-}k},\Omega-\omega)$, and replacing $Im\left(\frac{1}{-\omega+kv+i^{0+}}\right)$ by $-i\pi\delta(\omega-kv)$, we finally obtain the nonlinear force $G_{Nhz2}(\mathbf{K},\Omega)$ after a lengthy calculation retaining the dominant term only from I_1 and I_2 as

$$\begin{aligned} G_{Nhz2} &= -en_b\sqrt{\pi}\delta E_{lhi}\left(\frac{e}{m_i}\right)\frac{\Omega^2(K_{\parallel}-k_{\parallel})k_{\parallel}^2}{K_{\parallel}^2(v_Ak_{\parallel}+k_{\perp}V_F)^2|\mathbf{K\text{-}k}||k_{\parallel}|v_i}\left(E_{l\perp}\frac{\lambda}{\Omega_i}+2E_{l\parallel}\frac{v_Ak_{\parallel}+k_{\perp}V_F}{k_{\parallel}v_i^2}\right) \\ &\quad \times exp\left\{-\left(\frac{v_Ak_{\parallel}+k_{\perp}V_F}{v_ik_{\parallel}}\right)^2\right\} \end{aligned} \tag{33}$$

The expression of modulation electric field $\delta E_{lhi}(\mathbf{K\text{-}k})$ is obtained by using maxwell's equation

$$\nabla\times\mathbf{B}_{lh} = \frac{1}{c}\frac{\partial}{\partial t}\delta\mathbf{E}_{lhi}+\frac{4\pi}{c}J$$
$$J = -en_i\int \mathbf{v}f_{lhi}d\mathbf{v}$$

which is in the form after simlification

$$\delta E_{lh} = \frac{4\pi i'en_i(\Omega-\omega)}{\{(\Omega-\omega)^2-c^2|\mathbf{K\text{-}k}|^2\}}\int v_{\parallel}\delta f_{lhi}d\mathbf{v}$$

Now, we obtain δE_{lh} using the values of I_{lh}^1, I_{lh}^2 and I_{lh}^3 as

$$\begin{aligned} \delta E_{lhi} &= \frac{4\pi i'en_i(\Omega-\omega)}{R\{(\Omega-\omega)^2-c^2|\mathbf{K\text{-}k}|^2\}}\left(\frac{e}{m_i}\right)^2\delta E_h\int v_{\parallel}\left[\left[E_l\left[\left(1-\frac{K_{\parallel}v_{\parallel}}{\Omega}\right)f_{0i}\frac{m_i}{T_i}\frac{(K_{\parallel}v_{\parallel}-\Omega)}{\{(K_{\parallel}v_{\parallel}-\Omega)^2-\Omega_i{}^2\}}\right.\right.\right. \\ &- \left.\frac{K_{\parallel}}{\Omega}\frac{\partial f_{0i}}{\partial v_{\parallel}}\frac{1}{K_{\parallel}v_{\parallel}-\Omega}\right]N_{st}+E_{l\perp}\left[\left(1-\frac{K_{\parallel}v_{\parallel}}{\Omega}\right)f_{0i}\left\{\frac{m_i}{T_i}\frac{v_{\perp}cos\theta(K_{\parallel}v_{\parallel}-\Omega)}{\{(K_{\parallel}v_{\parallel}-\Omega)^2-\Omega_i{}^2\}}+\frac{\lambda}{\Omega_i(K_{\parallel}v_{\parallel}-\Omega)}\right\}\right. \\ &- \left.\left.\frac{K_{\parallel}v_{\perp}cos\theta}{\Omega}\frac{\partial f_{0j}}{\partial v_{\parallel}}\frac{1}{K_{\parallel}v_{\parallel}-\Omega}\right]\times\frac{m_i}{T_ik_{\perp}}\left[1+\left\{(\Omega-\omega)-(K_{\parallel}-k_{\parallel})v_{\parallel}-\frac{\lambda T_ik_{\perp}}{m_i\Omega_i}\right\}N_{st}\right]\right. \end{aligned}$$

