Degree Ratio Gourava Sombor Index of Certain Chemical Structures

Kulli V R[1], Gutman I[2]

[1]Department of Mathematics, Gulbarga University, Kalaburgi, India

[2]Faculty of Science, University Kragujevac, Kragujevac, Seerbia

Abstract: ***In this paper, we introduce the degree ratio Gourava Sombor and the reciprocal degree ratio Gourava Sombor indices of a graph. We compute these newly defined degree ratio Gourava Sombor indices for certain chemical structures.***

Keywords: degree ratio Gourava Sombor index, reciprocal degree ratio Gourava Sombor index, structure.

1. Introduction

The simple graphs which are finite, undirected, connected graphs without loops and multiple edges are considered. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u.

In [1], the degree ratio Sombor index of a graph G is defined as

$$DRSO(G)=\sum_{uv\in E(G)}\sqrt{\left(\frac{d_G(u)}{d_G(v)}\right)^2+\left(\frac{d_G(v)}{d_G(u)}\right)^2}.$$

The Gourava Sombor index [2] of a graph G is defined as

$$GSO(G)=\sum_{uv\in E(G)}\sqrt{[d_G(u)+d_G(v)]^2+d_G(u)^2d_G(v)^2}.$$

We define the degree ratio Gourava Sombor index of a graph G as

$$DRGSO(G)=\sum_{uv\in E(G)}\sqrt{\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right)^2+1}.$$

In view of the degree ratio Gourava Sombor index, we propose the degree ratio Gourava Sombor exponential of a graph G and it is defined as

$$DRGSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right)^2+1}}.$$

We define the reciprocal degree ratio Gourava Sombor index of a graph G as

$$RDRGSO(G)$$

$$=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)^2d_G(v)^2}{\left(d_G(u)^2+d_G(v)^2\right)^2+d_G(u)^2d_G(v)^2}}$$

Recently, some degree ratio indices were studied in [3-9].

In view of the reciprocal degree ratio Gourava Sombor index, we propose the reciprocal degree ratio Gourava Sombor exponential of a graph G and it is defined as

$$RDRGSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\frac{d_G(u)^2d_G(v)^2}{\left(d_G(u)^2+d_G(v)^2\right)^2+d_G(u)^2d_G(v)^2}}}.$$

In this research, we compute the degree ratio Gourava Sombor and reciprocal degree ratio Gourava Sombor indices for certain chemical structures.

2. Armchair Polyhex Nanotubes

Carbon polyhex nanotubes exist in nature with remarkable stability and possess very interesting electrical, thermal and mechanical properties. The molecular graph of armchair polyhex nanotube $TUAC_6\,[p, q]$ is shown in the below graph.

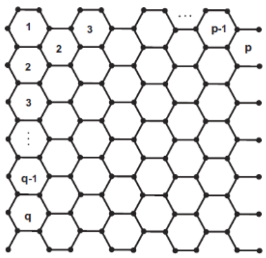

Figure 1

The graphs of armchair polyhex nanotubes have $2p(q+1)$ vertices and $3pq + 2p$ edges are shown in the above graph. Let $G= TUAC_6[p, q]$.

We obtain that $\{d(u), d(v): uv \in E(G)\}$ has three edge set partitions.

$d(u), d(v) \setminus uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	p	$2p$	$3pq-p$

Theorem 1. The degree ratio Sombor index of $TUAC_6\,[p, q]$ is given by

$$DRSO(G)=3\sqrt{2}pq+\frac{\sqrt{97}}{3}p.$$

Proof: Applying definition and edge partition of $TUAC_6\,[p, q]$, we conclude

$$DRSO(G)=\sum_{uv\in E(G)}\sqrt{\left(\frac{d_G(u)}{d_G(v)}\right)^2+\left(\frac{d_G(v)}{d_G(u)}\right)^2}$$

$$=p\left(\sqrt{\left(\frac{2}{2}\right)^2+\left(\frac{2}{2}\right)^2}\right)+2p\left(\sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{3}{2}\right)^2}\right)$$

$$+(3pq-p)\left(\sqrt{\left(\frac{3}{3}\right)^2+\left(\frac{3}{3}\right)^2}\right).$$

By solving the above equation, we get the desired result.

