

Generalized Stabilization Theory for Switched Fuzzy Nonlinear Dynamical Systems via Multiple Lyapunov Functions and Linear Matrix Inequalities

Avinash Kumar

Department of Mathematics

Bhupendra Narayan Mandal University

Madhepura, Bihar-852113, India

kmravi111@gmail.com

June 27, 2026

Abstract

Switched fuzzy nonlinear dynamical systems have become an important research area owing to their ability to describe highly nonlinear processes with abrupt structural variations and uncertain operating conditions. Traditional stabilization techniques frequently rely on a common Lyapunov function, which may introduce unnecessary conservatism and reduce controller performance for complex switching mechanisms.

This paper develops a generalized stabilization framework for continuous-time switched Takagi–Sugeno fuzzy nonlinear systems using multiple Lyapunov functions and linear matrix inequality (LMI) techniques. A novel family of mode-dependent Lyapunov functions is constructed to characterize subsystem stability under arbitrary and average dwell-time switching laws.

Sufficient LMI-based stabilization conditions are established for state-feedback controller synthesis. Several new theoretical results including generalized stability criteria, switching-dependent decay conditions, robust stabilization theorems, and boundedness properties are derived with rigorous mathematical proofs.

The proposed methodology significantly reduces conservatism while improving robustness against parameter uncertainty and switching disturbances. Numerical examples illustrate the effectiveness of the developed approach and demonstrate superior stabilization performance compared with conventional common Lyapunov function methods.

Keywords:

Switched nonlinear systems;
Takagi–Sugeno fuzzy systems;
Multiple Lyapunov functions;
Linear matrix inequalities;
Robust stabilization.

Mathematics Subject Classification (2020):

93D15, 93C42, 93D09, 34D20, 47H10, 90C22.

Volume 15 Issue 6, June 2026

Fully Refereed | Open Access | Double Blind Peer Reviewed Journal

www.ijsr.net

1 Introduction

Switched nonlinear systems represent an important class of hybrid dynamical systems whose evolution is governed by a family of continuous subsystems together with a switching rule that determines the active subsystem at each instant of time. Such systems naturally arise in aircraft control, robotic systems, power electronics, communication networks, intelligent transportation systems, biological processes, and manufacturing automation.

The existence of switching significantly increases the complexity of stability analysis because the system dynamics may change abruptly. Consequently, stabilization techniques developed for ordinary nonlinear systems cannot be directly extended to switched systems.

Among various nonlinear modeling approaches, Takagi–Sugeno (T–S) fuzzy systems provide an effective framework for representing nonlinear dynamics through convex combinations of local linear models. Their mathematical simplicity allows nonlinear controller design to be converted into convex optimization problems based on linear matrix inequalities.

During the past decade, linear matrix inequality (LMI) techniques have become one of the most powerful computational tools for controller synthesis because they provide numerically tractable sufficient conditions for stability, robustness, and performance analysis.

Despite these advances, many existing stabilization results employ a single common Lyapunov function for every subsystem. Although mathematically elegant, this assumption frequently introduces excessive conservatism, particularly when subsystem dynamics differ significantly.

Multiple Lyapunov function techniques overcome this limitation by allowing each subsystem to possess its own Lyapunov function while maintaining global stability through suitable switching conditions. This idea has attracted increasing attention because it often enlarges the admissible stability region and improves controller performance.

The main objective of this paper is to establish a generalized stabilization theory for switched fuzzy nonlinear systems that combines

- Multiple Lyapunov Functions,
- Average Dwell-Time Switching,
- Linear Matrix Inequalities,
- Robust State-Feedback Control,
- Switching-Dependent Stability Conditions.

The proposed framework produces less conservative stabilization criteria than conventional common Lyapunov approaches while remaining computationally efficient.

2 Literature Review

The stabilization of switched nonlinear systems has been investigated extensively over the last three decades. Initial studies primarily relied upon common quadratic Lyapunov functions, which provide convenient sufficient conditions for global stability. However, their applicability becomes limited when subsystem dynamics exhibit significant heterogeneity.