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$$
\begin{aligned}
&+ \quad E_{l\parallel}\frac{\partial}{\partial v_\parallel}\left[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)f_{0i}\left\{\frac{m_i}{T_i}\frac{v_\perp cos\theta(K_\parallel v_\parallel-\Omega)}{\{(K_\parallel v_\parallel-\Omega)^2-{\Omega_i}^2\}}+\frac{\lambda}{\Omega_i(K_\parallel v_\parallel-\Omega)}\right\}\right.\\
&- \quad \left.\frac{K_\parallel v_\perp cos\theta}{\Omega}\frac{\partial f_{0i}}{\partial v_\parallel}\frac{1}{K_\parallel v_\parallel-\Omega}\right]N_{st}\Bigg]+\left[\left(1-\frac{K_\parallel v_\parallel}{\Omega}\right)\frac{m_i}{T_i k_\perp}\left[1+\left\{(\Omega-\omega)-(K_\parallel-k_\parallel)v_\parallel\right.\right.\right.\\
&\qquad \left.\left.-\frac{\lambda T_i k_\perp}{m_i\Omega_i}\right\}N_{st}\right]\times\frac{m_i}{T_i k_\perp}\left[E_{l\perp}\left\{1-(k_\parallel v_\parallel-\omega-\frac{\lambda T_i k_\perp}{m_i\Omega_i})M_{ab}\right\}f_{0i}-E_{l\parallel}\frac{\partial f_{0i}}{\partial v_\parallel}M_{ab}\right]\\
&\qquad \left.+\frac{K_\parallel v_\perp cos\theta}{\Omega}\frac{\partial}{\partial v_\parallel}\left[\frac{m_i}{T_i k_\perp}E_{l\perp}\left\{1-(k_\parallel v_\parallel-\omega-\frac{\lambda T_i k_\perp}{m_i\Omega_i})M_{ab}\right\}f_{0i}-E_{l\parallel}\frac{\partial f_{0i}}{\partial v_\parallel}M_{ab}\right]N_{st}\right]\Bigg] \qquad (34)
\end{aligned}
$$

where

$$
\begin{aligned}
R \quad = \quad & 1+\frac{\omega_{pi}^2(\Omega-\omega)}{\{(\Omega-\omega)^2-c^2|\mathbf{K\text{-}k}|^2\}}\int v_\parallel\left[\left(\frac{k_\perp}{|\mathbf{K\text{-}k}|}+\frac{|\mathbf{K\text{-}k}|}{\Omega-\omega}v_\parallel\right)f_{0i}\right.\\
\times \quad & \frac{m_i}{T_i k_\perp}\left[1+\left\{(\Omega-\omega)-(K_\parallel-k_\parallel)v_\parallel-\frac{\lambda T_i k_\perp}{m_i\Omega_i}\right\}N_{st}\right]\\
& \left.+\left(\frac{K_\parallel k_\parallel}{|\mathbf{K\text{-}k}|}+\frac{|\mathbf{K\text{-}k}|}{\Omega-\omega}v_\perp cos\theta\right)\frac{\partial f_{0i}}{\partial v_\parallel}N_{st}\right]
\end{aligned} \qquad (35)
$$

After integrating, retaining the dominant term and using the value [Ref.]

$$
\frac{1}{R\{c^2|\mathbf{K\text{-}k}|^2-(\Omega-\omega)^2\}}\approx\frac{1}{c^2k_\parallel^2}
$$

we get

$$
\delta E_{lhi}=\frac{\sqrt{\pi}i'e}{m_i}\frac{\omega_{pi}^2(\Omega-\omega)\delta E_h E_{l\perp}K_\parallel}{c^2k_\parallel^2\Omega_i^2(\omega-\Omega-k_\perp V_F)} \qquad (36)
$$

Therefore,the nonlinear force G_{Nhz2} can be derived using the Eq.'s(32) and Eq.(35) as

$$
\begin{aligned}
G_{Nhz2} \quad = \quad & -en_b i'\pi\left(\frac{e}{m_i}\right)^2\delta E_h\frac{\omega_{pi}^2\Omega^2(\Omega-\omega)}{\Omega_i^2c^2v_i|k_\parallel|K_\parallel(v_Ak_\parallel+k_\perp V_F)^2(\omega-\Omega-k_\perp V_F)}E_{l\perp}\\
& \times\left(E_{l\perp}\frac{\lambda}{\Omega_i}+2E_{l\parallel}\frac{v_Ak_\parallel+k_\perp V_F}{k_\parallel v_i^2}\right)exp\left\{-\left(\frac{v_Ak_\parallel+k_\perp V_F}{v_ik_\parallel}\right)^2\right\}
\end{aligned} \qquad (37)
$$

