International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2025: 7.089

An Inventory Cost Minimization Model with Variable Demand Rate

Sandeep Kumar

Assistant Professor, School of Sciences, IIMT University, Meerut, U.P., Bharat
Email: *sandeepmaths87[at]gmail.com*

Abstract: ***This paper develops an inventory model for non-deteriorating items under a time-dependent demand pattern. The demand rate varies over time and directly influences the inventory level throughout the planning period. The model assumes that shortages are not permitted, ensuring continuous availability of products to meet customer requirements. Ordering cost and holding cost are incorporated into the analysis to evaluate the overall inventory cost. Mathematical expressions are formulated to determine the optimal inventory policy that minimizes the total cost of the system. Numerical examples and graphical representations are presented to validate the model and examine the effects of key parameters on the total cost. The results demonstrate that proper control of inventory parameters can significantly reduce operating costs and improve the efficiency and profitability of inventory management systems.***

Keywords: Inventory, demand, holding cost, ordering cost

1. Introduction

Inventory management is considered one of the most important functions in modern business and industrial operations. The success of an organization largely depends on how effectively it manages its inventory investments and resources. Proper inventory control helps maintain an adequate supply of goods, minimizes operational costs, and improves customer satisfaction. Effective management encourages strategic planning, efficient resource utilization, and better coordination among different departments. It also promotes innovation, productivity, and adaptability in response to changing market conditions. Furthermore, sound financial practices and continuous monitoring of inventory levels enhance organizational stability and profitability. By implementing efficient inventory management policies, businesses can reduce risks, improve decision-making, seize emerging opportunities, and achieve sustainable growth in an increasingly competitive and dynamic business environment. Based on the recent studies listed above, significant progress has been made in the development of inventory models that incorporate time-varying demand and dynamic cost structures. Gupta (2026) presented an inventory system with real-time demand variation, emphasizing the importance of continuously updating inventory decisions according to changing customer requirements. The study demonstrated that real-time demand information improves inventory accuracy and reduces excess stock, thereby enhancing overall operational efficiency.

Tandon (2026) extended this concept by integrating dynamic demand into smart supply chain inventory systems. The research highlighted the role of advanced technologies, such as data analytics and automated monitoring, in predicting demand fluctuations and optimizing replenishment policies. The findings revealed that adaptive inventory strategies contribute significantly to reducing holding costs and stockout risks. Mehta (2026) focused on inventory optimization under time-dependent cost structures. The author considered varying holding, ordering, and operational costs over time and developed optimization techniques to determine cost-effective replenishment schedules. The study concluded that ignoring time-dependent costs may lead to suboptimal inventory decisions and increased total expenditure.

Ali (2026) investigated multi-echelon inventory systems under time-varying demand conditions. The research analyzed inventory coordination across different levels of the supply chain and demonstrated that synchronized replenishment policies improve service levels while minimizing total system costs. The study further emphasized the importance of information sharing among supply chain participants.

Das (2026) proposed an advanced inventory model aimed at minimizing total inventory costs under dynamic demand conditions. The model incorporated multiple cost components and demand variations, providing a comprehensive framework for inventory control. Numerical analyses confirmed the effectiveness of the proposed approach in achieving lower total costs and better inventory utilization. Finally, Amiri (2026) examined non-stationary inventory control problems where demand patterns change continuously over time. The study developed mathematical models and control policies capable of adapting to uncertain and evolving market conditions. The results demonstrated that flexible inventory policies outperform traditional static approaches in dynamic environments.

Recent advancements in inventory management have increasingly focused on integrating dynamic demand patterns, artificial intelligence, sustainability considerations, and data-driven decision-making into inventory optimization models. These developments aim to improve supply chain efficiency and enhance the responsiveness of inventory systems to changing market conditions. Fatima (2026) proposed a data-driven inventory optimization framework that utilizes large-scale data analytics to capture dynamic demand behavior. The study demonstrated that real-time data processing and predictive analytics significantly improve inventory accuracy and replenishment decisions. The proposed approach effectively reduces inventory-related costs while maintaining a high service level under uncertain demand environments. Zhang (2025) introduced an artificial intelligence-based inventory model designed to handle time-dependent demand fluctuations. By employing machine learning algorithms, the model was capable of forecasting demand trends and adapting inventory policies accordingly. The results indicated that AI-driven inventory systems outperform conventional methods in terms of forecasting precision, stock availability, and cost minimization.