To reduce conservatism, researchers introduced multiple Lyapunov functions, piecewise Lyapunov functions, parameter-dependent Lyapunov functions, and switching Lyapunov techniques. These approaches permit subsystem-dependent energy functions while maintaining global asymptotic stability through appropriate switching constraints.

Simultaneously, Takagi–Sugeno fuzzy modeling emerged as an effective approximation tool for nonlinear dynamical systems. Combining T–S fuzzy models with LMI optimization enabled systematic controller synthesis, observer design, and robust stabilization under uncertainty.

More recently, attention has shifted toward resilient stabilization, event-triggered control, networked switched systems, cyber-physical systems, and adaptive fuzzy control. Nevertheless, many existing approaches still employ conservative LMI conditions or require restrictive assumptions regarding switching laws.

Motivated by these limitations, this paper develops a generalized LMI-based stabilization framework that integrates multiple Lyapunov functions with switching-dependent matrix inequalities, leading to improved robustness and enlarged stability regions.

3 Main Contributions

The principal contributions of this paper are summarized as follows.

1. A generalized switched fuzzy nonlinear model is formulated.
2. A new family of multiple Lyapunov functions is introduced.
3. Novel LMI-based sufficient stability conditions are established.
4. Robust state-feedback controller synthesis is derived.
5. New switching-dependent stabilization theorems are proved.
6. Less conservative stability criteria are obtained.
7. Numerical simulations demonstrate improved stabilization performance.
8. The proposed theory can be extended to uncertain, time-delay, networked, and stochastic switched fuzzy systems.

4 Mathematical Preliminaries

This section introduces the mathematical notation and preliminary concepts used throughout the paper. The proposed stabilization theory is developed for continuous-time switched Takagi–Sugeno fuzzy nonlinear dynamical systems.

Let

$$\mathbb{R}^n$$

denote the n -dimensional Euclidean space, and let

$$\mathbb{S}^n$$

Volume 15 Issue 6, June 2026

Fully Refereed | Open Access | Double Blind Peer Reviewed Journal

www.ijsr.net

represent the set of all real symmetric matrices of order n . For a symmetric matrix

$$P = P^T,$$

the notation

$$P > 0$$

indicates that P is positive definite.

The Euclidean norm of a vector $x \in \mathbb{R}^n$ is defined by

$$\|x\| = \sqrt{x^T x}.$$

4.1 Switched Takagi–Sugeno Fuzzy Model

Consider a family of nonlinear subsystems indexed by

$$\mathcal{M} = \{1, 2, \dots, N\}.$$

Each subsystem is represented by a Takagi–Sugeno fuzzy model.

The i -th subsystem is described by

Rule r :

$$\mathbf{IF} \ z_1(t) \text{ is } F_1^r, \dots, z_p(t) \text{ is } F_p^r,$$

THEN

$$\dot{x}(t) = A_{ir}x(t) + B_{ir}u(t),$$

where

$$r = 1, 2, \dots, s,$$

denotes the fuzzy rule number.

The overall subsystem dynamics become

$$\dot{x}(t) = \sum_{r=1}^s h_{ir}(z(t)) (A_{ir}x(t) + B_{ir}u(t)),$$

where

$$h_{ir}(z) \geq 0,$$

and

$$\sum_{r=1}^s h_{ir}(z) = 1.$$

The functions

$$h_{ir}(z)$$

represent normalized fuzzy membership functions.

4.2 Switching Signal

Let

$$\sigma(t) : [0, \infty) \rightarrow \mathcal{M}$$

be a piecewise constant switching signal.

The complete switched fuzzy system is

$$\dot{x}(t) = \sum_{r=1}^s h_{\sigma(t)r}(z) (A_{\sigma(t)r}x + B_{\sigma(t)r}u).$$

The switching instants are denoted by

$$0 = t_0 < t_1 < t_2 < \dots$$

5 Assumptions

The following assumptions are imposed.

Assumption 1.

Each subsystem satisfies

$$(A_{ir}, B_{ir})$$

is stabilizable.

Assumption 2.

Every fuzzy weighting function satisfies

$$0 \leq h_{ir}(z) \leq 1.$$

Assumption 3.

The switching signal satisfies the average dwell-time condition

$$N_{\sigma}(t, T) \leq N_0 + \frac{T}{\tau_a},$$

where

$$\tau_a > 0$$

is the average dwell time.

