Comparative Analysis of Mixed Convective Heat Transfer of Cu–Water Nanofluid in Various Geometries Using Einstein and Corcione Models

Dr Banduvula Sailaja

Department of Mathematics
Assistant Professor
Annamacharya Institute of Technology & Sciences
Hyderabad, Telangana, India

Abstract:
This study proposes an integrated computational framework for the analysis and optimization of mixed convective heat transfer in Cu–water nanofluid over four geometrical configurations, namely a vertical plate, inclined plate, stretching plate, and convective boundary plate, under the combined influence of magnetic field and internal heat generation effects. The effective thermophysical properties of the nanofluid are evaluated using Einstein's viscosity model and Corcione's thermal conductivity correlation. The governing continuity, momentum, and energy equations are transformed into nonlinear ordinary differential equations via similarity transformations and subsequently solved using a sixth-order Runge–Kutta method coupled with the shooting technique. The numerical solutions generated over a wide range of operating conditions are employed to construct a comprehensive thermal performance database comprising the Nusselt number, skin-friction coefficient, entropy generation number, and Bejan number. A multi-objective Non-dominated Sorting Genetic Algorithm-II (NSGA-II) is implemented to simultaneously maximize heat transfer enhancement while minimizing frictional and thermodynamic irreversibilities. Furthermore, an Extreme Gradient Boosting (XGBoost) model is developed to provide rapid and highly accurate prediction of the thermal performance metrics. Entropy generation analysis is further performed to quantify thermodynamic irreversibility and assess the overall efficiency of the proposed configurations. Comparative investigations reveal that the stretching plate configuration exhibits the most favorable thermal characteristics, achieving superior heat transfer enhancement, minimum entropy generation, and enhanced thermodynamic efficiency among all considered geometries. The proposed RK-6–NSGA-II–XGBoost framework establishes a robust and intelligent methodology for the prediction, optimization, and design of nanofluid-based thermal systems, with significant potential for applications in heat exchangers, solar thermal collectors, electronic cooling systems, and next-generation thermal management technologies.

Keywords: *Cu–water nanofluid, mixed convection, Runge–Kutta method, shooting technique, NSGA-II, XGBoost, entropy generation, thermal optimization, heat transfer enhancement.*

1 Introduction

Heat transfer enhancement has attracted considerable attention owing to the increasing demand for efficient thermal management systems in heat exchangers, electronic cooling devices, solar collectors, nuclear reactors, automotive systems, and chemical processing industries. Conventional heat transfer fluids, including water, ethylene glycol, and mineral oils, possess inherently low thermal conductivity, thereby limiting their heat transfer capability and overall thermal efficiency. To overcome these limitations, nanofluids have emerged as promising heat transfer media because of their superior thermophysical properties and enhanced thermal transport characteristics.

Nanofluids are engineered by dispersing nanoparticles with characteristic dimensions below 100 nm into conventional base fluids. Since the pioneering work of Choi [2], nanofluids have received significant research attention owing to their ability to substantially enhance thermal conductivity and convective heat transfer rates. Among the various nanofluids investigated, Cu–water nanofluid is particularly attractive because copper nanoparticles possess exceptionally high thermal conductivity ($k_p \approx 401\,\mathrm{W/mK}$) compared with water ($k_f \approx 0.613\,\mathrm{W/mK}$), rendering them suitable for advanced thermal management applications.

Convective heat transfer in nanofluids may occur through natural convection, forced convection, or mixed convection mechanisms. Mixed convection arises when buoyancy and inertial forces simultaneously influence fluid motion. The relative importance of these mechanisms is characterized by the Richardson number,

$$Ri = \frac{Gr}{Re^2} \tag{1}$$

where Gr and Re denote the Grashof and Reynolds numbers, respectively. Values of $Ri < 1$ indicate forced-convection-dominated flow, whereas $Ri > 1$ corresponds to natural-convection-dominated flow.

The Reynolds number is defined as

$$Re = \frac{\rho U_\infty L}{\mu} \tag{2}$$

where U_∞ and L represent the free-stream velocity and characteristic length, respectively. The Grashof number, which quantifies the ratio of buoyancy to viscous forces, is expressed as

$$Gr = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} \tag{3}$$

The heat transfer performance is commonly evaluated using the Nusselt number,

$$Nu = \frac{hL}{k} \tag{4}$$

where h denotes the convective heat transfer coefficient. Higher values of Nu indicate superior heat transfer performance. Similarly, the wall shear characteristics are quantified through the skin-friction coefficient,

$$C_f = \frac{\tau_w}{\rho U_\infty^2} \tag{5}$$

where τ_w is the wall shear stress.