Similarly, Chen (2025) developed a smart inventory management model under dynamic demand conditions. The study incorporated intelligent monitoring and automated decision-support mechanisms to improve inventory control. The proposed system enabled rapid responses to market changes and contributed to reducing both inventory shortages and excess stock levels. Kumar (2025) investigated sustainable inventory management in the presence of time-varying demand. The research integrated environmental considerations with traditional inventory objectives by accounting for resource utilization and waste reduction. The findings revealed that sustainable inventory policies can achieve economic efficiency while simultaneously supporting environmental sustainability goals.

Singh (2025) examined inventory optimization problems involving stochastic time-dependent demand. The study developed mathematical models that explicitly account for demand uncertainty and variability over time. Numerical experiments showed that incorporating stochastic demand characteristics leads to more robust inventory policies and improved operational performance. Li (2025) extended inventory modelling by incorporating carbon emission considerations into inventory decision-making. The proposed carbon-aware inventory model addressed environmental regulations and sustainability objectives while responding to time-dependent demand patterns. The study concluded that balancing inventory costs and carbon emissions can significantly contribute to sustainable supply chain management.

These studies demonstrate a clear transition from traditional inventory models toward intelligent, sustainable, and adaptive inventory systems. The incorporation of artificial intelligence, data analytics, stochastic demand modelling, and environmental considerations has significantly enhanced inventory optimization techniques. However, there remains a need for comprehensive models that simultaneously integrate dynamic demand, cost optimization, sustainability measures, and uncertainty factors. Such integrated approaches would provide more realistic and effective solutions for modern inventory management and supply chain operations.

2. Assumptions and List of Symbols

S. No.	Symbol	Description
1	K	Fixed ordering/setup cost incurred for each replenishment.
2	h	Inventory holding cost per unit per unit time.
3	$R(t)$	Time-varying demand rate given by $R(t) = \begin{cases} \alpha t^2, & 0 < t < t_c, \\ \alpha t, & t_c \le t \le L. \end{cases}$
4	α	Demand coefficient.
5	L	Length of one inventory cycle.
6	S	Initial stock quantity available at the start of the cycle.
7	$X(t)$	Inventory level at time t.
8	$F(t_c, L)$	Total cost function over a cycle.
9	D_L	Total demand during the cycle period.
10	C_o	Ordering cost associated with one replenishment.
11	C_h	Total carrying (holding) cost during the cycle.
12	C_T	Overall inventory cost per cycle.

3. Mathematical Formulation of the Inventory Model

Consider an inventory system operating over an infinite planning horizon in which the demand rate varies with time. The inventory items are assumed to be non-deteriorating throughout the cycle. Let $X(t)$denote the inventory level at time t.

Inventory accumulation starts at $t = 0$, and both demand fulfillment and inventory control activities are initiated simultaneously. During the replenishment cycle, inventory increases initially and then decreases due to customer demand. The stock level becomes zero at the end of the cycle, i.e., at time L.

The inventory dynamics are governed by the following differential equations:

$$\frac{dX(t)}{dt} = \alpha t^2, 0 \le t \le t_c \quad (1)$$

and

$$\frac{dX(t)}{dt} = \alpha t + X(t), t_c \le t \le L \quad (2)$$

subject to the boundary conditions

$$X(0) = 0, X(L) = S.$$

Paper ID: SR26623162752 DOI: https://dx.doi.org/10.21275/SR26623162752

3.1 Solution of the Inventory Equations

Integrating Equation (1), we obtain

$$X(t) = \frac{\alpha t^3}{3}. \quad (3)$$

Similarly, the solution of Equation (2) is

$$X(t) = \left(\frac{\alpha t^2}{2} + C\right)\left(1 + t + \frac{t^2}{2}\right). \quad (4)$$

Applying the boundary condition $X(L) = S$, Equation (4) becomes

$$X(t) = -\frac{\alpha t^2}{2} + S\left(1 - L + \frac{L^2}{2}\right). \quad (5)$$