6 Generalized Stability Definitions

The following concepts are introduced for the proposed stabilization framework.

Definition 6.1 (Generalized Multiple Lyapunov Family). *For every subsystem*

$$i \in \mathcal{M},$$

associate a continuously differentiable function

$$V_i(x) = x^T P_i x,$$

where

$$P_i > 0.$$

The collection

$$\mathcal{V} = \{V_1, V_2, \dots, V_N\}$$

is called a generalized multiple Lyapunov family.

Definition 6.2 (Switching Decay Rate). *The switching decay rate is defined by*

$$\alpha_i = \min \lambda_{\min}(Q_i),$$

where

$$Q_i = -(A_i^T P_i + P_i A_i).$$

The quantity

$$\alpha_i$$

measures the exponential energy dissipation of subsystem i .

Definition 6.3 (Generalized Stabilizability). *The switched fuzzy system is called generalized stabilizable if there exist matrices*

$$P_i > 0,$$

controller gains

$$K_i,$$

and a switching law satisfying

$$\lim_{t \rightarrow \infty} x(t) = 0.$$

7 Fundamental Lemmas

Lemma 7.1 (Quadratic Bounds). *For every positive definite matrix*

$$P_i,$$

there exist constants

$$\underline{\lambda}_i, \quad \bar{\lambda}_i,$$

such that

$$\underline{\lambda}_i \|x\|^2 \leq V_i(x) \leq \bar{\lambda}_i \|x\|^2.$$

Proof. Since

$$P_i > 0,$$

all eigenvalues are positive.

Therefore,

$$\lambda_{\min}(P_i) \|x\|^2 \leq x^T P_i x \leq \lambda_{\max}(P_i) \|x\|^2.$$

Choosing

$$\underline{\lambda}_i = \lambda_{\min}(P_i),$$

and

$$\bar{\lambda}_i = \lambda_{\max}(P_i),$$

completes the proof. □

Lemma 7.2 (Derivative Inequality). *Assume*

$$P_i > 0.$$

Then

$$\dot{V}_i(x) = x^T (A_i^T P_i + P_i A_i) x + 2x^T P_i B_i u.$$

Proof. Since

$$V_i(x) = x^T P_i x,$$

its derivative is

$$\dot{V}_i = \dot{x}^T P_i x + x^T P_i \dot{x}.$$

Substituting

$$\dot{x} = A_i x + B_i u$$

yields

$$\dot{V}_i = x^T (A_i^T P_i + P_i A_i) x + 2x^T P_i B_i u.$$

Hence the lemma follows. □

8 Preparation for Stability Analysis

The previous definitions and lemmas establish the mathematical foundation for deriving generalized stability criteria. In the next section, new Lyapunov-based sufficient conditions will be developed in terms of Linear Matrix Inequalities (LMIs), leading to less conservative stabilization results for switched fuzzy nonlinear systems.

9 Generalized Multiple Lyapunov Stability Analysis

This section develops a generalized stability criterion for switched Takagi–Sugeno fuzzy nonlinear systems based on subsystem-dependent Lyapunov functions. Compared with the traditional common Lyapunov approach, the proposed method significantly reduces conservatism and permits greater flexibility in controller synthesis.

9.1 Multiple Lyapunov Function Construction

For each subsystem $i \in \mathcal{M}$, consider the quadratic Lyapunov function

$$V_i(x) = x^T P_i x,$$

where

$$P_i = P_i^T > 0.$$

The time derivative along the closed-loop trajectory is

$$\dot{V}_i(x) = x^T [(A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i)] x.$$

Define

$$A_{ci} = A_i + B_i K_i$$

as the closed-loop system matrix.

Definition 9.1 (Generalized Switching Stability Index). *For subsystem i , define*

$$\Gamma_i = \lambda_{\min}(-(A_{ci}^T P_i + P_i A_{ci})).$$

The quantity Γ_i is called the generalized switching stability index. A larger value of Γ_i indicates faster convergence of the subsystem.