Theorem 2. The Gourava Sombor index of $TUAC_6\,[p, q]$ is

$$GSO(G)=9\sqrt{13}pq+4\sqrt{2}p+2\sqrt{61}p-3\sqrt{13}p.$$

Proof: Applying definition and edge partition of $TUAC_6\,[p, q]$, we conclude

$$GSO(G)$$

$$=\sum_{uv\in E(G)}\sqrt{[d_G(u)+d_G(v)]^2+d_G(u)^2d_G(v)^2}$$

$$=p\sqrt{[2+2]^2+2^2\times 2^2}+2p\sqrt{[2+3]^2+2^2\times 3^2}$$

$$+(3pq-p)\sqrt{[3+3]^2+3^2\times 3^2}.$$

By solving the above equation, we get the desired result.

Theorem 3. The degree ratio Gourava Sombor index of $TUAC_6\,[p, q]$ is given by

$$DRGSO(G)=3\sqrt{5}pq+\frac{\sqrt{133}}{3}p.$$

Proof: Applying definition and edge partition of $TUAC_6\,[p, q]$, we conclude

$$DRGSO(G)=\sum_{uv\in E(G)}\sqrt{\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right)^2+1}$$

$$=p\left(\sqrt{\left(\frac{2}{2}+\frac{2}{2}\right)^2+1}\right)+2p\left(\sqrt{\left(\frac{2}{3}+\frac{3}{2}\right)^2+1}\right)$$

$$+(3pq-p)\left(\sqrt{\left(\frac{3}{3}+\frac{3}{3}\right)^2+1}\right).$$

By solving the above equation, we get the desired result.

Theorem 4. The degree ratio Gourava Sombor exponential of $TUAC_6\,[p, q]$ is

$$DRGSO(G,x)=3pqx^{\sqrt{5}}+2px^{\frac{\sqrt{205}}{6}}.$$

Proof: Applying definition and edge partition of $TUAC_6\,[p, q]$, we conclude

$$DRGSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right)^2+1}}$$

$$=px^{\sqrt{\left(\frac{2}{2}+\frac{2}{2}\right)^2+1}}+2px^{\sqrt{\left(\frac{2}{3}+\frac{3}{2}\right)^2+1}}+(3pq-p)x^{\sqrt{\left(\frac{3}{3}+\frac{3}{3}\right)^2+1}}.$$

By solving the above equation, we get the desired result.

Theorem 5. The reciprocal degree ratio Gourava Sombor index of $TUAC_6\,[p, q]$ is given by

$$RDRGSO(G)=3\sqrt{\frac{1}{5}}pq+12\sqrt{\frac{1}{205}}p.$$

Proof: Applying definition and edge partition of $TUAC_6\,[p, q]$, we conclude

$$RDRGSO(G)$$

$$=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)^2d_G(v)^2}{\left(d_G(u)^2+d_G(v)^2\right)^2+d_G(u)^2d_G(v)^2}}$$

$$=p\sqrt{\frac{2^2\times 2^2}{\left(2^2+2^2\right)^2+2^2\times 2^2}}+2p\sqrt{\frac{2^2\times 3^2}{\left(2^2+3^2\right)^2+2^2\times 3^2}}$$

$$+(3pq-p)\sqrt{\frac{3^2\times 3^2}{\left(3^2+3^2\right)^2+3^2\times 3^2}}.$$

By solving the above equation, we get the desired result.

Theorem 6. The reciprocal degree ratio Gourava Sombor exponential of $TUAC_6\,[p, q]$ is

$$RDRGSO(G,x)=3pqx^{\sqrt{\frac{1}{5}}}+2px^{6\sqrt{\frac{1}{205}}}.$$

Proof: Applying definition and edge partition of $TUAC_6\,[p, q]$, we conclude

$$RDRGSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\frac{d_G(u)^2d_G(v)^2}{\left(d_G(u)^2+d_G(v)^2\right)^2+d_G(u)^2d_G(v)^2}}}$$

$$=px^{\sqrt{\frac{2^2\times 2^2}{\left(2^2+2^2\right)^2+2^2\times 2^2}}}+2px^{\sqrt{\frac{2^2\times 3^2}{\left(2^2+3^2\right)^2+2^2\times 3^2}}}$$

$$+\left(3pq-p\right)x^{\sqrt{\frac{3^2\times 3^2}{\left(3^2+3^2\right)^2+3^2\times 3^2}}}.$$

By solving the above equation, we get the desired result.