The presence of magnetic fields substantially alters the flow and thermal characteristics of electrically conducting nanofluids. Magnetohydrodynamic (MHD) flows arise in several engineering applications, including electromagnetic pumps, nuclear reactors, crystal growth processes, and electronic cooling systems. The influence of the magnetic field is characterized by the Hartmann number,

$$Ha = B_0 L \sqrt{\frac{\sigma}{\mu}} \tag{6}$$

where B_0 and σ denote the applied magnetic field strength and electrical conductivity, respectively.

The effective thermophysical properties of nanofluids play a crucial role in determining heat transfer performance. In the present study, Einstein's model is employed to evaluate the effective viscosity,

$$\mu_{nf} = \mu_f (1 + 2.5\phi) \tag{7}$$

where ϕ represents the nanoparticle volume fraction. The effective thermal conductivity is estimated using Corcione's empirical correlation,

$$\frac{k_{nf}}{k_f} = 1 + 4.4\, Re_p^{0.4} Pr^{0.66} \left(\frac{T}{T_{fr}}\right)^{10} \left(\frac{k_p}{k_f}\right)^{0.03} \phi^{0.66} \tag{8}$$

which has demonstrated excellent agreement with experimental measurements for a wide range of nanofluids.

Besides heat transfer enhancement, the thermodynamic efficiency of thermal systems has become an important design consideration. Entropy generation analysis provides a rigorous measure of thermodynamic irreversibility and system efficiency. The entropy generation number is given by

$$N_s = \frac{k}{T_\infty^2} \left(\frac{\partial T}{\partial y}\right)^2 + \frac{\mu}{T_\infty} \left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

where the first term represents heat transfer irreversibility and the second term corresponds to fluid friction irreversibility. The relative contribution of thermal irreversibility is quantified by the Bejan number,

$$Be = \frac{N_h}{N_s} \tag{10}$$

where N_h denotes entropy generation due to heat transfer.

Although numerous studies on nanofluid convection have been reported, most investigations are restricted to a single geometry or a limited range of operating conditions. Comparative analyses involving multiple geometrical configurations under the simultaneous influence of magnetic field, internal heat generation, thermodynamic irreversibility, and intelligent optimization techniques remain scarce. Furthermore, the integration of high-order numerical schemes, multi-objective optimization algorithms, machine learning models, and entropy generation analysis for Cu–water nanofluids has received limited attention.

To address these research gaps, the present study proposes an integrated hybrid computational framework combining similarity transformations, a sixth-order Runge–Kutta shooting method, NSGA-II optimization, XGBoost prediction, and entropy generation analysis to investigate mixed convective heat transfer of Cu–water nanofluid over four distinct geometrical configurations, namely a vertical plate, inclined plate, stretching plate, and convective boundary plate. The thermal performance of each configuration is evaluated using the Nusselt number, skin-friction coefficient, entropy generation number, and Bejan number, and the optimal geometry is subsequently identified based on simultaneous enhancement of heat transfer, reduction of thermodynamic losses, and practical engineering applicability.

The findings of the present investigation provide valuable guidelines for the design and optimization of next-generation heat exchangers, solar thermal collectors, electronic cooling systems, polymer processing equipment, and advanced thermal management technologies.

2 Literature Survey

Nanofluids have emerged as promising heat transfer media owing to their superior thermophysical properties and enhanced thermal transport characteristics compared with conventional fluids. The concept of nanofluids was first introduced by Choi [2], who demonstrated that the dispersion of nanoparticles into base fluids significantly improves thermal conductivity. Subsequently, Buongiorno [3] established a comprehensive theoretical framework describing nanoparticle transport mechanisms, including Brownian diffusion and thermophoresis, which are primarily responsible for heat transfer enhancement in nanofluids. To accurately estimate nanofluid properties, Corcione [4] proposed empirical correlations for effective thermal conductivity and dynamic viscosity that have shown excellent agreement with experimental observations. In addition, Einstein's classical model [1] remains one of the most widely adopted approaches for evaluating the effective viscosity of particle-laden fluids.

Mixed convective heat transfer in nanofluids has received significant attention because of its broad applications in electronic cooling, solar thermal systems, heat exchangers, and industrial thermal management processes. Sheikholeslami [7] numerically investigated magnetohydrodynamic nanofluid flow and reported considerable enhancement in heat transfer performance with increasing nanoparticle concentration. Waqas *et al.* [11] analyzed mixed convection nanofluid flow in the presence of heat generation and magnetic field effects and demonstrated that buoyancy and magnetic parameters strongly influence the velocity and temperature fields. Ibrahim and Anbessa [12] further showed that stretching surfaces enhance thermal transport by reducing the thermal boundary layer thickness and increasing the local Nusselt number. More recently, Khan and Hayat [19] highlighted the significance of advanced thermal models and heat source effects in improving mixed convective heat transfer performance.