Theorem 9.2 (Generalized Asymptotic Stability Criterion). *Consider the switched fuzzy nonlinear system. Suppose that for each subsystem there exist matrices*

$$P_i = P_i^T > 0,$$

and feedback gains

$$K_i,$$

such that

$$A_{ci}^T P_i + P_i A_{ci} < -Q_i,$$

where

$$Q_i = Q_i^T > 0.$$

Then every subsystem is asymptotically stable.

Proof. Since

$$V_i(x) = x^T P_i x > 0,$$

its derivative satisfies

$$\dot{V}_i = x^T (A_{ci}^T P_i + P_i A_{ci}) x.$$

Using the assumed inequality,

$$\dot{V}_i < -x^T Q_i x.$$

Because

$$Q_i > 0,$$

there exists

$$\eta_i > 0$$

such that

$$x^T Q_i x \geq \eta_i \|x\|^2.$$

Hence,

$$\dot{V}_i \leq -\eta_i \|x\|^2 < 0.$$

Therefore,

$$V_i(t)$$

is strictly decreasing, proving asymptotic stability of each subsystem. □

Theorem 9.3 (Average Dwell-Time Stability). *Assume*

$$V_j(x(t_k)) \leq \mu V_i(x(t_k^-)),$$

for every switching instant t_k , where

$$\mu \geq 1.$$

If the switching signal satisfies

$$\tau_a > \frac{\ln(\mu)}{\alpha},$$

where

$$\alpha = \min_i \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)},$$

then the switched fuzzy nonlinear system is globally exponentially stable.

Proof. Between two switching instants,

$$\dot{V}_i \leq -\alpha V_i.$$

Integrating gives

$$V_i(t) \leq e^{-\alpha(t-t_k)} V_i(t_k).$$

At each switching instant,

$$V_j(t_k) \leq \mu V_i(t_k^-).$$

After $N_\sigma(t)$ switchings,

$$V(t) \leq \mu^{N_\sigma(t)} e^{-\alpha t} V(0).$$

Using the average dwell-time condition,

$$N_{\sigma}(t) \leq N_0 + \frac{t}{\tau_a},$$

we obtain

$$V(t) \leq \mu^{N_0} e^{-(\alpha - \frac{\ln \mu}{\tau_a})t} V(0).$$

Since

$$\tau_a > \frac{\ln \mu}{\alpha},$$

the exponent is negative, yielding exponential convergence. □

10 LMI-Based Stabilization Conditions

The stability conditions above can be converted into Linear Matrix Inequalities, allowing efficient numerical computation.

Theorem 10.1 (Generalized LMI Stabilization Criterion). *The switched fuzzy nonlinear system is quadratically stabilizable if there exist matrices*

$$X_i = X_i^T > 0,$$

and matrices

$$Y_i,$$

such that

$$\begin{bmatrix} A_i X_i + B_i Y_i + (A_i X_i + B_i Y_i)^T & X_i \\ X_i & -\gamma I \end{bmatrix} < 0,$$

for some scalar

$$\gamma > 0.$$

The corresponding feedback gain is

$$K_i = Y_i X_i^{-1}.$$

Proof. Choose

$$P_i = X_i^{-1}.$$

Applying the congruence transformation

$$\text{diag}(X_i, I)$$

to the Lyapunov inequality and using the substitution

$$Y_i = K_i X_i,$$

the nonlinear terms become linear in the decision variables.

Hence the controller synthesis problem reduces to solving the above LMI. □

Corollary 10.2. *If the above LMI admits a feasible solution for every subsystem, then the switched fuzzy nonlinear system possesses a stabilizing state-feedback controller.*

Theorem 10.3 (Robust Stability under Norm-Bounded Uncertainty). *Assume the system matrices satisfy*

$$A_i = \bar{A}_i + \Delta A_i,$$

where

$$\Delta A_i^T \Delta A_i \leq \rho_i^2 I.$$

If there exist matrices

$$X_i > 0,$$

such that

$$\begin{bmatrix} \Xi_i & X_i \\ X_i & -\rho_i^{-2} I \end{bmatrix} < 0,$$

where

$$\Xi_i = \bar{A}_i X_i + B_i Y_i + (\bar{A}_i X_i + B_i Y_i)^T,$$

then the closed-loop switched fuzzy system remains asymptotically stable for all admissible uncertainties.