Therefore,

$$G_{Nhz2}(K,\Omega) = i' f_{h2} \delta E_h(K,\Omega)$$

where

$$\begin{aligned} f_{h2} &= -en_b\pi\left(\frac{e}{m_i}\right)^2 \frac{\omega_{pi}^2\Omega^2(\Omega-\omega)}{\Omega_i^2 c^2 v_i |k_\parallel| K_\parallel (v_A k_\parallel + k_\perp V_F)^2(\omega - \Omega - k_\perp V_F)} E_{l\perp}\left(E_{l\perp}\frac{\lambda}{\Omega_i} + 2E_{l\parallel}\frac{v_A k_\parallel + k_\perp V_F}{k_\parallel v_i^2}\right) \\ &\quad \times exp\left\{-\left(\frac{v_A k_\parallel + k_\perp V_F}{v_i k_\parallel}\right)^2\right\} \end{aligned} \tag{38}$$

4 Dispersion Relation

Maxwell equation governing the high frequency electromagnetic perturbation can be written as

$$\nabla \times \mathbf{E}_h = \frac{1}{c}\frac{\partial}{\partial t}\delta \mathbf{B}_h$$

$$\nabla \times \mathbf{B}_h = \frac{1}{c}\frac{\partial}{\partial t}\delta \mathbf{E}_h + \frac{4\pi}{c}\mathbf{J}_h$$

From these maxwell equation we then obtain

$$(\Omega^2 - c^2K^2)\mathbf{E}_h(\mathbf{K},\Omega) = -4\pi i \mathbf{J}_h(\mathbf{K},\Omega) \tag{39}$$

Following the method of chen[]we use the momentum equation with the nonlinear term as

$$mn\left(\frac{\partial\ \mathbf{v}}{\partial t} + \mathbf{v}.\nabla\mathbf{v}\right) = -en\delta\mathbf{E}_h(\mathbf{K},\Omega) + \mathbf{G}_{Nh}(\mathbf{K},\Omega) \tag{40}$$

Now,the linearized equation of Eq.(40) with the nonlinear high frequency force term as

$$mn_0\frac{\partial \mathbf{v}_h}{\partial t} = -en_0\delta\mathbf{E}_h(\mathbf{K},\Omega) + \mathbf{G}_{Nh}(\mathbf{K},\Omega) \tag{41}$$

By Fourier analysis we get

$$-imn_0\Omega\mathbf{v}_h = -en_0\delta\mathbf{E}_h(\mathbf{K},\Omega) + if_h\delta\mathbf{E}_h(\mathbf{K},\Omega)$$

$$\Rightarrow \mathbf{v}_h = \frac{i}{mn_0\Omega}\left(if_h - en_0\right)\delta\mathbf{E}_h(\mathbf{K},\Omega) \tag{42}$$

Now substituting the current density equation $\mathbf{J}_h(\mathbf{K},\Omega) = -en_0\mathbf{v}_h$ in Eq.(39) we have

$$\mathbf{v}_h = \frac{\Omega^2 - c^2K^2}{4\pi i e n_0\Omega}\delta\mathbf{E}_h(\mathbf{K},\Omega) \tag{43}$$

Therefore from Eq.(42) and Eq.(43) we get the dispersion relation of electromagnetic whistler mode in burning plasma in presence of kinetic Alfven wave turbulence field as

$$\Omega^2 - c^2K^2 - \omega_{pi}^2 = \frac{-4\pi e i}{m_i}f_h \tag{44}$$

5 Growth Rate

Now to estimate the growth rate electromagnetic instability in presence of kinetic Alfven wave turbulence we put $\Omega = \Omega_r + i\gamma$ in Eq.(44) to Ω_r and γ are the real frequency and the growth rate respectively
gives

$$\Omega_r = \left(\omega_{pi}^2 + c^2K^2\right)^{\frac{1}{2}} \tag{45}$$

$$\gamma = \frac{2\pi e f_h}{m_i\Omega_r} \tag{46}$$

Finally substituting the values of f_h we obtain the growth rate of whistler mode as

$$\begin{aligned}\frac{\gamma}{\Omega} \quad &= \quad \frac{1}{2}\pi\frac{\omega_{pi}^4\Omega(\Omega-\omega)}{\Omega_i^3|k_\parallel|c^2K_\parallel v_i(v_Ak_\parallel + k_\perp V_F)^2}\left(\frac{e}{m_i}\right)^2 E_{l\perp}\left(E_{l\perp}\frac{\lambda}{\Omega_i} + 2E_{l\parallel}\frac{v_Ak_\parallel + k_\perp V_F}{k_\parallel v_i^2}\right) \\ &\quad \times exp\left\{-\left(\frac{v_Ak_\parallel + k_\perp V_F}{v_ik_\parallel}\right)^2\right\}\end{aligned} \tag{47}$$