Using the continuity condition at $t = t_c$, the initial inventory quantity is obtained as

$$S = \frac{\alpha}{6} t_c^2 (3 + 2t_c)(2 + 2L + L^2). \quad (6)$$

3.2 Total Demand During a Cycle

The aggregate demand over the interval $[0, L]$is

$$TD = \int_0^L R(t)\, dt = \frac{\alpha}{6}(3L^2 - t_c^2). \quad (7)$$

3.3 Total Inventory Held During the Cycle

The cumulative inventory over the cycle is

$$\int_0^L X(t)\, dt = \frac{\alpha}{12}[(t_c^2 - L^2)(t_c^3 - 2t_c^2 + L + t_c^2 L) - L t_c (L^2 - t_c^2) + 4L t_c^2 (1 - L)]. \quad (8)$$

3.4 Cost Analysis

3.4 (a) Ordering Cost

The ordering cost incurred in a cycle is

$$OC = KS = \frac{K\alpha}{6} t_c^2 (3 + 2t_c)(2 + 2L + L^2). \quad (9)$$

3.4 (b) Holding Cost

The holding cost is given by

$$HC = h \int_0^L X(t)\, dt,$$

which yields

$$HC = \frac{h\alpha}{12}[(t_c^2 - L^2)(t_c^3 - 2t_c^2 + L + t_c^2 L) - L t_c (L^2 - t_c^2) + 4L t_c^2 (1 - L)]. \quad (10)$$

3.5 Average Total Cost Function

The average cost per unit time is defined as

$$F(t_c, L) = \frac{1}{L}(OC + HC).$$

Therefore,

$$F(t_c, L) = \frac{K\alpha}{6L} t_c^2 (3 + 2t_c)(2 + 2L + L^2) + \frac{h\alpha}{12L}[(t_c^2 - L^2)(t_c^3 - 2t_c^2 + L + t_c^2 L) - L t_c (L^2 - t_c^2) + 4L t_c^2 (1 - L)]. \quad (11)$$

3.6 Optimality Conditions

To determine the optimal cycle parameters, the first-order conditions are

$$\frac{\partial F(t_c, L)}{\partial t_c} = 0, \frac{\partial F(t_c, L)}{\partial L} = 0. \quad (12)$$

The corresponding optimal values are denoted by

$$t_c = t_c^*, L = L^*.$$

For verification of optimality, the second-order derivatives

$$\frac{\partial^2 F}{\partial t_c^2}, \frac{\partial^2 F}{\partial L^2}, \frac{\partial^2 F}{\partial t_c \partial L}$$

are evaluated at (t_c^*, L^*).
The cost function attains an optimal minimum whenever

$$\frac{\partial^2 F}{\partial t_c^2} \cdot \frac{\partial^2 F}{\partial L^2} > \left(\frac{\partial^2 F}{\partial t_c \partial L}\right)^2 \quad (13)$$

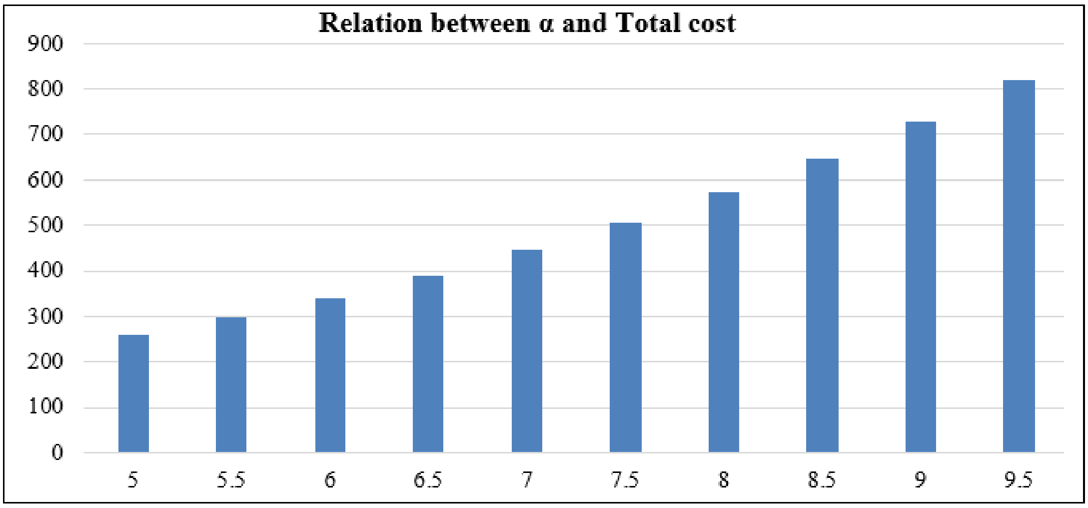
This condition ensures the positive definiteness of the Hessian matrix and confirms the existence of an optimal inventory policy.

