International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2025: 7.089

Diminished Euler Sombor Index of Certain Chemical Drugs

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Abstract: ***In this paper, we introduce the diminished Euler Sombor and the reciprocal diminished Euler Sombor indices of a graph. We compute these newly defined diminished Euler Sombor indices for certain chemical drugs.***

Keywords: diminished Euler Sombor index, reciprocal diminished Euler Sombor index, drug

1. Introduction

The simple graphs which are finite, undirected, connected graphs without loops and multiple edges are considered. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u.

The Euler Sombor index [1] or Nirmala alpha Gourava index [2] of a graph is defined as

$$EU(G)=\sum_{uv\in E(G)}\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}.$$

Recently, some Sombor indices were studied in [3-6].

The diminished Sombor index [7] of a graph G is defined as

$$D(G)=\sum_{uv\in E(G)}\frac{\sqrt{d_G(u)^2+d_G(v)^2}}{d_G(u)+d_G(v)}.$$

In view of the diminished Sombor index, we propose the diminished Sombor exponential of a graph G and it is defined as

$$D(G,x)=\sum_{uv\in E(G)}x^{\frac{\sqrt{d_G(u)^2+d_G(v)^2}}{d_G(u)+d_G(v)}}.$$

We define the diminished Euler Sombor index of a graph G as

$$DEU(G)=\sum_{uv\in E(G)}\frac{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}{d_G(u)+d_G(v)}.$$

In view of the diminished Euler Sombor index, we propose the diminished Euler Sombor exponential of a graph G and it is defined as

$$DEU(G,x)=\sum_{uv\in E(G)}x^{\frac{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}{d_G(u)+d_G(v)}}.$$

We define the reciprocal diminished Euler Sombor index of a graph G as

$$RDEU(G)=\sum_{uv\in E(G)}\frac{d_G(u)+d_G(v)}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}.$$

In view of the diminished Euler Sombor index, we propose the diminished Euler Sombor exponential of a graph G and it is defined as

$$RDEU(G,x)=\sum_{uv\in E(G)}x^{\frac{d_G(u)+d_G(v)}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}}.$$

In this research, we compute the diminished Euler Sombor and reciprocal diminished Euler Sombor indices for certain chemical drugs.

2. Results and Discussion: Chloroquine

Let G be the molecular structure of chloroquine. Clearly G has 21 vertices and 23 edges, see Figure 1.

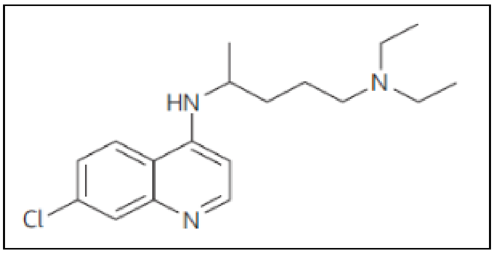

Figure 1

The edge set of G can be divided into five partitions based on the degree of end vertices of each edge as given in Table 1.

Table 1: Edge partition of G

$d_G(u), d_G(v)\backslash uv\in E(G)$	(1,2)	(1,3)	(2,2)	(2,3)	(3,3)
No. of edges	2	2	5	12	2

Theorem 1. Let G be the chemical structure of chloroquine. Then

$$D(G)=\frac{2\sqrt{5}}{3}+\frac{\sqrt{10}}{2}+3\sqrt{2}+\frac{12\sqrt{13}}{5}.$$

Proof: By using the definition and edge partition of G, we deduce

$$D(G)=\sum_{uv\in E(G)}\frac{\sqrt{d_G(u)^2+d_G(v)^2}}{d_G(u)+d_G(v)}$$
$$=2\frac{\sqrt{1^2+2^2}}{1+2}+2\frac{\sqrt{1^2+3^2}}{1+3}+5\frac{\sqrt{2^2+2^2}}{2+2}$$
$$+12\frac{\sqrt{2^2+3^2}}{2+3}+2\frac{\sqrt{3^2+3^2}}{3+3}$$
$$=\frac{2\sqrt{5}}{3}+\frac{\sqrt{10}}{2}+3\sqrt{2}+\frac{12\sqrt{13}}{5}.$$

Theorem 2. Let G be the chemical structure of chloroquine. Then

$$D(G,x)=2x^{\frac{\sqrt{5}}{3}}+2x^{\frac{\sqrt{10}}{4}}+7x^{\frac{\sqrt{2}}{2}}+12x^{\frac{\sqrt{13}}{5}}.$$