6 Conclusion

In an earlier investigation amplification of whistler wave in presence of kinetic wave turbulence through nonlinear wave particle interaction has considered in magnetosphere plasma. In the above study it has been predicted that growth rate increases with increase of density gradient parameter. It suggest that spatial gradient of plasma in magnetosphere may contributed to faster growth of whistler wave. In this present paper we are consider interaction of whistler wave with kinetic Alfven mode in burning plasma. Here our study is based on distribution of particles considered inhomogeneity of burning plasma. The particle distribution consider here include spatial gradient along y-axis with confining field.The transverse polarization of kinetic Alfven waves implies that the parallel component of electric fields $E_{l\|}$ are must smaller than the transverse component $E_{l\perp}$. so it is reasonable to take $|E_{l\perp}| \gg |E_{l\|}|$

For weak density gradient, we put $\lambda \sim 0$ in Eq.(47) and using $\frac{E_{l\|}}{E_{l\perp}} = \frac{k_\|}{k_\perp}$ the growth rate for such region is estimated as

$$\begin{aligned} \frac{\gamma_p}{\Omega} &= \frac{\pi\omega_{pi}^4\Omega(\Omega-\omega)k_\perp}{\Omega_i^3|k_\||c^2k_\|^2K_\|v_i^3(v_Ak_\|+k_\perp V_F)(\omega-\Omega-k_\perp V_F)}\left(\frac{e}{m_i}\right)^2E_{l\|}^2 \\ &\quad\times exp\left\{-\left(\frac{v_Ak_\|+k_\perp V_F}{v_ik_\|}\right)^2\right\} \end{aligned} \tag{48}$$

For inhomogeneous plasma with density gradient λ, considering $|E_{l\perp}| \gg |E_{l\|}|$ and with the condition $(\frac{\lambda v_i^2 k_\|}{2\Omega_i(v_Ak_\|+k_\perp V_F)} > 1$ the growth rate is estimated from Eq(47) we obtain as

$$\begin{aligned} \frac{\gamma_p}{\Omega} &= \frac{1}{4}\pi\frac{\omega_{pi}^4\Omega(\Omega-\omega)k_\perp^2v_i}{\Omega_i^4|k_\||c^2k_\|K_\|(v_Ak_\|+k_\perp V_F)^3(\omega-\Omega-k_\perp V_F)}\left(\frac{e}{m_i}\right)^2E_{l\|}^2\lambda \\ &\quad\times exp\left\{-\left(\frac{v_Ak_\|+k_\perp V_F}{v_ik_\|}\right)^2\right\} \end{aligned} \tag{49}$$

By using the Helimak experiment observational data: [28]
$\omega_{pi} \sim 1.05\times10^7Hz, \Omega \sim 2.8\times10^9Hz, \Omega_i \sim 3.80\times10^4Hz, k_\perp \sim 7.3m^{-1}, k_\| \sim 10^{-1}m^{-1}$ $v_A \sim 1.09\times10^6ms^{-1}, v_i \sim 2.68\times10^2ms^{-1}, \frac{e}{m} \sim 4.8\times10^7Ckg^{-1}$, $E_{l\perp} \sim 10^{-5}vm^{-1}, V_F = 1.3\times10^{-2}ms^{-1}, K_\| \sim 10^6m^{-1}$, $\lambda \sim 10m^{-1}$ to estimate the growth rate for whistler mode from Eq.(48) as

$$\frac{\gamma_p}{\Omega} \sim 10^{-2}$$

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For non-negligible density gradient,the estimated growth rate of whistler mode for burning plasma using Eq.(49) as

$$\frac{\gamma_p}{\Omega} \sim 10^{-7}\lambda$$

For tokamak plasma, we put $\lambda \sim 10$

The estimated growth rate of whistler mode for inhomogeneous plasma using Eq.(49) as

$$\frac{\gamma_p}{\Omega} \sim 10^{-6}$$

which is significant for explaining drift wave instabilities [29] in burning plasma in tokamak region.

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