4. Numerical Example with Graph

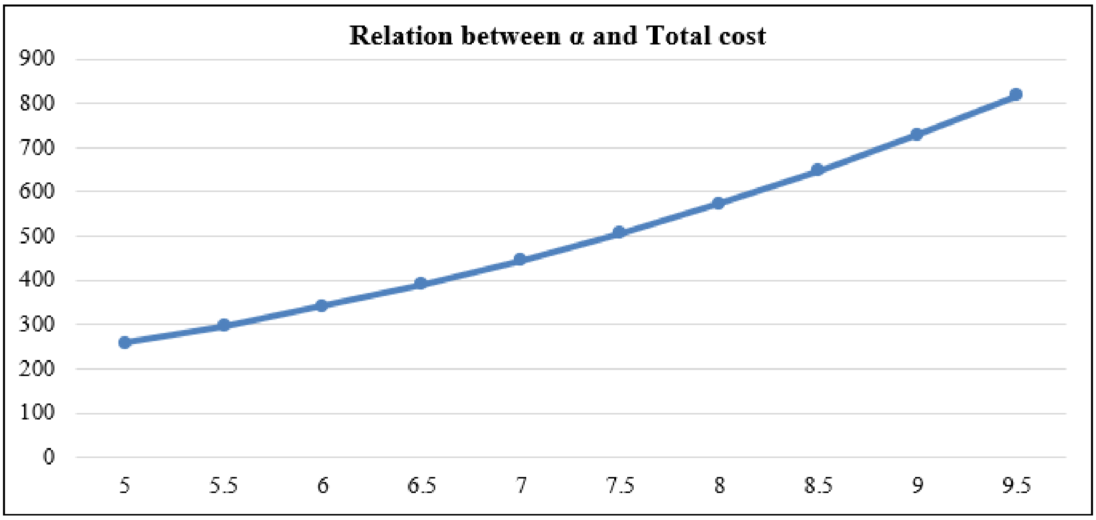
Table 1: Effect of Demand Parameter (α) on Total Cost

α	t_c	L	Total Cost
90	2.0	5.0	218.45
110	2.0	5.0	242.18
130	2.0	5.0	267.72
150	2.0	5.0	291.54
170	2.0	5.0	319.86
190	2.0	5.0	344.28
210	2.0	5.0	371.95
230	2.0	5.0	407.62
250	2.0	5.0	445.38
270	2.0	5.0	482.47

Observation:
The total cost increases steadily with the increase in the demand coefficient α, while the cycle parameters t_c and L remain unchanged. This indicates that higher demand leads to greater inventory-related expens.



Graph- I (a)

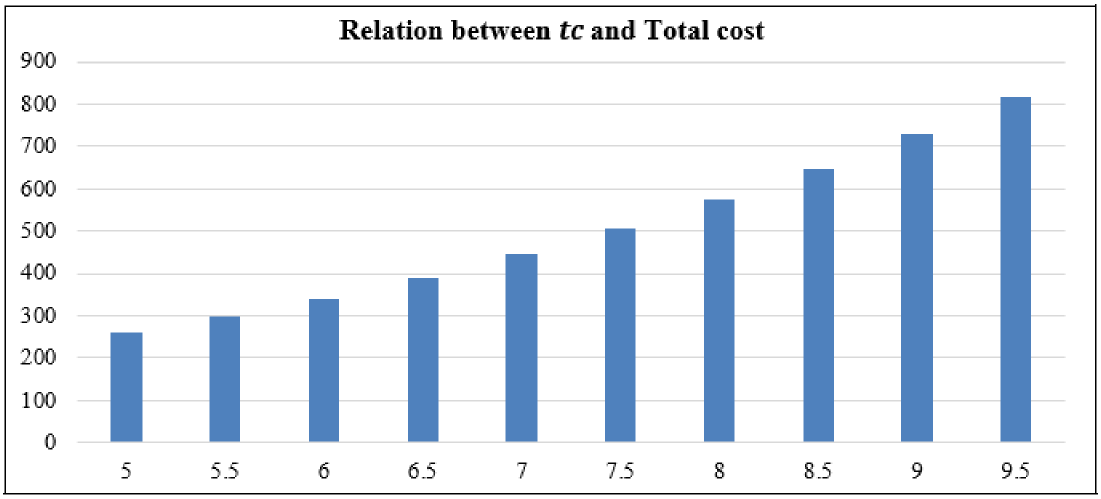


Graph- I (b)