Proof: By using the definition and edge partition of G, we deduce

$$D(G,x)=\sum_{uv\in E(G)}x^{\frac{\sqrt{d_G(u)^2+d_G(v)^2}}{d_G(u)+d_G(v)}}$$
$$=2x^{\frac{\sqrt{1^2+2^2}}{1+2}}+2x^{\frac{\sqrt{1^2+3^2}}{1+3}}+5x^{\frac{\sqrt{2^2+2^2}}{2+2}}$$
$$+12x^{\frac{\sqrt{2^2+3^2}}{2+3}}+2x^{\frac{\sqrt{3^2+3^2}}{3+3}}$$
$$=2x^{\frac{\sqrt{5}}{3}}+2x^{\frac{\sqrt{10}}{4}}+7x^{\frac{\sqrt{2}}{2}}+12x^{\frac{\sqrt{13}}{5}}.$$

Theorem 3. Let G be the chemical structure of chloroquine. Then

$$DEU(G)=\frac{2\sqrt{7}}{3}+\frac{\sqrt{13}}{2}+3\sqrt{3}+\frac{12\sqrt{19}}{5}.$$

Proof: By using the definition and edge partition of G, we deduce

$$DEU(G)=\sum_{uv\in E(G)}\frac{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$$
$$=2\frac{\sqrt{1^2+2^2+1\times 2}}{1+2}+2\frac{\sqrt{1^2+3^2+1\times 3}}{1+3}+5\frac{\sqrt{2^2+2^2+2\times 2}}{2+2}$$
$$+12\frac{\sqrt{2^2+3^2+2\times 3}}{2+3}+2\frac{\sqrt{3^2+3^2+3\times 3}}{3+3}$$
$$=\frac{2\sqrt{7}}{3}+\frac{\sqrt{13}}{2}+3\sqrt{3}+\frac{12\sqrt{19}}{5}.$$

Theorem 4. Let G be the chemical structure of chloroquine. Then

$$DEU(G,x)=2x^{\frac{\sqrt{7}}{3}}+2x^{\frac{\sqrt{13}}{4}}+7x^{\frac{\sqrt{3}}{2}}+12x^{\frac{\sqrt{19}}{5}}.$$

Proof: By using the definition and edge partition of G, we deduce

$$DEU(G,x)=\sum_{uv\in E(G)}x^{\frac{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}{d_G(u)+d_G(v)}}$$
$$=2x^{\frac{\sqrt{1^2+2^2+1\times 2}}{1+2}}+2x^{\frac{\sqrt{1^2+3^2+1\times 3}}{1+3}}+5x^{\frac{\sqrt{2^2+2^2+2\times 2}}{2+2}}$$
$$+12x^{\frac{\sqrt{2^2+3^2+2\times 3}}{2+3}}+2x^{\frac{\sqrt{3^2+3^2+3\times 3}}{3+3}}$$
$$=2x^{\frac{\sqrt{7}}{3}}+2x^{\frac{\sqrt{13}}{4}}+7x^{\frac{\sqrt{3}}{2}}+12x^{\frac{\sqrt{19}}{5}}.$$

Theorem 5. Let G be the chemical structure of chloroquine. Then

$$RDEU(G)=\frac{6}{\sqrt{7}}+\frac{8}{\sqrt{13}}+\frac{14}{\sqrt{3}}+\frac{60}{\sqrt{19}}+\frac{12\sqrt{19}}{5}.$$

Proof: By using the definition and edge partition of G, we deduce

$$RDEU(G)=\sum_{uv\in E(G)}\frac{d_G(u)+d_G(v)}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}$$
$$=2\frac{1+2}{\sqrt{1^2+2^2+1\times 2}}+2\frac{1+3}{\sqrt{1^2+3^2+1\times 3}}+5\frac{2+2}{\sqrt{2^2+2^2+2\times 2}}$$
$$+12\frac{2+3}{\sqrt{2^2+3^2+2\times 3}}+2\frac{3+3}{\sqrt{3^2+3^2+3\times 3}}$$
$$=\frac{6}{\sqrt{7}}+\frac{8}{\sqrt{13}}+\frac{14}{\sqrt{3}}+\frac{60}{\sqrt{19}}+\frac{12\sqrt{19}}{5}.$$

