Modified Hyperbolic Sombor and Modified Euler Hyperbolic Sombor Indices of Certain Chemical Structures

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Abstract: ***In this paper, we introduce the modified hyperbolic Sombor and the modified Euler hyperbolic Sombor indices of a graph. We compute these newly defined the hyperbolic Sombor indices for certain chemical structures.***

Keywords: modified hyperbolic Sombor index, modified Euler hyperbolic Sombor index, chemical structure

1. Introduction

The simple graphs which are finite, undirected, connected graphs without loops and multiple edges are considered. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u.

In [1], the hyperbolic Sombor index of a graph G is defined as

$$HSO(G)=\sum_{uv\in E(G)}\frac{\sqrt{d_G(u)^2+d_G(v)^2}}{\min\{d_G(u),d_G(v)\}}$$

We define the modified hyperbolic Sombor index of a graph G as

$${}^{m}HSO(G)=\sum_{uv\in E(G)}\frac{\min\{d_G(u),d_G(v)\}}{\sqrt{d_G(u)^2+d_G(v)^2}}$$

In [2], the Euler hyperbolic Sombor index of a graph G is defined as

$$EHSO(G)=\sum_{uv\in E(G)}\frac{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}{\min\{d_G(u),d_G(v)\}}$$

Recently, some hyperbolic Sombor indices were studied in [3, 4, 5, 6, 7].

We define the modified Euler hyperbolic Sombor index of a graph G as

$${}^{m}EHSO(G)$$
$$=\sum_{uv\in E(G)}\frac{\min\{d_G(u),d_G(v)\}}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}$$

In this research, we compute the modified hyperbolic Sombor and modified Euler hyperbolic Sombor indices for certain chemical structures.

2. Armchair Polyhex Nanotubes

Carbon polyhex nanotubes exist in nature with remarkable stability and possess very interesting electrical, thermal and mechanical properties. The molecular graph of armchair polyhex nanotube $TUAC_6$ [p, q] is shown in the below graph.

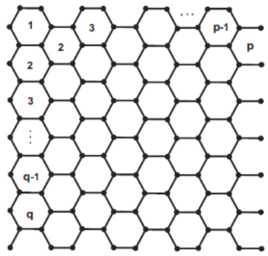

Figure 1

The graphs of armchair polyhex nanotubes have $2p(q+1)$ vertices and $3pq + 2p$ edges are shown in the above graph. Let $G= TUAC_6[p, q]$.

We obtain that $\{d(u), d(v): uv \in E(G)\}$ has three edge set partitions.

$d(u), d(v) \setminus uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	p	$2p$	$3pq - p$

Theorem 1. The hyperbolic Sombor index of $TUAC_6$ [p, q] is given by

$$HSO(G)=3\sqrt{2}pq+\sqrt{13}p.$$

Proof: Applying definition and edge partition of $TUAC_6$ [p, q], we conclude

$$HSO(G)=\sum_{uv\in E(G)}\frac{\sqrt{d_G(u)^2+d_G(v)^2}}{\min\{d_G(u),d_G(v)\}}$$
$$=p\frac{\sqrt{2^2+2^2}}{2}+2p\frac{\sqrt{2^2+3^2}}{2}+(3pq-p)\frac{\sqrt{3^2+3^2}}{3}.$$

By solving the above equation, we get the desired result.

Theorem 2. The modified hyperbolic Sombor index of $TUAC_6\,[p, q]$ is

$$^{m}HSO(G)=\frac{3}{\sqrt{2}}pq+\frac{1}{\sqrt{13}}p.$$

Proof: Applying definition and edge partition of $TUAC_6\,[p, q]$, we conclude

$$^{m}HSO(G)=\sum_{uv\in E(G)}\frac{\min\{d_G(u),d_G(v)\}}{\sqrt{d_G(u)^2+d_G(v)^2}}$$

$$=p\frac{2}{\sqrt{2^2+2^2}}+2p\frac{2}{\sqrt{2^2+3^2}}+(3pq-p)\frac{3}{\sqrt{3^2+3^2}}.$$

By solving the above equation, we get the desired result.

Theorem 3. The Euler hyperbolic Sombor index of $TUAC_6\,[p, q]$ is

$$EHSO(G)=3\sqrt{3}pq+\sqrt{19}p.$$

Proof: Applying definition and edge partition of $TUAC_6\,[p, q]$, we conclude

$$EHSO(G)$$

$$=\sum_{uv\in E(G)}\frac{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}{\min\{d_G(u),d_G(v)\}}$$

$$=p\frac{\sqrt{2^2+2^2+2\times 2}}{2}+2p\frac{\sqrt{2^2+3^2+2\times 3}}{2}$$

$$+(3pq-p)\frac{\sqrt{3^2+3^2+3\times 3}}{3}.$$

By solving the above equation, we get the desired result.