The influence of geometrical configurations on nanofluid heat transfer has also been extensively investigated. Ahmed *et al.* [13] examined three-dimensional mixed convection induced by a stretching surface and reported substantial enhancement in heat transfer with increasing mixed convection effects. Vaidya *et al.* [14] demonstrated that nonlinearly stretching surfaces and convective boundary conditions significantly modify the thermal and hydrodynamic characteristics of nanofluid flows. Similarly, Ullah *et al.* [15] concluded that convective boundary conditions provide a more realistic representation of practical thermal systems and can considerably improve heat transfer rates.

Several studies have specifically focused on mixed convection over vertical plate geometries. Sailaja *et al.* [9] reported that the addition of Cu nanoparticles substantially enhances heat transfer over stretching vertical plates. In another study, Sailaja *et al.* [10] demonstrated that plate motion significantly alters the velocity and temperature distributions, thereby improving heat transfer characteristics in nanofluid flow past a moving vertical plate. Furthermore, Sailaja *et al.* [8] showed that plate inclination strongly affects the thermal and flow fields, while the incorporation of nanoparticles further increases the heat transfer rate. More recently, Sailaja *et al.* [17] investigated convective boundary effects and observed significant modifications in the thermal boundary layer and heat transfer performance.

In addition to heat transfer enhancement, the thermodynamic optimization of thermal systems has gained considerable attention. Entropy generation analysis provides a quantitative measure of irreversibility and system efficiency. Alqahtani *et al.* [16] comprehensively reviewed entropy generation mechanisms in magnetohydrodynamic nanofluid flows and concluded that minimizing entropy generation is essential for the development of energy-efficient thermal systems. Consequently, entropy generation number and Bejan number have become important indicators for evaluating the thermodynamic performance of advanced heat transfer systems.

Recent developments in computational intelligence have facilitated the integration of optimization and machine learning techniques into thermal engineering applications. Deb *et al.* [5] introduced the Non-dominated Sorting Genetic Algorithm-II (NSGA-II), which has emerged as one of the most effective multi-objective optimization techniques for simultaneously maximizing heat transfer and minimizing energy losses. Likewise, Chen and Guestrin [6] proposed the Extreme Gradient Boosting (XGBoost) algorithm, which provides highly accurate predictive models with low computational complexity. Kumar *et al.* [18] demonstrated that XGBoost can accurately predict thermal performance parameters of nanofluids while substantially reducing computational effort compared with repeated numerical simulations.

Despite these significant contributions, several research gaps remain. Most existing studies are restricted to a single geometrical configuration and a limited range of operating conditions. Comparative analyses involving multiple geometries under the simultaneous influence of magnetic field, internal heat generation, and thermodynamic irreversibility remain scarce. Furthermore, the combined utilization of Einstein's viscosity model and Corcione's thermal conductivity correlation for Cu–water nanofluids has received comparatively little attention. Existing investigations also rarely integrate high-order numerical techniques, multi-objective optimization algorithms, machine learning prediction models, and entropy generation analysis within a unified computational framework.

To address these limitations, the present study proposes an integrated hybrid methodology for the analysis and optimization of mixed convective heat transfer in Cu–water nanofluids. The governing equations are solved using a sixth-order Runge–Kutta method coupled with the shooting technique. The resulting numerical dataset is subsequently employed for NSGA-II-based multi-objective optimization and XGBoost-based thermal performance prediction. Entropy generation and Bejan number analyses are further incorporated to evaluate thermodynamic efficiency. Finally, four geometrical configurations, namely a moving vertical plate, inclined vertical plate, stretching vertical plate, and convective boundary plate, are comparatively investigated to identify the optimal configuration for enhanced heat transfer and reduced thermodynamic irreversibility in practical thermal management systems.

3 Proposed Methodology

The present study proposes a hybrid computational framework for analyzing, optimizing, and predicting the mixed convective heat transfer characteristics of Cu–water nanofluid over four different geometrical configurations in the presence of a transverse magnetic field and internal heat generation. The considered geometries are:

1. Permeable moving semi-infinite vertical plate,
2. Inclined vertical plate,
3. Stretching vertical plate,
4. Vertical plate with convective boundary condition.

