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Photon Trajectories in the Vicinity of Black Holes

Arnav Reddy Visavaram

Manthan International School, Hyderabad, India
Email: *arnavreddyv[at]gmail.com*

Abstract: ***In this paper, we investigate photon trajectories in the Schwarzschild metric, focusing on how the curvature of spacetime influences the paths of light near a non-rotating, spherically symmetric black hole. The Schwarzschild metric is a solution to Einstein's field equations that describes a non-rotating spherical black hole. The black hole used in this paper has a mass of one solar mass and a gravitational radius of three kilometers. The photon trajectories are described by a second-order differential equation derived from the geodesic equation for null particles. We solve this equation numerically, simulating the behavior of photons in the strong gravitational field near the black hole. From our simulations, we estimate the opening angle of a cone within which photons can escape to infinity. These results provide insights into the dynamics of photon propagation in curved spacetime and the boundary between capture by the black hole and escape from the black hole, which can be helpful in gravitational lensing, interpreting black hole images and understanding accretion disk emissions. The results confirm the existence of a photon sphere. While this research paper is limited to the Schwarzschild metric, an extension to rotating non-symmetric black holes and inclusion of quantum effects can provide even more insight into astrophysical environments that are more realistic. This work offers a foundation for further research into black hole imaging and relativistic light propagation.***

Keywords: Photon trajectories, Black holes, Schwarzschild metric, Photon sphere

1. Introduction

The study of photon trajectories in curved spacetime plays a critical role in understanding the behavior of light in the presence of strong gravitational fields, such as those near black holes. The Schwarzschild metric, describing the spacetime around a non-rotating, spherically symmetric black hole, provides an ideal framework for investigating the motion of massless particles, including photons. Given the profound effects of gravitational curvature, photon paths in such environments deviate significantly from those in flat spacetime, leading to phenomena such as gravitational lensing and the formation of event horizons. A key aspect of these dynamics is the delineation between photons that escape the gravitational pull and those that are captured by the black hole. In this paper, we focus on the photon trajectories near a Schwarzschild black hole by formulating their motion through the geodesic equation for null particles. This leads to a second-order differential equation that governs the paths of photons in the Schwarzschild metric. Solving this equation numerically allows us to explore a range of initial conditions and identify key features of photon dynamics, such as the critical angle at which photons either escape to infinity or spiral inward to the event horizon. The research confines itself to black holes without angular momentum and those that exhibit spherical symmetry, which can be adequately characterized by the Schwarzschild metric, thus excluding their rotation, charge, and quantum effects. These idealizations enable us to deepen our understanding of the basic behavior of light paths in the bending of space-time caused by gravity. One of the primary goals of this investigation is to estimate the opening angle of a "cone of escape"—the angular region within which photons can avoid being trapped by the black hole and instead propagate outward to infinity. This concept is important for understanding the observable characteristics of light from accretion disks, jets, or other astrophysical processes occurring near black holes. Our work contributes to the broader study of photon dynamics in strong gravitational fields, offering both theoretical insights and numerical results that illuminate the interplay between gravity and light.

2. Literature Survey

Schwarzschild derived the first exact solution to Einstein's field equations in 1916, describing the spacetime around a non-rotating, spherically symmetric mass. This solution remains the standard framework for analysing photon paths near a black hole and is the metric used throughout this paper.

Chandrasekhar later worked out the geodesic equations for photons in this metric in detail, and showed that the effective potential governing radial motion has a single unstable circular orbit at r = 1.5 r_s. This is the photon sphere referenced in Section 4: a photon displaced even slightly from this orbit will either escape to infinity or fall into the black hole, the same instability captured numerically in Figure 1 of this paper.

Darwin later investigated light deflection in strong gravitational fields and provided some of the earliest quantitative descriptions of photon trajectories near compact objects. This bending is the basis of gravitational lensing, and it is the same effect that, taken to its strong-field limit, produces the escape cone calculated in this paper.

Bardeen, Press, and Teukolsky extended this analysis to rotating black holes described by the Kerr metric, where frame dragging breaks the spherical symmetry assumed here and makes the photon sphere angle-dependent. Cunningham and Bardeen used this framework to model the appearance of a star orbiting close to an extremal Kerr black hole, one of the first attempts to predict what such an object would actually look like to a distant observer.