Theorem 6. Let G be the chemical structure of chloroquine. Then

$$RDEU(G,x)=2x^{\frac{3}{\sqrt{7}}}+2x^{\frac{4}{\sqrt{13}}}+7x^{\frac{2}{\sqrt{3}}}+12x^{\frac{5}{\sqrt{19}}}.$$

Proof: By using the definition and edge partition of G, we deduce

$$RDEU(G,x)=\sum_{uv\in E(G)}x^{\frac{d_G(u)+d_G(v)}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}}$$
$$=2x^{\frac{1+2}{\sqrt{1^2+2^2+1\times 2}}}+2x^{\frac{1+3}{\sqrt{1^2+3^2+1\times 3}}}+5x^{\frac{2+2}{\sqrt{2^2+2^2+2\times 2}}}$$
$$+12x^{\frac{2+3}{\sqrt{2^2+3^2+2\times 3}}}+2x^{\frac{3+3}{\sqrt{3^2+3^2+3\times 3}}}$$
$$=2x^{\frac{3}{\sqrt{7}}}+2x^{\frac{4}{\sqrt{13}}}+7x^{\frac{2}{\sqrt{3}}}+12x^{\frac{5}{\sqrt{19}}}.$$

3. Results and Discussion: hydroxychloroquine

Let H be the molecular structure of hydroxychloroquine. Clearly H has 22 vertices and 24 edges, see Figure 2.

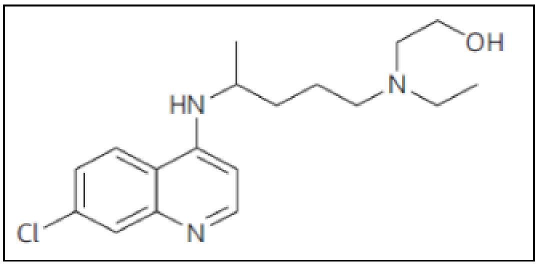

Figure 2

In H, the edge set of $E(H)$ can be divided into five partitions based on the degree of end vertices of each edge as given in Table 2:

Table 2: Edge partition of H

$d_H(u), d_H(v) \backslash uv \in E(H)$	(1,2)	(1,3)	(2,2)	(2,3)	(3,3)
No. of edges	2	2	6	12	2

Theorem 7. Let H be the chemical structure of hydroxychloroquine. Then

$$D(H)=\frac{2\sqrt{5}}{3}+\frac{\sqrt{10}}{2}+4\sqrt{2}+\frac{12\sqrt{13}}{5}.$$

Proof: By using the definition and edge partition of H, we deduce

$$D(H)=\sum_{uv\in E(H)}\frac{\sqrt{d_H(u)^2+d_H(v)^2}}{d_H(u)+d_H(v)}$$
$$=2\frac{\sqrt{1^2+2^2}}{1+2}+2\frac{\sqrt{1^2+3^2}}{1+3}+6\frac{\sqrt{2^2+2^2}}{2+2}$$
$$+12\frac{\sqrt{2^2+3^2}}{2+3}+2\frac{\sqrt{3^2+3^2}}{3+3}$$
$$=\frac{2\sqrt{5}}{3}+\frac{\sqrt{10}}{2}+4\sqrt{2}+\frac{12\sqrt{13}}{5}.$$

Theorem 8. Let H be the chemical structure of hydroxychloroquine. Then

$$D(H,x)=2x^{\frac{\sqrt{5}}{3}}+2x^{\frac{\sqrt{10}}{4}}+8x^{\frac{\sqrt{2}}{2}}+12x^{\frac{\sqrt{13}}{5}}.$$

Proof: By using the definition and edge partition of H, we deduce

$$D(H,x)=\sum_{uv\in E(G)}x^{\frac{\sqrt{d_H(u)^2+d_H(v)^2}}{d_H(u)+d_H(v)}}$$
$$=2x^{\frac{\sqrt{1^2+2^2}}{1+2}}+2x^{\frac{\sqrt{1^2+3^2}}{1+3}}+6x^{\frac{\sqrt{2^2+2^2}}{2+2}}$$
$$+12x^{\frac{\sqrt{2^2+3^2}}{2+3}}+2x^{\frac{\sqrt{3^2+3^2}}{3+3}}$$
$$=2x^{\frac{\sqrt{5}}{3}}+2x^{\frac{\sqrt{10}}{4}}+8x^{\frac{\sqrt{2}}{2}}+12x^{\frac{\sqrt{13}}{5}}.$$