Theorem 4. The modified Euler hyperbolic Sombor of $TUAC_6\,[p, q]$ is

$$^{m}EHSO(G)=\frac{3}{\sqrt{3}}pq+\frac{4}{\sqrt{19}}p.$$

Proof: Applying definition and edge partition of $TUAC_6\,[p, q]$, we conclude

$$^{m}EHSO(G)$$

$$=\sum_{uv\in E(G)}\frac{\min\{d_G(u),d_G(v)\}}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}$$

$$=p\frac{2}{\sqrt{2^2+2^2+2\times 2}}+2p\frac{2}{\sqrt{2^2+3^2+2\times 3}}$$

$$+(3pq-p)\frac{3}{\sqrt{3^2+3^2+3\times 3}}.$$

By solving the above equation, we get the desired result.

3. ZigZag Polyhex Nanotubes

The molecular graph of zigzag polyhex nanotube $TUZC_6\,[p, q]$ is depicted in below graph.

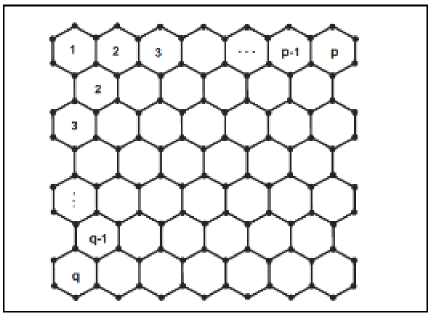

Figure 2

The graphs of zigzag polyhex nanotubes have $2p(q+1)$ vertices and $3pq + 2p$ edges are shown in the above graph. Let $G = TUZC_6[p, q]$.

We obtain that $\{d(u), d(v): uv \in E(G)\}$ has three edge set partitions.

$d(u), d(v) \setminus uv \in E(G)$	(2, 3)	(3, 3)
Number of edges	$4p$	$3pq - 2p$

Theorem 5. The hyperbolic Sombor index of $TUZC_6\,[p, q]$ is given by

$$HSO(G)=3\sqrt{2}pq+2\sqrt{13}p-2\sqrt{2}p.$$

Proof: Applying definition and edge partition of $TUZC_6\,[p, q]$, we conclude

$$HSO(G)=\sum_{uv\in E(G)}\frac{\sqrt{d_G(u)^2+d_G(v)^2}}{\min\{d_G(u),d_G(v)\}}$$

$$=4p\frac{\sqrt{2^2+3^2}}{2}+(3pq-2p)\frac{\sqrt{3^2+3^2}}{3}.$$

By solving the above equation, we get the desired result.

Theorem 6. The modified hyperbolic Sombor index of $TUZC_6\,[p, q]$ is

$$^{m}HSO(G)=\frac{3}{\sqrt{2}}pq+\frac{8}{\sqrt{13}}p-\frac{2}{\sqrt{2}}p.$$

Proof: Applying definition and edge partition of $TUZC_6\,[p, q]$, we conclude

$$^{m}HSO(G)=\sum_{uv\in E(G)}\frac{\min\{d_G(u),d_G(v)\}}{\sqrt{d_G(u)^2+d_G(v)^2}}$$

$$=4p\frac{2}{\sqrt{2^2+3^2}}+(3pq-2p)\frac{3}{\sqrt{3^2+3^2}}.$$

By solving the above equation, we get the desired result.

Theorem 7. The Euler hyperbolic Sombor index of $TUZC_6$ $[p, q]$ is

$$EHSO(G) = 3\sqrt{3}pq + 2\sqrt{19}p - 2\sqrt{3}p.$$

Proof: Applying definition and edge partition of $TUZC_6$ $[p, q]$, we conclude

$$EHSO(G)$$
$$= \sum_{uv\in E(G)} \frac{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}{\min\{d_G(u), d_G(v)\}}$$
$$= 4p\frac{\sqrt{2^2+3^2+2\times 3}}{2} + (3pq - 2p)\frac{\sqrt{3^2+3^2+3\times 3}}{3}.$$

By solving the above equation, we get the desired result.

Theorem 8. The modified Euler hyperbolic Sombor of $TUZC_6$ $[p, q]$ is

$$^{m}EHSO(G) = \frac{3}{\sqrt{3}}pq + \frac{8}{\sqrt{19}}p - \frac{2}{\sqrt{3}}p.$$

Proof: Applying definition and edge partition of $TUZC_6$ $[p, q]$, we conclude

$$^{m}EHSO(G)$$
$$= \sum_{uv\in E(G)} \frac{\min\{d_G(u), d_G(v)\}}{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}$$
$$= 4p\frac{2}{\sqrt{2^2+3^2+2\times 3}} + (3pq - 2p)\frac{3}{\sqrt{3^2+3^2+3\times 3}}.$$

By solving the above equation, we get the desired result.

