

D₂ & D₂* Values for Weibull Distribution

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Abstract: The d₂ and d₂* values are tabulated for normal distribution. This article obtains d₂ and d₂* values for Weibull distribution. The scale (α) and the shape (β) parameters are calculated from which variance and hence standard deviation is calculated. Average range and variance are used to calculate the d₂ and d₂* values.

Keywords: Weibull distribution, Scale and Shape parameters, Newton Raphson's method, Gamma function, statistical process control, d₂ and d₂* values

Section 1:

α (Scale Parameter): Represents the characteristic life. At x = α, approximately 63.2% of the population will have failed regardless of the shape parameter. ("Weibull Distribution")

β (Shape Parameter): Dictates the failure behaviour: ("Weibull Distribution")

- β < 1: Indicates a *decreasing* failure rate (early-life or "infant mortality" failures).
- β = 1: Indicates a *constant* failure rate (random, useful life failures).
- β > 1: Indicates an *increasing* failure rate (wear-out or aging)

$$\frac{1}{\beta} + \frac{\sum x^{\beta} \ln(x)}{\sum x^{\beta}} + \frac{\ln \sum(x)}{n} = 0 \quad \dots [2]$$

$$\alpha = \left\{ \sum_{i=1}^n x^{\beta} i \right\}^{1/\beta} \dots [2]$$

Maximum likelihood Estimation is used to calculate value of β and hence α. For non-linear equations like above Newton-Raphson's Method can be used as shown below.

$$X(n+1) = X(n) - \frac{f(Xn)}{f'(Xn)}$$

After finding α and β, variance is calculated using following formula,

$$\sigma^2 = \alpha^2 \left[\Gamma \left(1 + \frac{2}{\beta} \right) - \left(\Gamma \left(1 + \frac{1}{\beta} \right) \right)^2 \right]$$

$$\Gamma(n) = (n-1)!$$

If n is a fraction Gamma function needs to be used,

$$\Gamma(n) = \int_0^{\infty} t^{n-1} e^{-t} dt \dots \text{integral goes from 0 to } \infty.$$

From variance, standard deviation can be calculated taking square root.

$$d_2 = \frac{Rbar}{\sigma}$$

d₂ is the expected value of the sample range for n independent observations from a normal distribution with σ = 1. It assumes a large (theoretically infinite) number of subgroups (k = ∞).

[3]

d₂* is a modified version of d₂ used when you only have a small, finite number of subgroups (k). It accounts for the additional variation caused by a small number of samples. [4]

Section 2

20	21	19	22	20
19	18	19	17	19
21	22	23	18	22
23	20	22	20	21
20	19	22	23	20
22	21	20	21	20

Subgroups = 5

Each subgroup has 6 values

Average Range = 4.2

α = 64.1

β = 3

σ = 5.8

d₂ = 0.71

The d₂ value can be calculated for various subgroup sizes.

Conclusion

The d₂ and d₂* values are already tabulated for normal distribution. This article obtains d₂ and d₂* values for Weibull distribution. The scale (α) and the shape (β) parameters are calculated from which variance and hence standard deviation is calculated. Average range and variance are used to calculate the d₂ and d₂* values.

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