This question of appearance is what connects the theory to observation. Gralla, Holz, and Wald related the photon ring structure predicted by these geodesic calculations to the black hole shadow imaged by the Event Horizon Telescope, giving the escape-cone and photon-sphere results computed here a direct observational counterpart.

3. Methods

To describe the photon trajectory in a Schwarzschild metric, we use polar coordinates, where r ≥ 0 is the radial distance and φ ∈ [0, 2π] is the angular coordinate. These polar coordinates are related to Cartesian coordinates through the following transformations:

$$x = r\cos\varphi,\ y = r\sin\varphi$$

The photon trajectory is governed by a second-order differential equation, given by

$$u'' = 3u^2 - u,$$

Where $u = 0.5\frac{r_s}{r}$, with $r_s \approx 3 \times 10^5 \left(\frac{M}{M_\odot}\right)$ cm is the Schwarzschild radius of a black hole of mass M, and u″ represents the second derivative of u with respect to φ.

To numerically calculate the photon trajectory, we need the following initial conditions:

- The black hole with mass M,
- The initial coordinates of the photon (r_0, φ_0),
- The initial angle of motion of the photon, $\alpha_0 \in [0, 2\pi]$, which defines the initial direction of photon propagation.

3.1 Steps for Calculating the Photon Trajectory:

1) **Initialization:** Select the black hole mass M, and set initial conditions for the photon trajectory: r0, φ0, α0. Choose a small step size Δφ, which improves the accuracy of the numerical calculation.

2) **Initial Calculations:** Set $i = 0$, and compute the initial value of $u_0 = 0.5\frac{r_s}{r_0}$, along with the first derivative u_0', determined by the initial angle α_0:

$$u_0' = -0.5\left(\frac{r_s}{r_0}\right)\frac{1 + \tan\alpha_0 \tan\varphi_0}{\tan\alpha_0 - \tan\varphi_0}$$

3) **First Step:** Calculate $u_1 = u_0 + u_0'\Delta\varphi$ and the corresponding radial distance $r_1 = 0.5\, r_s/u_1$.

4) **Iteration Setup:** Set $i = 1$.

5) **Recursion:** For each subsequent step, calculate the second derivative $u_i'' = \boldsymbol{3u_i^2 - u_i}$

6) **Update Photon Position:** Compute the next value of u using the numerical scheme:

$$u_{i+1} = 2u_i - u_{i-1} + u_i''(\Delta\varphi)^2$$

and calculate the corresponding radial distance as $r_{i+1} = 0.5\, r_s/u_{i+1}$

7) Iterate: Increment the index $i = i + 1$, and repeat steps (5) and (6) until the photon either escapes or falls into the black hole.

The result of this iterative scheme is a set of points (r_i, φ_i), which collectively form the photon's trajectory in polar coordinates. These trajectories allow us to study the behavior of photons near the black hole.

3.2 Software and Numerical Methods

The numerical calculations in this study were implemented using MATLAB(version r2024a). The complexity of photon motion near black holes, governed by general relativity, necessitates the use of high-precision numerical methods to accurately simulate these trajectories. This combination of analytical derivation and numerical simulation provides crucial insights into photon dynamics in strong gravitational fields.

Our approach involves the development of efficient algorithms that balance accuracy and computational efficiency. These methods are essential for simulating and visualizing the behavior of photons near black holes, contributing to the broader understanding of astrophysical phenomena in regions of extreme gravity.

4. Results

This paper's plots help visualize the photon trajectory in the vicinity of black holes. The results also show that the critical angle, where the photon collides with the black hole, increases with the increase in distance from the black hole. In this case, the size and strength of the black hole's gravity source significantly affect the actual behavior and characteristics of the photon particles, which adds to the understanding of the processes occurring in the vicinity of black holes.