3. ZigZag Polyhex Nanotubes

The molecular graph of zigzag polyhex nanotube $TUZC_6$ [p, q] is depicted in below graph.

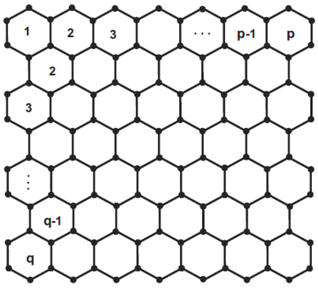

Figure 2

The graphs of zigzag polyhex nanotubes have $2p(q+1)$ vertices and $3pq + 2p$ edges are shown in the above graph. Let $B = TUZC_6[p, q]$.

We obtain that $\{d(u), d(v): uv \in E(B)\}$ has three edge set partitions.

$d(u), d(v) \setminus uv \in E(B)$	(2, 3)	(3, 3)
Number of edges	$4p$	$3pq - 2p$

Theorem 7. The degree ratio Sombor index of $TUZC_6$ [p, q] is given by

$$DRSO(G)=3\sqrt{2}pq+\frac{2\sqrt{97}}{3}p-2\sqrt{2}p.$$

Proof: Applying definition and edge partition of $TUZC_6$ [p, q], we conclude

$$DRSO(G)=\sum_{uv\in E(G)}\sqrt{\left(\frac{d_G(u)}{d_G(v)}\right)^2+\left(\frac{d_G(v)}{d_G(u)}\right)^2}$$

$$=4p\left(\sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{3}{2}\right)^2}\right)+\left(3pq-2p\right)\left(\sqrt{\left(\frac{3}{3}\right)^2+\left(\frac{3}{3}\right)^2}\right).$$

By solving the above equation, we get the desired result.

Theorem 8. The Gourava Sombor index of $TUZC_6$ [p, q] is

$$GSO(G)=9\sqrt{13}pq+4\sqrt{61}p-6\sqrt{13}p.$$

Proof: Applying definition and edge partition of $TUZC_6$ [p, q], we conclude

$$GSO(G)=\sum_{uv\in E(G)}\sqrt{[d_G(u)+d_G(v)]^2+d_G(u)^2 d_G(v)^2}$$

$$=4p\sqrt{[2+3]^2+2^2\times 3^2}+\left(3pq-2p\right)\sqrt{[3+3]^2+3^2\times 3^2}.$$

By solving the above equation, we get the desired result.

Theorem 9. The degree ratio Gourava Sombor index of $TUZC_6$ [p, q] is

$$DRGSO(G)=3\sqrt{5}pq+\frac{\sqrt{133}}{3}p-2\sqrt{5}p.$$

Proof: Applying definition and edge partition of $TUZC_6$ [p, q], we conclude

$$DRGSO(G)=\sum_{uv\in E(G)}\sqrt{\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right)^2+1}$$

$$=4p\sqrt{\left(\frac{2}{3}+\frac{3}{2}\right)^2+1}+\left(3pq-2p\right)\sqrt{\left(\frac{3}{3}+\frac{3}{3}\right)^2+1}.$$

By solving the above equation, we get the desired result.

Theorem 10. The degree ratio Gourava Sombor exponential of $TUAC_6$ [p, q] is

$$DRGSO(G,x)=4px^{\frac{\sqrt{205}}{6}}+\left(3pq-2p\right)x^{\sqrt{5}}.$$

Proof: Applying definition and edge partition of $TUAC_6$ [p, q], we conclude

$$DRGSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right)^2+1}}$$

$$=4px^{\sqrt{\left(\frac{2}{3}+\frac{3}{2}\right)^2+1}}+\left(3pq-2p\right)x^{\sqrt{\left(\frac{3}{3}+\frac{3}{3}\right)^2+1}}.$$

By solving the above equation, we get the desired result.