Theorem 9. Let H be the chemical structure of hydroxychloroquine. Then

$$DEU(H)=\frac{2\sqrt{7}}{3}+\frac{\sqrt{13}}{2}+4\sqrt{3}+\frac{12\sqrt{19}}{5}.$$

Proof: By using the definition and edge partition of H, we deduce

$$DEU(H)=\sum_{uv\in E(H)}\frac{\sqrt{d_H(u)^2+d_H(v)^2+d_H(u)d_H(v)}}{d_H(u)+d_H(v)}$$
$$=2\frac{\sqrt{1^2+2^2+1\times2}}{1+2}+2\frac{\sqrt{1^2+3^2+1\times3}}{1+3}+6\frac{\sqrt{2^2+2^2+2\times2}}{2+2}$$
$$+12\frac{\sqrt{2^2+3^2+2\times3}}{2+3}+2\frac{\sqrt{3^2+3^2+3\times3}}{3+3}$$
$$=\frac{2\sqrt{7}}{3}+\frac{\sqrt{13}}{2}+4\sqrt{3}+\frac{12\sqrt{19}}{5}.$$

Theorem 10. Let H be the chemical structure of hydroxychloroquine. Then

$$DEU(H,x)=2x^{\frac{\sqrt{7}}{3}}+2x^{\frac{\sqrt{13}}{4}}+8x^{\frac{\sqrt{3}}{2}}+12x^{\frac{\sqrt{19}}{5}}.$$

Proof: By using the definition and edge partition of H, we deduce

$$DEU(H,x)=\sum_{uv\in E(H)}x^{\frac{\sqrt{d_H(u)^2+d_H(v)^2+d_H(u)d_H(v)}}{d_H(u)+d_H(v)}}$$
$$=2x^{\frac{\sqrt{1^2+2^2+1\times2}}{1+2}}+2x^{\frac{\sqrt{1^2+3^2+1\times3}}{1+3}}+6x^{\frac{\sqrt{2^2+2^2+2\times2}}{2+2}}$$
$$+12x^{\frac{\sqrt{2^2+3^2+2\times3}}{2+3}}+2x^{\frac{\sqrt{3^2+3^2+3\times3}}{3+3}}$$
$$=2x^{\frac{\sqrt{7}}{3}}+2x^{\frac{\sqrt{13}}{4}}+8x^{\frac{\sqrt{3}}{2}}+12x^{\frac{\sqrt{19}}{5}}.$$

Theorem 11. Let H be the chemical structure of hydroxychloroquine. Then

$$RDEU(H)=\frac{6}{\sqrt{7}}+\frac{8}{\sqrt{13}}+\frac{16}{\sqrt{3}}+\frac{60}{\sqrt{19}}+\frac{12\sqrt{19}}{5}.$$

Proof: By using the definition and edge partition of H, we deduce

$$RDEU(H)=\sum_{uv\in E(H)}\frac{d_H(u)+d_H(v)}{\sqrt{d_H(u)^2+d_H(v)^2+d_H(u)d_H(v)}}$$
$$=2\frac{1+2}{\sqrt{1^2+2^2+1\times2}}+2\frac{1+3}{\sqrt{1^2+3^2+1\times3}}+6\frac{2+2}{\sqrt{2^2+2^2+2\times2}}$$
$$+12\frac{2+3}{\sqrt{2^2+3^2+2\times3}}+2\frac{3+3}{\sqrt{3^2+3^2+3\times3}}$$
$$=\frac{6}{\sqrt{7}}+\frac{8}{\sqrt{13}}+\frac{16}{\sqrt{3}}+\frac{60}{\sqrt{19}}+\frac{12\sqrt{19}}{5}.$$

Theorem 12. Let H be the chemical structure of hydroxychloroquine. Then

$$RDEU(H,x)=2x^{\frac{3}{\sqrt{7}}}+2x^{\frac{4}{\sqrt{13}}}+8x^{\frac{2}{\sqrt{3}}}+12x^{\frac{5}{\sqrt{19}}}.$$