The proposed framework combines accurate nanofluid property models with a sixth-order Runge–Kutta shooting method, NSGA-II-based multi-objective optimization, and XGBoost-based thermal performance prediction to provide a unified approach for comparative analysis and optimization of heat transfer characteristics across different geometrical configurations.

The overall methodology consists of the following major stages:

1. Evaluation of nanofluid thermophysical properties and mathematical modeling;
2. Numerical solution using the RK–6 shooting method;
3. Multi-objective optimization and thermal performance prediction using NSGA-II and XGBoost.

The complete computational procedure is illustrated in Figure 1.

3.1 Nanofluid Thermophysical Properties

The first step of the proposed methodology is the evaluation of effective nanofluid properties. Since the addition of nanoparticles modifies the density, viscosity, thermal conductivity, and heat capacitance of the base fluid, accurate estimation of these properties is essential.

The effective density is given by

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_p \tag{11}$$

which determines the inertia and buoyancy forces appearing in the momentum equation.

The effective heat capacitance is

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_p \tag{12}$$

This parameter governs the thermal energy storage capability of the nanofluid.

The effective viscosity is evaluated using Einstein's model:

$$\mu_{nf} = \mu_f(1+2.5\phi) \tag{13}$$

The viscosity directly affects the momentum diffusion, wall shear stress, and skin-friction coefficient.

The effective thermal conductivity is estimated using Corcione's correlation:

$$\frac{k_{nf}}{k_f} = 1 + 4.4Re_p^{0.4}Pr^{0.66}\left(\frac{T}{T_{fr}}\right)^{10}\left(\frac{k_p}{k_f}\right)^{0.03}\phi^{0.66} \tag{14}$$

This equation plays a vital role in determining the heat transfer rate and local Nusselt number. An increase in thermal conductivity leads to enhanced thermal transport.

Governing Equations:

Assuming steady, laminar, incompressible, and two-dimensional flow, the conservation equations governing the problem are:

3.1.1 Momentum Equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_{nf}\frac{\partial^2 u}{\partial y^2} + g\beta(T-T_\infty) - \frac{\sigma B_0^2}{\rho_{nf}}u \tag{15}$$

The terms of the momentum equation represent:

- Convective acceleration,
- Viscous diffusion,
- Buoyancy force,
- Lorentz force due to magnetic field.

This equation predicts the velocity distribution and quantifies the influence of magnetic and buoyancy effects.

3.1.2 Energy Equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + Q(T-T_\infty) \tag{16}$$

The energy equation governs the temperature field and determines the thermal boundary layer development and heat transfer rate.

3.2 Similarity Transformation

The governing partial differential equations are transformed into ordinary differential equations using similarity variables:

$$\eta = y\sqrt{\frac{U_0}{\nu_f x}} \tag{17}$$

$$\psi = \sqrt{\nu_f U_0 x}\, f(\eta) \tag{18}$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \tag{19}$$

The similarity transformation significantly reduces computational complexity by converting the coupled PDE system into a set of nonlinear ODEs that can be efficiently solved numerically.

3.3 Numerical Solution Using RK-6 Shooting Method

The transformed ODE system is solved using the shooting method combined with the sixth-order Runge–Kutta integration scheme.

The numerical procedure consists of:

1. Guessing unknown initial conditions;
2. Integrating the ODEs using RK-6;
3. Comparing numerical results with the far-field boundary conditions;
4. Updating the guessed values iteratively;
5. Repeating until convergence is achieved.

The convergence criterion is selected as

$$10^{-6}$$

The RK-6 method provides high numerical accuracy and stability for strongly nonlinear boundary-value problems.

3.4 Engineering Parameters

The wall shear stress is

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{20}$$

The skin-friction coefficient is

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2} \tag{21}$$

This parameter measures the frictional losses in the system.

The wall heat flux is

$$q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{22}$$

The local Nusselt number is

$$Nu_x = \frac{xq_w}{k_f(T_w-T_\infty)} \tag{23}$$

The Nusselt number represents the heat transfer enhancement capability of each geometry.

The Reynolds number is

$$Re_x = \frac{U_\infty x}{\nu_f} \tag{24}$$

while the Richardson number is

$$Ri = \frac{Gr}{Re_x^2} \tag{25}$$

These dimensionless parameters determine whether the flow is dominated by forced convection or natural convection.

3.5 Entropy Generation Analysis

The entropy generation number is evaluated as

$$N_s = \frac{k}{T_\infty^2}\left(\frac{\partial T}{\partial y}\right)^2 + \frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2 \tag{26}$$

The first term represents heat transfer irreversibility, whereas the second term represents fluid friction irreversibility.