Theorem 11. The reciprocal degree ratio Gourava Sombor of $TUAC_6$ [p, q] is

$$RDRGSO(G)=3\sqrt{\frac{1}{5}}pq+24\sqrt{\frac{1}{205}}-2\sqrt{\frac{1}{5}}p.$$

Proof: Applying definition and edge partition of $TUAC_6$ [p, q], we conclude

$$RDRGSO(G)$$

$$=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)^2 d_G(v)^2}{\left(d_G(u)^2+d_G(v)^2\right)^2+d_G(u)^2 d_G(v)^2}}$$

$$=4p\sqrt{\frac{2^2\times 3^2}{\left(2^2+3^2\right)^2+2^2\times 3^2}}+\left(3pq-2p\right)\sqrt{\frac{3^2\times 3^2}{\left(3^2+3^2\right)^2+3^2\times 3^2}}.$$

By solving the above equation, we get the desired result.

Theorem 12. The reciprocal degree ratio Gourava Sombor exponential of $TUAC_6$ [p, q] is

$$RDRGSO(G,x)=4px^{6\sqrt{\frac{1}{205}}}+\left(3pq-2p\right)x^{\sqrt{\frac{1}{5}}}.$$

Proof: Applying definition and edge partition of $TUAC_6$ [p, q], we conclude

$$RDRGSO(G,x)=\sum_{uv\in E(G)} x^{\sqrt{\frac{d_G(u)^2 d_G(v)^2}{\left(d_G(u)^2+d_G(v)^2\right)^2+d_G(u)^2 d_G(v)^2}}}$$

$$=4px^{\sqrt{\frac{2^2\times 3^2}{(2^2+3^2)^2+2^2\times 3^2}}}+(3pq-2p)x^{\sqrt{\frac{3^2\times 3^2}{(3^2+3^2)^2+3^2\times 3^2}}}.$$

By solving the above equation, we get the desired result.

4. Carbon Nanocone Networks

The molecular graph of pentagonal nanocone network CNC_5 [n] is depicted in below graph.

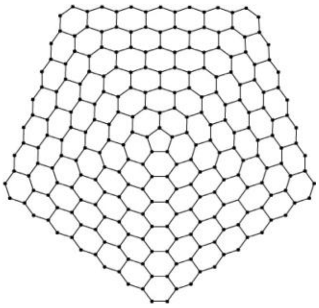

Figure 3

The graphs of pentagonal nanocone networks have $5(n+1)^2$ vertices and $\frac{15}{2}n^2+\frac{25}{2}n+5$ edges. Let $G = CNC_5[n]$.

We obtain that {$d(u)$, $d(v)$: $uv \in E(G)$} has three edge set partitions.

$d(u), d(v) \backslash uv\in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	5	$10n$	$\frac{15}{2}n^2+\frac{5}{2}n$

Theorem 13. The degree ratio Sombor index of CNC_5 [n] is given by

$$DRSO(G)=\frac{15}{\sqrt{2}}n^2+\frac{5}{\sqrt{2}}n+\frac{5\sqrt{97}}{3}n+5\sqrt{2}.$$

Proof: Applying definition and edge partition of CNC_5 [n], we conclude

$$DRSO(G)=\sum_{uv\in E(G)}\sqrt{\left(\frac{d_G(u)}{d_G(v)}\right)^2+\left(\frac{d_G(v)}{d_G(u)}\right)^2}$$

$$=5\left(\sqrt{\left(\frac{2}{2}\right)^2+\left(\frac{2}{2}\right)^2}\right)+10n\left(\sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{3}{2}\right)^2}\right)$$

$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\left(\sqrt{\left(\frac{3}{3}\right)^2+\left(\frac{3}{3}\right)^2}\right).$$

By solving the above equation, we get the desired result.

Theorem 14. The Gourava Sombor index of CNC_5 [n] is

$$GSO(G)=\frac{45\sqrt{13}}{2}n^2+10\sqrt{61}n+\frac{15\sqrt{13}}{2}n+20\sqrt{2}.$$

Proof: Applying definition and edge partition of CNC_5 [n], we conclude

$$GSO(G)=\sum_{uv\in E(G)}\sqrt{[d_G(u)+d_G(v)]^2+d_G(u)^2 d_G(v)^2}$$

$$=5\sqrt{[2+2]^2+2^2\times 2^2}+10n\sqrt{[2+3]^2+2^2\times 3^2}$$

$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\sqrt{[3+3]^2+3^2\times 3^2}.$$

By solving the above equation, we get the desired result.