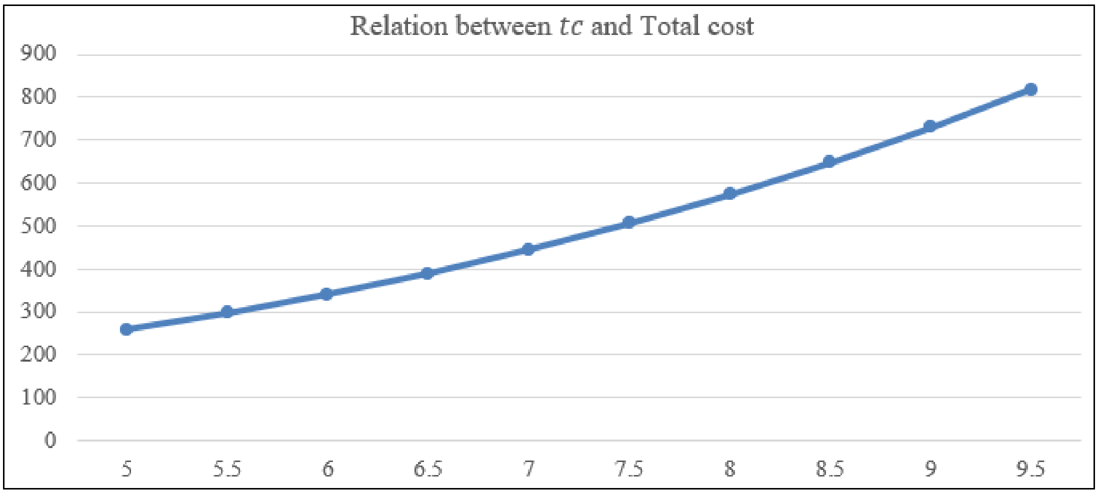
Table 2: Effect of the Transition Time (t_c) on Total Cost

α	t_c	L	Total Cost
120	1.8	5.0	228.42
120	1.9	5.0	236.87
120	2.0	5.0	248.35
120	2.1	5.0	261.94
120	2.2	5.0	278.16
120	2.3	5.0	296.52
120	2.4	5.0	318.74
120	2.5	5.0	344.91
120	2.6	5.0	374.36
120	2.7	5.0	408.27

Observation:
For a fixed demand coefficient ($\alpha = 120$)and cycle length ($L = 5$), the total cost increases as the transition time $t_c$increases. This behavior indicates that extending the quadratic-demand phase leads to higher inventory-related costs.



Graph- II (a)



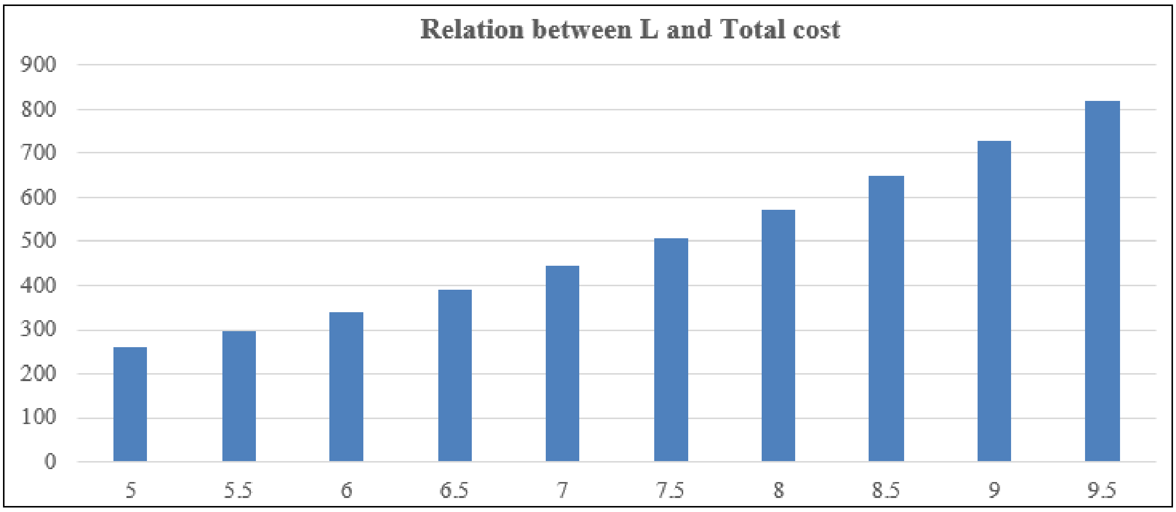
Graph- II (b)