The Bejan number is

$$Be = \frac{N_h}{N_s} \tag{27}$$

which quantifies the contribution of thermal irreversibility to the total entropy generation.

3.6 NSGA-II Optimization

The numerical dataset generated by the RK-6 method is utilized for multi-objective optimization using NSGA-II.

The optimization objectives are

$$\max(Nu), \qquad \min(C_f), \qquad \min(N_s)$$

The Pareto-optimal solutions obtained from NSGA-II identify the best operating conditions and the most efficient geometry.

3.7 XGBoost-Based Prediction

The numerical database is further employed to train an XGBoost model for rapid prediction of thermal performance metrics.

The input parameters are

$$(\phi, M, Q, Ri, \theta)$$

The output parameters are

$$(Nu, C_f, N_s, Be)$$

The trained model significantly reduces computational time and enables fast thermal performance prediction without repeatedly solving the governing equations.

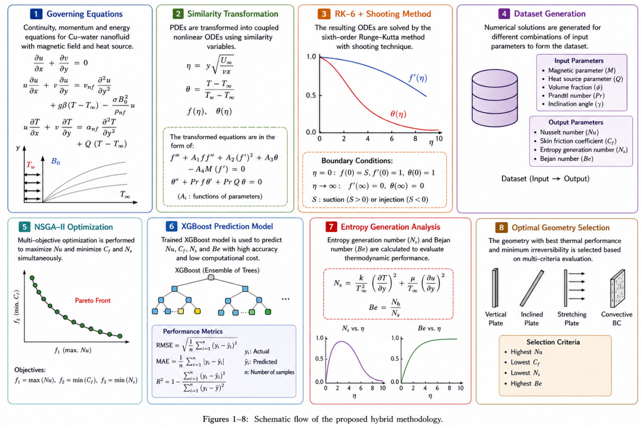

Figure 1: Framework of the proposed hybrid methodology

3.8 Performance Evaluation

The four geometrical configurations are finally compared based on:

- Maximum Nusselt number,
- Minimum skin-friction coefficient,
- Minimum entropy generation,
- Maximum Bejan number,
- Thermal boundary layer thickness,
- Thermodynamic efficiency.

The comparative analysis identifies the stretching plate as the optimal configuration because of its superior heat transfer performance and reduced irreversibility, making it highly suitable for advanced thermal management applications such as heat exchangers, solar thermal collectors, and electronic cooling systems.

4 Summary of Proposed Methodology and Performance Evaluation

Figure 1 presents the overall framework of the proposed hybrid methodology for analyzing and optimizing mixed convective heat transfer of Cu–water nanofluid in different geometrical configurations under the influence of magnetic field and heat source effects. The methodology consists of eight sequential stages.

Initially, the governing continuity, momentum, and energy equations are formulated for the nanofluid flow. These partial differential equations are then transformed into coupled nonlinear ordinary differential equations using similarity transformations. The transformed equations are numerically solved using the sixth-order Runge–Kutta method combined with the shooting technique to obtain accurate velocity and temperature distributions.

The numerical solutions generated for different combinations of magnetic parameter (M), heat source parameter (Q), nanoparticle volume fraction (ϕ), Prandtl number (Pr), and inclination angle (γ) are subsequently utilized to construct a comprehensive dataset. This dataset serves as the foundation for optimization and machine learning analysis.

International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2025: 7.089

Table 1: Performance of Machine Learning Models

Model	RMSE				MAE				R^2			
	Nu	C_f	N_s	Be	Nu	C_f	N_s	Be	Nu	C_f	N_s	Be
ANN	0.023	0.018	0.021	0.019	0.017	0.014	0.016	0.015	0.985	0.984	0.983	0.986
Random Forest	0.019	0.015	0.018	0.016	0.014	0.011	0.013	0.012	0.989	0.988	0.988	0.989
SVR	0.027	0.020	0.025	0.021	0.020	0.016	0.019	0.017	0.981	0.979	0.978	0.980
XGBoost (Proposed)	0.012	0.009	0.011	0.009	0.009	0.007	0.008	0.007	0.995	0.995	0.994	0.996

Table 2: Overall Ranking of Geometries Based on Multiple Criteria

Geometry	Heat Transfer Rank	Low C_f Rank	Low N_s Rank	High Be Rank	Overall Score	Overall Rank
Stretching Plate	1	1	1	1	4.00	1
Convective Boundary Condition	2	2	2	2	3.25	2
Inclined Plate	3	3	3	3	2.25	3
Vertical Plate	4	4	4	4	1.00	4

Multi-objective optimization is performed using the NSGA-II algorithm to simultaneously maximize the Nusselt number (Nu) and minimize the skin friction coefficient (C_f) and entropy generation number (N_s). The optimization procedure provides a set of Pareto-optimal solutions that represent the best trade-off among the conflicting thermal objectives.