Proof: By using the definition and edge partition of H, we deduce

$$RDEU(H,x)=\sum_{uv\in E(H)}x^{\frac{d_H(u)+d_H(v)}{\sqrt{d_H(u)^2+d_H(v)^2+d_H(u)d_H(v)}}}$$
$$=2x^{\frac{1+2}{\sqrt{1^2+2^2+1\times2}}}+2x^{\frac{1+3}{\sqrt{1^2+3^2+1\times3}}}+6x^{\frac{2+2}{\sqrt{2^2+2^2+2\times2}}}$$
$$+12x^{\frac{2+3}{\sqrt{2^2+3^2+2\times3}}}+2x^{\frac{3+3}{\sqrt{3^2+3^2+3\times3}}}$$
$$=2x^{\frac{3}{\sqrt{7}}}+2x^{\frac{4}{\sqrt{13}}}+8x^{\frac{2}{\sqrt{3}}}+12x^{\frac{5}{\sqrt{19}}}.$$

4. Results and Discussion: Remdesivir

Let R be the molecular structure of remdesivir. Clearly R has 41 vertices and 44 edges, see Figure 3.

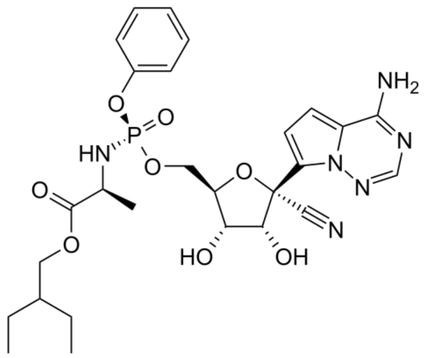

Figure 3

In R, the edge set $E(R)$ can be divided into eight partitions based on the degree of end vertices of each edge as given in Table 3:

Table 3: Edge partition of R

$d_G(u)$, $d_G(v)$\ $uv\in E(R)$	(1, 2)	(1, 3)	(1, 4)	(2, 2)	(2, 3)	(2, 4)	(3,3)	(3,4)
No. of edges	2	5	2	9	14	4	6	2

Theorem 13. Let R be the chemical structure of remdesivir. Then

$$D(R)=2\sqrt{5}+\frac{5\sqrt{10}}{4}+\frac{2\sqrt{17}}{5}+\frac{9\sqrt{2}}{2}+\frac{14\sqrt{13}}{5}+3\sqrt{2}+\frac{10}{7}.$$

Proof: By using the definition and edge partition of R, we deduce

$$D(R)=\sum_{uv\in E(R)}\frac{\sqrt{d_R(u)^2+d_R(v)^2}}{d_R(u)+d_R(v)}$$
$$=2\frac{\sqrt{1^2+2^2}}{1+2}+5\frac{\sqrt{1^2+3^2}}{1+3}+2\frac{\sqrt{1^2+4^2}}{1+4}+9\frac{\sqrt{2^2+2^2}}{2+2}$$
$$+14\frac{\sqrt{2^2+3^2}}{2+3}+4\frac{\sqrt{2^2+4^2}}{2+4}+6\frac{\sqrt{3^2+3^2}}{3+3}+2\frac{\sqrt{3^2+4^2}}{3+4}$$
$$=2\sqrt{5}+\frac{5\sqrt{10}}{4}+\frac{2\sqrt{17}}{5}+\frac{9\sqrt{2}}{2}+\frac{14\sqrt{13}}{5}+3\sqrt{2}+\frac{10}{7}.$$

Theorem 14. Let R be the chemical structure of remdesivir. Then

$$D(R,x)=6x^{\frac{\sqrt{5}}{3}}+5x^{\frac{\sqrt{10}}{4}}+2x^{\frac{\sqrt{17}}{5}}+15x^{\frac{\sqrt{2}}{2}}+14x^{\frac{\sqrt{13}}{5}}+2x^{\frac{5}{7}}.$$