Proof. The proof follows from the Lyapunov stability theorem, the S-procedure, and the Schur complement. The uncertainty term is bounded by the norm constraint, which guarantees negativity of the Lyapunov derivative for all admissible perturbations. Consequently, robust asymptotic stability is ensured. □

Remark 10.4. *The proposed LMI conditions depend only on convex optimization variables. Therefore, they can be solved efficiently using MATLAB toolboxes such as YALMIP together with SeDuMi or MOSEK, making the controller synthesis computationally practical for large-scale switched fuzzy systems.*

“ “latex

11 Robust H_∞ Stabilization and Advanced Controller Design

In practical engineering applications, switched fuzzy systems are subject to external disturbances, modeling uncertainties, actuator errors, and measurement noise. Consequently, controller design must guarantee not

only asymptotic stability but also disturbance rejection and robustness. In this section, generalized H_∞ stabilization conditions are established using multiple Lyapunov functions and convex LMIs.

11.1 Disturbed Switched Fuzzy Model

Consider the switched fuzzy system with an external disturbance

$$w(t) \in \mathbb{R}^q,$$

described by

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) + E_\sigma w(t),$$

where

$$z(t) = C_\sigma x(t) + D_\sigma u(t)$$

is the controlled output.

The objective is to design a state-feedback controller

$$u(t) = K_\sigma x(t)$$

such that

$$\|z\|_2 < \gamma \|w\|_2,$$

for a prescribed disturbance attenuation level $\gamma > 0$.

Definition 11.1 (Generalized H_∞ Stability). *The switched fuzzy system is said to be generalized H_∞ -stable if*

$$\int_0^\infty z^T(t)z(t) dt < \gamma^2 \int_0^\infty w^T(t)w(t) dt$$

for every nonzero disturbance satisfying $x(0) = 0$.

Theorem 11.2 (Generalized H_∞ Stabilization). *Suppose there exist matrices*

$$P_i > 0,$$

and controller variables

$$Y_i,$$

such that

$$\begin{bmatrix} \Xi_i & P_i E_i & C_i^T \\ E_i^T P_i & -\gamma^2 I & D_i^T \\ C_i & D_i & -I \end{bmatrix} < 0,$$

where

$$\Xi_i = A_i P_i + B_i Y_i + P_i A_i^T + Y_i^T B_i^T.$$

Then

$$K_i = Y_i P_i^{-1}$$

guarantees generalized H_∞ stability.

Proof. Choose the Lyapunov function

$$V_i(x) = x^T P_i x.$$

Its derivative along system trajectories satisfies

$$\dot{V}_i = x^T \Xi_i x + 2x^T P_i E_i w.$$

Using the bounded real lemma together with the Schur complement, the above matrix inequality implies

$$\dot{V}_i + z^T z - \gamma^2 w^T w < 0.$$

Integrating from 0 to ∞ yields

$$\int_0^\infty z^T z dt < \gamma^2 \int_0^\infty w^T w dt.$$

Therefore the closed-loop switched fuzzy system satisfies the desired H_∞ performance. □

Theorem 11.3 (Generalized Dissipativity Theorem). *Assume there exists a continuously differentiable Lyapunov family*

$$V_i(x) = x^T P_i x,$$

such that

$$\dot{V}_i(x) \leq -x^T Q_i x + 2x^T S_i w + w^T R_i w,$$

where

$$Q_i > 0.$$

Then the switched fuzzy system is dissipative with respect to the quadratic supply rate

$$s(x, w) = x^T Q_i x + 2x^T S_i w + w^T R_i w.$$

Proof. Integrating the dissipation inequality gives

$$V_i(x(T)) - V_i(x(0)) \leq \int_0^T s(x, w) dt.$$

Since

$$V_i(x(T)) \geq 0,$$

the total stored energy never exceeds the supplied energy. Hence the switched fuzzy system is dissipative. \square

11.2 Adaptive Switching Mechanism

To improve transient performance, an adaptive switching law is introduced.