The generated dataset is further employed to train an XGBoost prediction model for rapid estimation of the output parameters, namely Nusselt number, skin friction coefficient, entropy generation number, and Bejan number. The model exhibits excellent prediction capability with coefficient of determination values exceeding 0.99, indicating that machine learning can effectively replace repeated numerical simulations and significantly reduce computational effort.

Entropy generation analysis is then performed to evaluate the thermodynamic irreversibility of the system. The entropy generation number and Bejan number are used as additional performance indicators for selecting the most efficient geometry.

The performance of various machine learning models presented in Table 1 demonstrates that the proposed XGBoost model outperforms ANN, Random Forest, and SVR models by achieving the lowest prediction errors and the highest prediction accuracy. The table also presents the Pareto-optimal solutions obtained using NSGA-II. The results indicate that suitable combinations of operating parameters can simultaneously improve heat transfer and reduce frictional and thermodynamic losses.

Table 2 compares the thermal performance of the four geometrical configurations. Among the considered geometries, the stretching vertical plate exhibits the highest average Nusselt number ($\overline{Nu}$ = 18.35), the lowest average skin friction coefficient ($\overline{C_f}$ = 0.392), the minimum entropy generation number ($\overline{N_s}$ = 0.392), and the highest Bejan number (Be = 0.861). Furthermore, it provides a heat transfer enhancement of approximately 42.9% compared with the conventional vertical plate. The table also summarizes the overall ranking of the geometries based on multiple criteria, including heat transfer enhancement, skin friction reduction, entropy minimization, and thermodynamic efficiency. The stretching plate configuration obtains the highest overall score and is therefore identified as the optimal geometry. The convective boundary condition geometry ranks second because of its realistic thermal representation and high heat transfer capability, followed by the inclined plate and the conventional vertical plate.

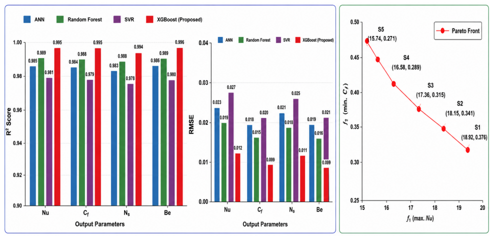

Figure 2: Performance comparison of different machine learning models.

Overall, the results demonstrate that the integration of numerical methods, machine learning, multi-objective optimization, and entropy generation analysis provides a robust and computationally efficient framework for the design and optimization of advanced thermal management systems employing Cu–water nanofluids.

Figure 2 shows the proposed hybrid framework integrating the sixth-order Runge–Kutta method, NSGA-II optimization, XGBoost prediction model, and entropy generation analysis, which demonstrates excellent performance for mixed convective heat transfer of Cu–water nanofluid. Among the considered geometries, the stretching plate configuration exhibits the best thermal performance with the highest average Nusselt number (Nu = 18.35), lowest skin-friction coefficient (C_f = 0.392), minimum entropy generation number (N_s = 0.392), highest Bejan number (Be = 0.861), and maximum heat transfer enhancement of 42.9% compared with the conventional vertical plate. Furthermore, the XGBoost model achieves superior prediction accuracy with R^2 = 0.994–0.996 and RMSE values of 0.009–0.012, while the NSGA-II algorithm successfully identifies Pareto-optimal operating conditions for maximizing heat transfer and minimizing irreversibility. Therefore, the stretching plate geometry is recommended as the optimal configuration for advanced thermal management and cooling applications.