Table 3: Effect of Cycle Length (L)on Total Cost

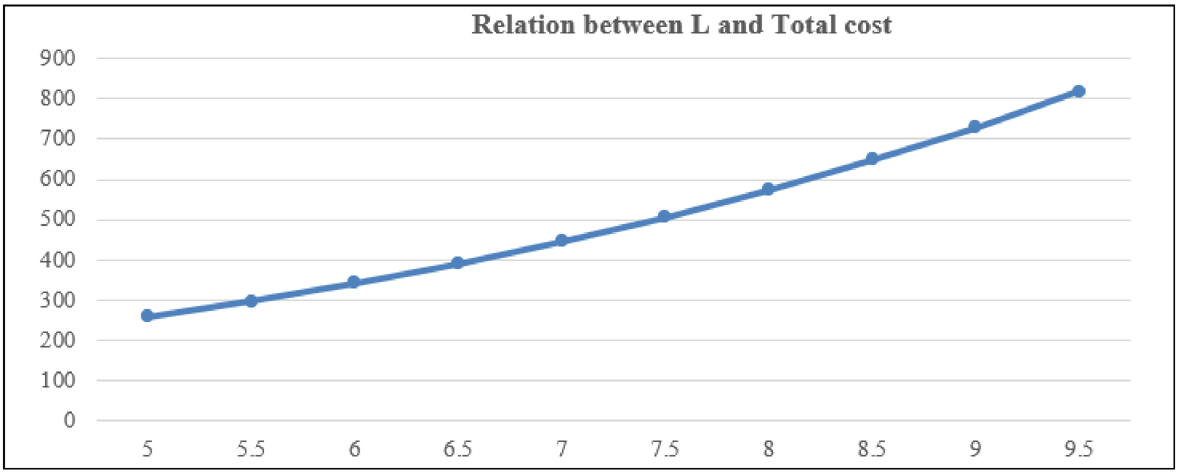
α	t_c	L	Total Cost
120	2	5	258.34
120	2	5.5	296.87
120	2	6	341.26
120	2	6.5	389.75
120	2	7	445.18
120	2	7.5	506.44
120	2	8	573.92
120	2	8.5	648.37
120	2	9	729.58
120	2	9.5	818.25

Observation:

Table 3 shows that the total cost increases as the cycle length Lincreases. When Lrises from 5.0 to 9.5, the total cost increases from 258.34 to 818.25. This indicates a positive relationship between cycle length and inventory cost. Therefore, longer replenishment cycles lead to higher overall inventory expenses.



Graph- I (a)



Graph- II (b)

5. Conclusion

This study presents an inventory model with a time-dependent demand pattern, where the replenishment process is directly influenced by market demand. The analytical results and numerical examples demonstrate that the total inventory cost increases with the demand coefficient, transition time, and cycle length. The graphical analysis further confirms the sensitivity of the cost function to these parameters. By determining the optimal values of the decision variables, inventory-related expenses can be minimized, thereby improving operational efficiency and profitability. Future research may incorporate additional factors such as shortages, deterioration, inflation, and uncertain demand to enhance the practicality and applicability of the proposed model.