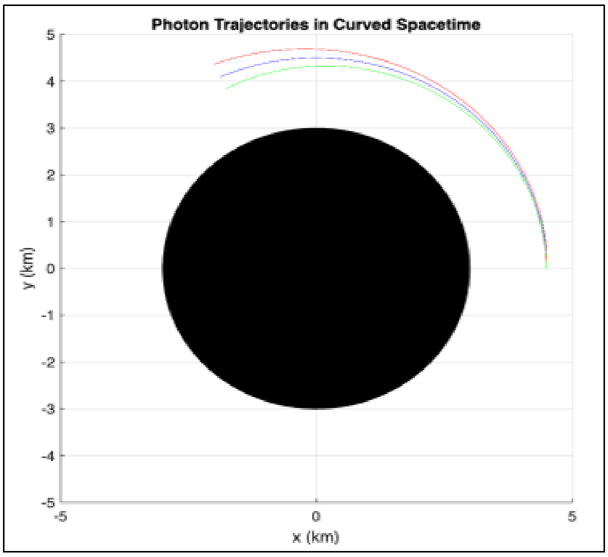

Figure 1: This graph illustrates three distinct photon trajectories. The blue trajectory represents a circular path at an angle of π/2, while the red trajectory, angled at 1 degree above π/2, escapes to infinity. In contrast, the green trajectory, at 1 degree below π/2, ultimately collides with the black hole. The black hole, depicted as a black circle, has a mass of 1 solar mass, corresponding to a gravitational radius of 3 kilometers.

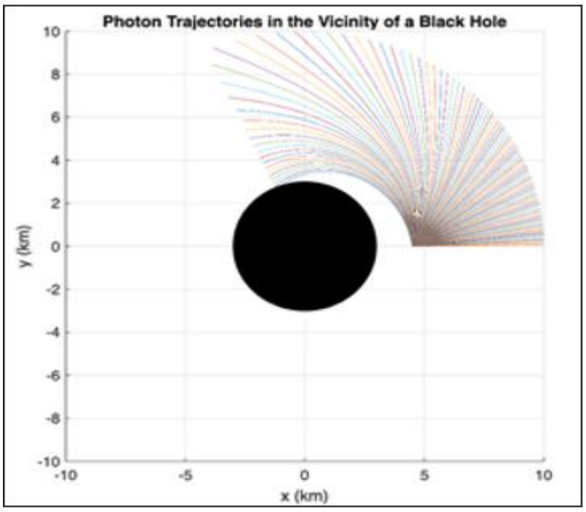

Figure 2: This graph displays various angles of photon trajectories, illustrating the conditions under which a photon ultimately collides with the black hole. The collision occurs at an angle of 98 degrees. The black hole, represented by the black circle, has a mass of 1 solar mass, corresponding to a gravitational radius of 3 kilometers.

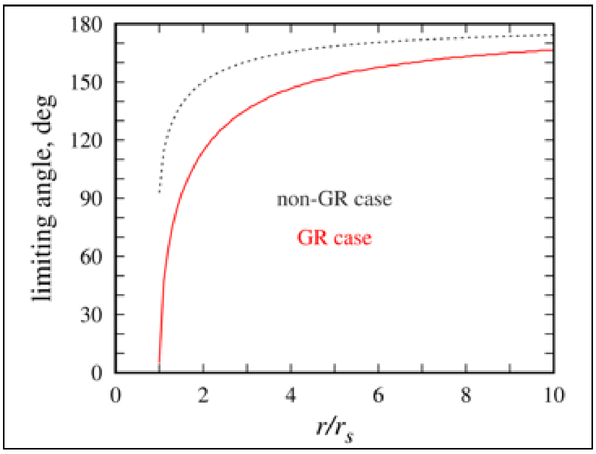

Figure 3: This graph illustrates the critical angles in both curved and flat spacetime. As the distance from the photon sphere decreases, the critical angles for both spacetime geometries converge, indicating a similarity in photon behavior under strong gravitational influence. The black hole represented in this analysis has a mass of 1 solar mass, corresponding to a gravitational radius of 3 kilometers.

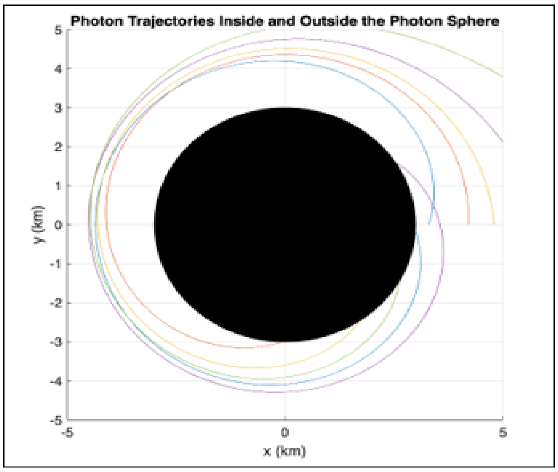

Figure 4: Photon trajectories for emission radii of 1.1, 1.4, 1.6, 2.0, and 3.0r_s. The corresponding critical angles are approximately 73.5°, 86.5°, 94.0°, 107.0°, and 129.5°, respectively. The black hole, represented by the black circle, has a mass of 1 solar mass, corresponding to a gravitational radius of 3 kilometers.