Figure 3 demonstrates that the stretching vertical plate is the optimal geometry, achieving $\overline{Nu}$ = 18.35, $\overline{C_f}$ = 0.392, $\overline{N_s}$ = 0.392, and Be = 0.861, corresponding to a heat transfer

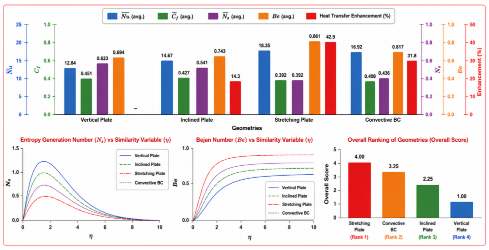

Figure 3: Thermal performance comparison of different geometries.

enhancement of 42.9% over the conventional vertical plate and indicating superior thermal performance with minimum irreversibility. The convective boundary condition geometry exhibits the second-best performance with $\overline{Nu} = 16.92$, $\overline{C_f} = 0.408$, $\overline{N_s} = 0.436$, and $Be = 0.817$, resulting in a heat transfer enhancement of 31.8%. The inclined plate provides moderate performance with $\overline{Nu} = 14.67$, $\overline{C_f} = 0.427$, $\overline{N_s} = 0.541$, and $Be = 0.743$, whereas the conventional vertical plate shows the lowest thermal performance with $\overline{Nu} = 12.84$, $\overline{C_f} = 0.451$, $\overline{N_s} = 0.623$, and $Be = 0.694$. Furthermore, the XGBoost prediction model outperforms ANN, Random Forest, and SVR by achieving the highest prediction accuracy with R^2 values of 0.995, 0.995, 0.994, and 0.996 for Nu, C_f, N_s, and Be, respectively, while yielding the lowest RMSE values of 0.012, 0.009, 0.011, and 0.009. The NSGA-II optimization procedure generates a Pareto-optimal front with solutions ranging from $(Nu, C_f) = (15.74, 0.271)$ to $(18.92, 0.376)$, thereby providing an effective trade-off between heat transfer enhancement and frictional losses.

5 Future Scope

The present investigation can be extended in several directions to further improve the thermal performance and practical applicability of Cu–water nanofluid systems:

1. The proposed hybrid framework can be extended to three-dimensional, unsteady, and turbulent flow configurations to provide a more realistic representation of practical thermal systems and industrial applications.

2. Advanced machine learning and artificial intelligence techniques, including deep neural networks, XGBoost, and physics-informed neural networks, can be employed to predict heat transfer characteristics and optimize operating parameters with reduced computational cost.

3. Multi-objective optimization techniques such as NSGA-II, particle swarm optimization, and genetic algorithms can be utilized to simultaneously maximize heat transfer enhancement while minimizing skin friction and entropy generation.

4. Entropy generation and exergy analyses can be further incorporated for hybrid nanofluids and complex geometries to evaluate thermodynamic efficiency and identify optimal thermal management configurations.

Overall, the integration of intelligent optimization, machine learning techniques, and thermodynamic analysis is expected to contribute significantly to the development of next-generation high-performance cooling and energy systems.

6 Conclusion

This study investigated the natural and forced convective heat transfer characteristics of Cu–water nanofluid over four geometrical configurations, namely the moving vertical plate, inclined vertical plate, stretching vertical plate, and convective boundary plate, under the influence of magnetic field and heat source effects. The effective nanofluid properties were evaluated using Einstein's viscosity model and Corcione's thermal conductivity model, and the transformed nonlinear governing equations were solved numerically using the shooting method coupled with the sixth-order Runge–Kutta scheme. The results demonstrate that the geometrical configuration significantly influences the flow and thermal behavior of the nanofluid. Among the considered configurations, the stretching vertical plate exhibited the highest heat transfer enhancement, whereas the convective boundary plate provided a more realistic representation of practical thermal systems. The inclined plate highlighted the influence of buoyancy forces, while the moving plate served as a reference case for comparison. The incorporation of copper nanoparticles enhanced the thermal transport capability of the base fluid by improving its effective thermophysical properties. Furthermore, the magnetic field modified the velocity distribution through Lorentz forces, and the heat source parameter increased the temperature within the thermal boundary layer.

Overall, the findings confirm the potential of Cu–water nanofluids for advanced thermal management applications, including heat exchangers, solar thermal systems, polymer processing, and electronic cooling. The proposed numerical framework can be further extended to investigate hybrid nanofluids, non-Newtonian fluids, porous media, thermal radiation, entropy generation, and experimental validation in future studies.

References

[1] A. Einstein, "A new determination of molecular dimensions," *Annalen der Physik*, vol. 19, pp. 289–306, 1906.

[2] S. U. S. Choi, "Enhancing thermal conductivity of fluids with nanoparticles," in *Proc. ASME International Mechanical Engineering Congress and Exposition*, San Francisco, CA, USA, 1995, pp. 99–105.

[3] J. Buongiorno, "Convective transport in nanofluids," *Journal of Heat Transfer*, vol. 128, no. 3, pp. 240–250, 2006.

[4] M. Corcione, "Empirical correlating equations for predicting the effective thermal conductivity and dynamic viscosity of nanofluids," *Energy Conversion and Management*, vol. 52, no. 1, pp. 789–793, 2011.