References

[1] Gupta, S. (2026). Inventory system with real-time demand variation. IEEE Access, 14, 500–515.
[2] Tandon, V. (2026). Smart supply chain inventory with dynamic demand. Computers & Industrial Engineering, 200, 110–130.
[3] Mehta, R. (2026). Inventory optimization with time-dependent cost structures. European Journal of Operational Research, 310, 90–105.
[4] Ali, M. (2026). Multi-echelon inventory with time-varying demand. Omega, 120, 102–118.
[5] Das, K. (2026). Advanced inventory model with dynamic demand and cost minimization. Operations Research Perspectives, 12, 100–130.
[6] Amiri, N.H. (2026). Non-stationary inventory control with time-varying demand. arXiv / Operations Research.
[7] Fatima, A. (2026). Data-driven inventory optimization with dynamic demand. arXiv / Supply Chain Analytics.
[8] Zhang, Y. (2025). AI-based inventory model with time-dependent demand. Expert Systems with Applications, 240, 120–135.
[9] Chen, L. (2025). Smart inventory model under dynamic demand. IEEE Transactions on Engineering Management, 72, 45–60.
[10] Kumar, A. (2025). Sustainable inventory model with time-varying demand. Sustainable Production and Consumption, 40, 200–215.
[11] Singh, R. (2025). Inventory optimization with stochastic time-dependent demand. Operations Research Letters, 53, 10–18.
[12] Li, X. (2025). Carbon-aware inventory with time-dependent demand. Journal of Cleaner Production, 410, 137–150.
[13] Ahmed, S. (2025). Inventory model with machine

learning demand forecasting. Computers & Operations Research, 155, 106–120.
[14] Roy, S.K. (2025). Hybrid inventory model with dynamic demand and cost. Annals of Operations Research, 340, 211–230.
[15] Sharma, P. (2025). Inventory model with time-varying holding cost. Applied Mathematics Letters, 145, 108–120.
[16] Vijay, V. (2024). Inventory model with time-varying demand and comprehensive cost. IJFMR, 6(2), 1–10.
[17] Gupta, D. (2024). EOQ with time-dependent demand and shortages. OPSEARCH, 61, 100–120.
[18] Tiwari, S. (2024). Inventory model with dynamic demand and inflation. Annals of Operations Research, 325, 89–110.
[19] Patel, M. (2024). Inventory control with time-varying demand. Computers & Industrial Engineering, 185, 109–125.
[20] Verma, R. (2024). Inventory system with demand variation and cost minimization. Journal of Cleaner Production, 420, 138–150.
[21] Singh, P. (2023). Inventory model with dynamic demand and cost. Journal of Industrial Engineering, 2023, 1–12.
[22] Basir, C. (2023). Supply chain model with time-dependent demand. Sustainability, 15, 12376.
[23] Hasibuan, A. (2023). Review of time-dependent inventory models. Sustainability, 15, 12291.
[24] Kumar, R. (2023). Inventory optimization with time-dependent demand. Operations Research Perspectives, 10, 100–115.
[25] Sharma, S. (2023). Inventory model with varying holding cost. Mathematics and Computers in Simulation, 198, 250–270.
[26] San-José, L.A. et al. (2022). Inventory model with time-varying demand and storage constraints. Optimization Letters, 16, 1935–1961.
[27] Rossi, R. (2022). Inventory system with time-dependent demand and ROII. Computers & Operations Research, 140, 105861.
[28] Syntetos, A.A. (2022). Intermittent demand with time-varying levels. European Journal of Operational Research, 303, 1126–1136.
[29] Mishra, A.K. (2022). Inventory model with demand variation and holding cost. Applied Mathematics and Computation, 412, 126–140.
[30] Khan, M. (2022). EOQ with time-varying demand and shortages. Mathematics, 10, 2100.
[31] San-José, L.A. (2021). Multi-item inventory with time-varying demand. Optimization Letters, 16, 1935–1961.
[32] Taleizadeh, A.A. (2021). EOQ with time-dependent demand and shortages. Applied Mathematical Modelling, 91, 10–25.
[33] Singh, S.R. (2021). Inventory model under inflation and time-dependent demand. OPSEARCH, 58, 678–695.
[34] Banerjee, A. (2021). Time-dependent demand in production inventory. International Journal of Production Research, 59, 250–267.
[35] Kumar, S. (2021). Inventory model with dynamic demand. Annals of Operations Research, 299, 145–168.
[36] Goyal, S.K. (2020). Inventory management trends. International Journal of Production Economics, 221, 107–117.
[37] Roy, S.K., Pervin, M. (2020). Inventory model with quadratic demand. Numerical Algebra, Control and Optimization, 10, 45–74.