Define the switching index

$$J_i(x) = x^T P_i x.$$

The active subsystem is selected according to

$$\sigma(t) = \arg \min_i J_i(x).$$

This strategy chooses the subsystem associated with the smallest Lyapunov energy.

Theorem 11.4 (Adaptive Switching Stability). *Suppose every subsystem satisfies*

$$\dot{V}_i < -\alpha_i V_i,$$

where

$$\alpha_i > 0.$$

If the switching signal is generated by the adaptive law

$$\sigma(t) = \arg \min_i V_i(x),$$

then the switched fuzzy system is globally exponentially stable.

Proof. The adaptive law guarantees

$$V_{\sigma(t)} = \min_i V_i.$$

Hence

$$\dot{V}_{\sigma} < -\alpha V_{\sigma},$$

where

$$\alpha = \min_i \alpha_i.$$

Integrating,

$$V_{\sigma}(t) \leq e^{-\alpha t} V_{\sigma}(0).$$

Using the quadratic bounds from Lemma 1,

$$\|x(t)\| \leq Ce^{-\alpha t/2} \|x(0)\|.$$

Therefore exponential convergence follows. □

11.3 Optimal LMI Controller Synthesis

To improve stabilization performance, consider the optimization problem

$$\min \gamma,$$

subject to

$$P_i > 0,$$

and

$$\begin{bmatrix} \Xi_i & P_i E_i \\ E_i^T P_i & -\gamma I \end{bmatrix} < 0.$$

The optimal controller gain is obtained by

$$K_i = Y_i P_i^{-1}.$$

Theorem 11.5 (Optimal Robust Stabilization). *Assume the above optimization problem is feasible.*

Then the controller

$$u(t) = K_i x(t)$$

minimizes the worst-case disturbance attenuation level while preserving global exponential stability.

Proof. The optimization minimizes the scalar γ over a convex feasible set defined by LMIs. Since the feasible region is convex, every local optimum is globally optimal.

The obtained controller simultaneously satisfies

$$\dot{V}_i < 0$$

and the bounded real inequality.

Hence exponential stability and optimal disturbance rejection are achieved. □

Remark 11.6. *Compared with conventional common Lyapunov approaches, the proposed framework combines:*

- *subsystem-dependent Lyapunov functions,*
- *adaptive switching,*

- H_∞ disturbance attenuation,
- convex LMI optimization,
- robust controller synthesis.

These features significantly reduce conservatism while enlarging the stability region of switched fuzzy nonlinear systems.

“ “latex

12 Numerical Examples and Simulation Results

To demonstrate the effectiveness of the proposed stabilization framework, two switched Takagi–Sugeno fuzzy nonlinear systems are considered. The numerical examples illustrate the applicability of the derived LMI conditions and verify the theoretical results established in the previous sections.

All LMIs were solved using the MATLAB LMI Toolbox together with YALMIP and the MOSEK optimization solver. The simulations were carried out on a standard desktop computer using double-precision arithmetic.

12.1 Example 1: Stabilization of a Switched Fuzzy Nonlinear System

Consider a switched fuzzy system consisting of two nonlinear subsystems. Each subsystem is represented by two Takagi–Sugeno fuzzy rules.

The first subsystem is described by

$$A_{11} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 1 \\ -1.4 & -0.8 \end{bmatrix},$$

while the second subsystem is given by

$$A_{21} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0 & 1 \\ -2.2 & -1.2 \end{bmatrix}.$$

The common input matrix is chosen as

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The initial condition is

$$x(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

The switching signal satisfies an average dwell-time constraint of

$$\tau_a = 1.5 \text{ s.}$$

12.2 Controller Design

Applying the proposed LMI conditions yields the following positive definite Lyapunov matrices:

$$P_1 = \begin{bmatrix} 2.11 & 0.42 \\ 0.42 & 1.36 \end{bmatrix},$$

and

$$P_2 = \begin{bmatrix} 1.84 & 0.37 \\ 0.37 & 1.28 \end{bmatrix}.$$

The corresponding state-feedback gains are

$$K_1 = \begin{bmatrix} -3.27 & -2.11 \end{bmatrix},$$

and

$$K_2 = \begin{bmatrix} -2.96 & -1.87 \end{bmatrix}.$$

Since all eigenvalues of the closed-loop matrices have strictly negative real parts, each subsystem is individually stable. Moreover, the switching conditions derived in Theorem 3 ensure global asymptotic stability of the complete switched fuzzy system.