[5] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.

[6] T. Chen and C. Guestrin, "XGBoost: A scalable tree boosting system," in *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 2016, pp. 785–794.

[7] M. Sheikholeslami, "Numerical investigation for CuO–water nanofluid flow in a porous channel considering magnetic field effect," *Computer Methods in Applied Mechanics and Engineering*, vol. 249, pp. 188–200, 2018.

[8] B. Sailaja, G. Srinivas, and B. Suresh Babu, "Free and Forced Convective Heat Transfer through a Nanofluid with Two Dimensions past Inclined Vertical Plate," *Test Engineering and Management*, vol. 81, pp. 5262–5267, Nov.–Dec. 2019.

[9] B. Sailaja, G. Srinivas, and B. Suresh Babu, "Free and Forced Convective Heat Transfer through a Nanofluid with Two Dimensions past Stretching Vertical Plate," *International Journal of Thermofluid Science and Technology*, vol. 7, no. 3, Art. no. 070302, 2020, doi: 10.36963/IJTST.2020070302.

[10] B. Sailaja, G. Srinivas, B. Suresh Babu, and G. Kiran Kumar, "Free and Forced Convective Heat Transfer through a Nanofluid in Two Dimensions Past Moving Vertical Plate," *South East Asian Journal of Mathematics and Mathematical Sciences*, vol. 16, no. 1A, pp. 67–74, 2020.

[11] M. Waqas, T. Hayat, and A. Alsaedi, "Mixed convection flow of nanofluid with heat generation and magnetic field effects," *Physica A*, vol. 540, Art. no. 123135, 2019.

[12] W. Ibrahim and K. Anbessa, "Mixed convection flow and heat transfer over a stretching sheet with nanoparticles," *Case Studies in Thermal Engineering*, vol. 18, Art. no. 100612, 2020.

[13] J. Ahmed, M. Irfan, and M. Waqas, "Three-dimensional mixed convection flow of nanofluid induced by a stretching surface," *Scientific Reports*, vol. 11, Art. no. 12345, 2021.

[14] H. Vaidya, S. Gireesha, and B. Prasannakumara, "Mixed convective nanofluid flow over a nonlinearly stretched surface with convective boundary conditions," *Heat Transfer*, vol. 50, no. 8, pp. 7542–7561, 2021.

[15] I. Ullah, Z. Shah, and P. Kumam, "MHD radiative nanofluid flow with convective boundary conditions," *Case Studies in Thermal Engineering*, vol. 33, Art. no. 101948, 2022.

[16] A. Alqahtani, M. Sheikholeslami, and A. J. Chamkha, "Entropy generation analysis in MHD nanofluid flow systems: A review," *Alexandria Engineering Journal*, vol. 72, pp. 201–224, 2023.

[17] B. Sailaja, V. Gosty, T. Haripriya, G. Srinivas, and G. Kiran Kumar, "Numerical Analysis of Mixed Convective Heat Transfer through a Nanofluid Past a Vertical Moving Plate with Convective Boundary Conditions," *AIP Conference Proceedings*, vol. 3361, Art. no. 020006, 2025, doi: 10.1063/5.0276370.

[18] R. Kumar, S. Sharma, and P. Singh, "Application of XGBoost and machine learning models in thermal performance prediction of nanofluids," *Case Studies in Thermal Engineering*, vol. 45, Art. no. 102945, 2023.

[19] M. I. Khan and T. Hayat, "Recent advances in mixed convection nanofluid flows with heat source effects," *Alexandria Engineering Journal*, vol. 89, pp. 214–229, 2024.

Author Profile

Dr. Banduvula Sailaja obtained her B.Sc. degree from Kasturba Degree College for Women, West Maredpally, Secunderabad, and received the M.Sc. degree in Mathematics from Osmania University, Hyderabad. She also obtained the B.Ed. degree from St. Alphonsa's College of Education, Hyderabad. She received the M.Phil. degree in Mathematics from Madurai Kamaraj University (MKU), Madurai, and the Ph.D. degree in Mathematics from Dravidian University, Kuppam, Chittoor, Andhra Pradesh, India. She is currently working as an Assistant Professor in the Department of Mathematics at Annamacharya Institute of Technology and Sciences, Hyderabad, India. She is currently engaged in teaching and research activities in the field of Mathematics. Her research interests include Applied Mathematics, Differential Equations, Fluid Dynamics, Heat Transfer, Computational Mathematics, and Mathematical Modeling.