Theorem 15. The degree ratio Gourava Sombor index of CNC_5 [n] is given by

$$DRGSO(G)=\frac{15\sqrt{5}}{2}n^2+\frac{5\sqrt{205}}{3}n+\frac{5\sqrt{5}}{2}n+5\sqrt{5}.$$

Proof: Applying definition and edge partition of CNC_5 [n], we conclude

$$DRGSO(G)=\sum_{uv\in E(G)}\sqrt{\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right)^2+1}$$

$$=5\left(\sqrt{\left(\frac{2}{2}+\frac{2}{2}\right)^2+1}\right)+10n\left(\sqrt{\left(\frac{2}{3}+\frac{3}{2}\right)^2+1}\right)$$

$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\left(\sqrt{\left(\frac{3}{3}+\frac{3}{3}\right)^2+1}\right).$$

By solving the above equation, we get the desired result.

Theorem 16. The degree ratio Gourava Sombor exponential of $TUAC_6$ [p, q] is

$$DRGSO(G,x)=5x^{\sqrt{5}}+10nx^{\frac{\sqrt{205}}{6}}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{\sqrt{5}}.$$

Proof: Applying definition and edge partition of $TUAC_6$ [p, q], we conclude

$$DRGSO(G,x)=\sum_{uv\in E(G)} x^{\sqrt{\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right)^2+1}}$$

$$=5x^{\sqrt{\left(\frac{2}{2}+\frac{2}{2}\right)^2+1}}+10nx^{\sqrt{\left(\frac{2}{3}+\frac{3}{2}\right)^2+1}}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{\sqrt{\left(\frac{3}{3}+\frac{3}{3}\right)^2+1}}.$$

By solving the above equation, we get the desired result.

Theorem 17. The reciprocal degree ratio Gourava Sombor index of CNC_5 [n] is given by

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$$RDRGSO(G)=\frac{15}{2}\sqrt{\frac{1}{5}}n^2+60\sqrt{\frac{1}{205}}n+\frac{5}{2}\sqrt{\frac{1}{5}}n+5\sqrt{\frac{1}{5}}.$$

Proof: Applying definition and edge partition of CNC_5 [n], we conclude

$$RDRGSO(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)^2 d_G(v)^2}{\left(d_G(u)^2+d_G(v)^2\right)^2+d_G(u)^2 d_G(v)^2}}$$
$$=5\sqrt{\frac{2^2\times 2^2}{\left(2^2+2^2\right)^2+2^2\times 2^2}}+10n\sqrt{\frac{2^2\times 3^2}{\left(2^2+3^2\right)^2+2^2\times 3^2}}$$
$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\sqrt{\frac{3^2\times 3^2}{\left(3^2+3^2\right)^2+3^2\times 3^2}}.$$

By solving the above equation, we get the desired result.

Theorem 18. The reciprocal degree ratio Gourava Sombor exponential of $TUAC_6$ [p, q] is

$$RDRGSO(G,x)=5x^{\sqrt{\frac{1}{5}}}+10nx^{6\sqrt{\frac{1}{205}}}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{\sqrt{\frac{1}{5}}}.$$

Proof: Applying definition and edge partition of $TUAC_6$ [p, q], we conclude

$$RDRGSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\frac{d_G(u)^2 d_G(v)^2}{\left(d_G(u)^2+d_G(v)^2\right)^2+d_G(u)^2 d_G(v)^2}}}$$
$$=5x^{\sqrt{\frac{2^2\times 2^2}{\left(2^2+2^2\right)^2+2^2\times 2^2}}}+10nx^{\sqrt{\frac{2^2\times 3^2}{\left(2^2+3^2\right)^2+2^2\times 3^2}}}$$
$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{\sqrt{\frac{3^2\times 3^2}{\left(3^2+3^2\right)^2+3^2\times 3^2}}}.$$

By solving the above equation, we get the desired result.

5. Conclusion

We have introduced the degree ratio Gourava Sombor and reciprocal degree ratio Gourava Sombor indices of a graph. Also, we have determined these newly defined the degree ratio Gourava Sombor indices of armchair polyhex nanotubes, zigzag polyhex nanotubes and carbon nanocone networks.

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