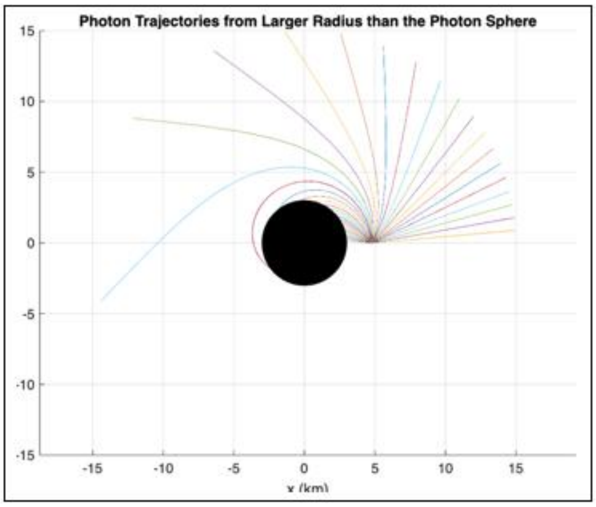

Figure 5: These trajectories illustrate the behavior of photons emitted from outside the photon sphere. The black hole, represented by the black circle, has a mass of 1 solar mass, corresponding to a gravitational radius of 3 kilometers.

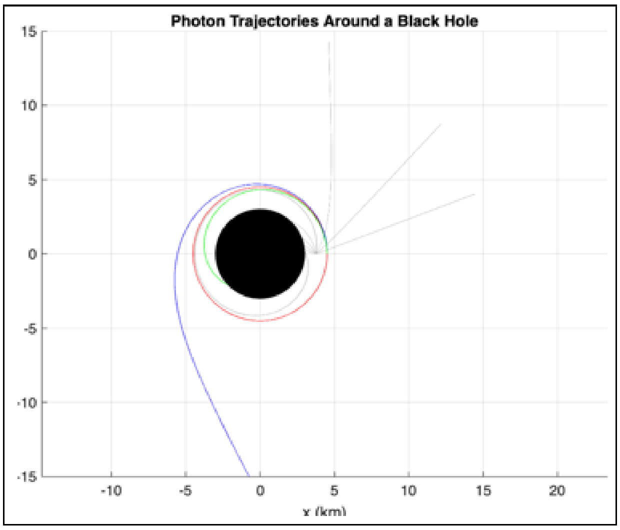

Figure 6: Photon trajectories near the photon sphere of a 1-solar-mass Schwarzschild black hole (r_s = 3 km). The red trajectory represents an unstable circular orbit, the blue trajectory escapes to infinity, and the green trajectory is captured after spiraling around the black hole. Grey trajectories emitted within the photon sphere are also captured.

5. Discussion

The findings confirm the existence of the photon sphere—a region around the black hole where photons can orbit in circular paths. For a Schwarzschild black hole, this occurs at a radius of $1.5r_s$, where r_s is the Schwarzschild radius. However, these orbits are not stable, and any perturbation will cause photons to either escape or be captured by the black hole. The study of photon trajectories near black holes has significant implications for both science and technology. These include fostering greater investment and international collaboration in astronomical research, as well as enhancing educational programs in astrophysics and general relativity. Understanding photon behavior near black holes also helps in the interpretation of observations from telescopes like the Event Horizon Telescope, which has captured images of black hole shadows.

This research builds on a solid theoretical foundation, incorporating advanced computational methods, numerical simulations, and data analysis. However, the study is limited by certain assumptions, such as the idealized nature of the Schwarzschild black hole, which excludes rotation or charge, and the neglect of quantum effects. Future work should focus on more realistic astrophysical scenarios by considering rotating black holes (Kerr metric), incorporating quantum effects, and improving simulations to better reflect the complex environments near black holes. Such research would provide deeper insights into black hole physics and the nature of gravity in extreme conditions.

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