4. Carbon Nanocone Networks

The molecular graph of pentagonal nanocone network CNC_5 $[n]$ is depicted in below graph.

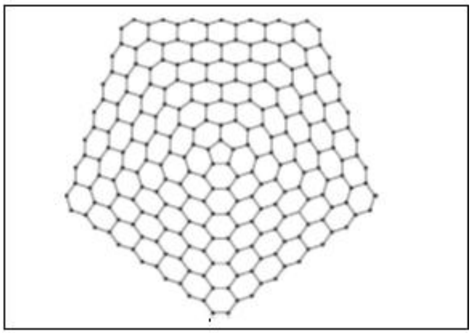

Figure 3

The graphs of pentagonal nanocone networks have $5(n+1)^2$ vertices and $\frac{15}{2}n^2 + \frac{25}{2}n + 5$ edges. Let $G = CNC_5[n]$.

We obtain that $\{d(u), d(v): uv \in E(G)\}$ has three edge set partitions.

$d(u), d(v) \backslash uv\in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	5	$10n$	$\frac{15}{2}n^2 + \frac{5}{2}n$

Theorem 9. The hyperbolic Sombor index of CNC_5 $[n]$ is given by

$$HSO(G) = \frac{15}{\sqrt{2}}n^2 + 5\sqrt{13}n + \frac{5}{\sqrt{2}}n + 5\sqrt{2}.$$

Proof: Applying definition and edge partition of CNC_5 $[n]$, we conclude

$$HSO(G) = \sum_{uv\in E(G)} \frac{\sqrt{d_G(u)^2 + d_G(v)^2}}{\min\{d_G(u), d_G(v)\}}$$

$$= 5\frac{\sqrt{2^2+2^2}}{2} + 10n\frac{\sqrt{2^2+3^2}}{2}$$
$$+ \left(\frac{15}{2}n^2 + \frac{5}{2}n\right)\frac{\sqrt{3^2+3^2}}{3}.$$

By solving the above equation, we get the desired result.

Theorem 10. The modified hyperbolic Sombor index of CNC_5 $[n]$ is

$$^{m}HSO(G) = \frac{15}{2\sqrt{2}}n^2 + \frac{20}{\sqrt{13}}n + \frac{5}{2\sqrt{2}}n + \frac{5}{\sqrt{2}}.$$

Proof: Applying definition and edge partition of CNC_5 $[n]$, we conclude

$$^{m}HSO(G) = \sum_{uv\in E(G)} \frac{\min\{d_G(u), d_G(v)\}}{\sqrt{d_G(u)^2 + d_G(v)^2}}$$
$$= 5\frac{2}{\sqrt{2^2+2^2}} + 10n\frac{2}{\sqrt{2^2+3^2}}$$
$$+ \left(\frac{15}{2}n^2 + \frac{5}{2}n\right)\frac{3}{\sqrt{3^2+3^2}}.$$

By solving the above equation, we get the desired result.

Theorem 11. The Euler hyperbolic Sombor index of CNC_5 $[n]$ is

$$EHSO(G) = \frac{15\sqrt{3}}{2}n^2 + 5\sqrt{19}n + \frac{5\sqrt{3}}{2}n + 5\sqrt{3}.$$

Proof: Applying definition and edge partition of CNC_5 $[n]$, we conclude

$$EHSO(G)$$
$$= \sum_{uv\in E(G)} \frac{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}{\min\{d_G(u), d_G(v)\}}$$

$$= 5\frac{\sqrt{2^2+2^2+2\times 2}}{2}+10n\frac{\sqrt{2^2+3^2+2\times 3}}{2}$$
$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\frac{\sqrt{3^2+3^2+3\times 3}}{3}.$$

By solving the above equation, we get the desired result.

Theorem 12. The modified Euler hyperbolic Sombor of $CNC_5[n]$ is

$$^{m}EHSO(G)=\frac{15}{2\sqrt{3}}n^2+\frac{20}{\sqrt{19}}n+\frac{5}{2\sqrt{3}}n+\frac{5}{\sqrt{3}}.$$

Proof: Applying definition and edge partition of $CNC_5[n]$, we conclude

$$^{m}EHSO(G)$$
$$=\sum_{uv\in E(G)}\frac{\min\{d_G(u),d_G(v)\}}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}$$
$$=5\frac{2}{\sqrt{2^2+2^2+2\times 2}}+10n\frac{2}{\sqrt{2^2+3^2+2\times 3}}$$
$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\frac{3}{\sqrt{3^2+3^2+3\times 3}}.$$

By solving the above equation, we get the desired result.

5. Conclusion

We have introduced the modified hyperbolic Sombor and modified Euler hyperbolic Sombor indices of a graph. Also we have determined these newly defined the modified hyperbolic Sombor indices of armchair polyhex nanotubes, zigzag polyhex nanotubes and carbon nanocone networks.

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