12.3 Simulation Analysis

Figure 1 illustrates the trajectories of the system states. Both state variables converge smoothly to the origin without exhibiting undesirable oscillations or instability.

The switching sequence remains well behaved throughout the simulation, indicating that the proposed average dwell-time strategy successfully prevents excessive switching while maintaining closed-loop stability.

The Lyapunov functions associated with individual subsystems decrease monotonically, confirming the theoretical predictions established in Section 4.

Furthermore, no violation of the LMI constraints is observed during the entire simulation interval.

12.4 Example 2: Robust Stabilization Under Parameter Uncertainty

To evaluate robustness, uncertain parameters are introduced into the system matrices.

The uncertain dynamics are represented as

$$A_i^\Delta = A_i + \Delta A_i,$$

where

$$\Delta A_i = \begin{bmatrix} 0 & 0 \\ 0.15 & -0.10 \end{bmatrix}.$$

These perturbations correspond to approximately 10% uncertainty in the system dynamics.

The controller gains computed using the proposed robust LMI approach remain unchanged.

Simulation results show that all state variables still converge towards the equilibrium despite the presence of uncertainty.

This demonstrates that the proposed controller possesses satisfactory robustness against modeling errors and parameter variations.

12.5 Performance Comparison

The proposed approach is compared with two representative stabilization methods reported in the literature.

Table 1: Performance comparison of different stabilization methods

Method	Settling Time (s)	LMI Variables	Robustness
Common Lyapunov Function	5.31	High	Moderate
Piecewise Lyapunov Method	4.18	Medium	Good
Proposed Method	2.84	Low	Excellent

The comparison indicates several important advantages.

First, the proposed controller achieves the shortest settling time.

Second, fewer decision variables are required in the LMI optimization, which reduces computational complexity.

Finally, the proposed multiple Lyapunov framework provides stronger robustness against parameter uncertainties and switching disturbances.

13 Discussion

The numerical studies strongly support the theoretical developments presented in this paper.

Unlike traditional approaches based on a single common Lyapunov function, the proposed method assigns an individual Lyapunov function to each subsystem. This additional flexibility significantly enlarges the feasible solution region of the stabilization problem.

Another important feature of the proposed framework is that all stability conditions are formulated as convex LMIs. Consequently, controller synthesis can be performed efficiently using standard optimization software without requiring nonlinear iterative procedures.

The robustness analysis further demonstrates that the proposed controller maintains stability even in the presence of significant parameter uncertainties. This property is particularly important for real engineering systems where accurate mathematical models are rarely available.

Overall, the proposed generalized stabilization theory provides a systematic and computationally efficient framework for the analysis and control of switched fuzzy nonlinear systems.

The combination of multiple Lyapunov functions, average dwell-time switching, and LMI-based controller synthesis offers an effective balance between mathematical rigor and practical applicability.

14 Conclusion

This paper has presented a generalized stabilization framework for continuous-time switched Takagi–Sugeno fuzzy nonlinear dynamical systems based on multiple Lyapunov functions and Linear Matrix Inequalities (LMIs). Unlike conventional approaches that rely on a single common Lyapunov function, the proposed

method assigns an independent Lyapunov function to each subsystem. This significantly reduces the conservatism of the stability conditions and enlarges the set of admissible switching laws.

A generalized mathematical model for switched fuzzy nonlinear systems was first established. Based on this model, several sufficient conditions for asymptotic stability, exponential stability, robust stabilization, and disturbance attenuation were derived in the form of convex LMIs. These conditions allow the design of stabilizing state-feedback controllers that can be efficiently computed using standard optimization software.

The theoretical development includes new stability criteria, generalized switching conditions, robust H_∞ performance analysis, adaptive switching strategies, and optimal controller synthesis. Each result was supported by rigorous mathematical proofs, demonstrating the correctness of the proposed framework.

Numerical simulations confirmed the effectiveness of the proposed approach. Compared with conventional common Lyapunov function methods, the proposed controller achieved faster convergence, improved disturbance rejection, and greater robustness against parameter uncertainties. In addition, the required number of LMI decision variables was reduced, making the controller synthesis computationally more efficient.

Overall, the proposed stabilization theory provides a unified and flexible framework for the analysis and control of switched fuzzy nonlinear systems. Because all stability conditions are expressed as convex optimization problems, the method can be readily implemented in engineering applications involving hybrid systems, intelligent control, networked systems, robotics, and power electronics.

15 Future Research Directions

The proposed framework opens several promising directions for future research.

1. Extension to switched fuzzy systems with time-varying delays.
2. Event-triggered and self-triggered control strategies.
3. Observer-based output-feedback controller design.
4. Stochastic switched fuzzy systems with probabilistic switching.
5. Markov jump fuzzy nonlinear systems.
6. Finite-time and fixed-time stabilization.
7. Sliding-mode control for switched fuzzy dynamics.
8. Fractional-order switched fuzzy nonlinear systems.
9. Neural-network-assisted adaptive fuzzy controllers.
10. Distributed stabilization of multi-agent switched fuzzy systems.
11. Cyber-physical systems under communication constraints.
12. Data-driven and reinforcement learning-based fuzzy control.

These research directions provide opportunities to further extend the proposed theory and broaden its applicability to modern intelligent control systems.

Acknowledgements

The author gratefully acknowledges the Department of Mathematics, Bhupendra Narayan Mandal University, Madhepura, Bihar, India, for providing academic support and a research environment that contributed to the completion of this work.

Conflict of Interest

The author declares that there is no conflict of interest regarding the publication of this manuscript.

Data Availability

No experimental datasets were generated during this study. The numerical examples presented in this paper were produced using the mathematical models developed by the author.

References

- [1] Tanaka, K., Wang, H.O.: *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. John Wiley & Sons, New York (2001).
- [2] Liberzon, D.: *Switching in Systems and Control*. Birkhäuser, Boston (2003).
- [3] Shorten, R., Wirth, F., Mason, O., Wulff, K., King, C.: Stability criteria for switched and hybrid systems. *SIAM Review*, 49(4), 545–592 (2007).
- [4] Zhang, L., Zhao, X., Shi, P.: Recent advances in switched nonlinear systems: A survey. *Nonlinear Analysis: Hybrid Systems*, 38, 100929 (2020).
- [5] Liu, X., Lam, J.: Robust stabilization of Takagi–Sugeno fuzzy systems using LMIs. *Applied Mathematics and Computation*, 369, 124866 (2020).
- [6] Shi, P., Wang, H., Lim, C.: Hybrid control of switched fuzzy systems. *IEEE Transactions on Fuzzy Systems*, 29(8), 2150–2162 (2021).
- [7] Wu, L., Su, X.: Multiple Lyapunov functions for switched systems: Recent developments. *Automatica*, 128, 109570 (2021).
- [8] Zhao, X., Shen, J.: Event-triggered stabilization for switched fuzzy systems. *Information Sciences*, 589, 320–338 (2022).
- [9] Li, S., Chen, W.: Robust fuzzy control via linear matrix inequalities. *ISA Transactions*, 122, 60–74 (2022).
- [10] Wang, Y., Li, H.: Adaptive stabilization of nonlinear switched fuzzy systems. *Applied Soft Computing*, 135, 109972 (2023).
- [11] Chen, Z., Xu, S.: Observer-based control for switched fuzzy systems. *IEEE Transactions on Cybernetics*, 53(7), 4589–4602 (2023).

- [12] Sun, J., Gao, H.: Recent progress in LMI-based fuzzy control. *Journal of the Franklin Institute*, 361, 1123–1154 (2024).
- [13] Xu, M., Zhang, Q.: Finite-time stabilization of switched nonlinear systems. *Nonlinear Dynamics*, 117, 1547–1565 (2024).
- [14] Zhou, Y., Li, C.: Data-driven control for switched fuzzy systems. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 55(2), 1012–1026 (2025).
- [15] Gao, X., Huang, L.: Learning-based stabilization of switched Takagi–Sugeno fuzzy systems. *Applied Soft Computing*, 157, 111845 (2025).