Proof: By using the definition and edge partition of R, we deduce

$$D(R,x)=\sum_{uv\in E(R)}x^{\frac{\sqrt{d_R(u)^2+d_R(v)^2}}{d_R(u)+d_R(v)}}$$
$$=2x^{\frac{\sqrt{1^2+2^2}}{1+2}}+5x^{\frac{\sqrt{1^2+3^2}}{1+3}}+2x^{\frac{\sqrt{1^2+4^2}}{1+4}}+9x^{\frac{\sqrt{2^2+2^2}}{2+2}}$$
$$+14x^{\frac{\sqrt{2^2+3^2}}{2+3}}+4x^{\frac{\sqrt{2^2+4^2}}{2+4}}+6x^{\frac{\sqrt{3^2+3^2}}{3+3}}+2x^{\frac{\sqrt{3^2+4^2}}{3+4}}$$
$$=6x^{\frac{\sqrt{5}}{3}}+5x^{\frac{\sqrt{10}}{4}}+2x^{\frac{\sqrt{17}}{5}}+15x^{\frac{\sqrt{2}}{2}}$$
$$+14x^{\frac{\sqrt{13}}{5}}+2x^{\frac{5}{7}}.$$

Theorem 15. Let R be the chemical structure of remdesivir. Then

$$DEU(R)=2\sqrt{7}+\frac{5\sqrt{13}}{4}+\frac{2\sqrt{21}}{5}+\frac{15\sqrt{3}}{2}+\frac{14\sqrt{19}}{5}+\frac{2\sqrt{37}}{7}.$$

Proof: By using the definition and edge partition of R, we deduce

$$DEU(R)=\sum_{uv\in E(R)}\frac{\sqrt{d_R(u)^2+d_R(v)^2+d_R(u)d_R(v)}}{d_R(u)+d_R(v)}$$
$$=2\frac{\sqrt{1^2+2^2+1\times 2}}{1+2}+5\frac{\sqrt{1^2+3^2+1\times 3}}{1+3}$$
$$+2\frac{\sqrt{1^2+4^2+1\times 4}}{1+4}+9\frac{\sqrt{2^2+2^2+2\times 2}}{2+2}$$
$$+14\frac{\sqrt{2^2+3^2+2\times 3}}{2+3}+4\frac{\sqrt{2^2+4^2+2\times 4}}{2+4}$$
$$+6\frac{\sqrt{3^2+3^2+3\times 3}}{3+3}+2\frac{\sqrt{3^2+4^2+3\times 4}}{3+4}$$
$$=2\sqrt{7}+\frac{5\sqrt{13}}{4}+\frac{2\sqrt{21}}{5}+\frac{15\sqrt{3}}{2}+\frac{14\sqrt{19}}{5}+\frac{2\sqrt{37}}{7}.$$

Theorem 16. Let R be the chemical structure of remdesivir. Then

$$DEU(R,x)=2x^{\frac{\sqrt{7}}{3}}+5x^{\frac{\sqrt{13}}{4}}+2x^{\frac{\sqrt{21}}{5}}+9x^{\frac{\sqrt{3}}{2}}+14x^{\frac{\sqrt{19}}{5}}+4x^{\frac{\sqrt{7}}{3}}+6x^{\frac{\sqrt{3}}{3}}+2x^{\frac{\sqrt{37}}{7}}.$$

Proof: By using the definition and edge partition of R, we deduce

$$DEU(R,x)=\sum_{uv\in E(R)}x^{\frac{\sqrt{d_R(u)^2+d_R(v)^2+d_R(u)d_R(v)}}{d_R(u)+d_R(v)}}$$
$$=2x^{\frac{\sqrt{1^2+2^2+1\times 2}}{1+2}}+5x^{\frac{\sqrt{1^2+3^2+1\times 3}}{1+3}}+2x^{\frac{\sqrt{1^2+4^2+1\times 4}}{1+4}}$$
$$+9x^{\frac{\sqrt{2^2+2^2+2\times 2}}{2+2}}+14x^{\frac{\sqrt{2^2+3^2+2\times 3}}{2+3}}+4x^{\frac{\sqrt{2^2+4^2+2\times 4}}{2+4}}$$
$$+6x^{\frac{\sqrt{3^2+3^2+3\times 3}}{3+3}}+2x^{\frac{\sqrt{3^2+4^2+3\times 4}}{3+4}}$$
$$=2x^{\frac{\sqrt{7}}{3}}+5x^{\frac{\sqrt{13}}{4}}+2x^{\frac{\sqrt{21}}{5}}$$
$$+9x^{\frac{\sqrt{3}}{2}}+14x^{\frac{\sqrt{19}}{5}}+4x^{\frac{\sqrt{7}}{3}}$$
$$+6x^{\frac{\sqrt{3}}{3}}+2x^{\frac{\sqrt{37}}{7}}$$

Theorem 17. Let R be the chemical structure of remdesivir. Then

$$RDEU(R)=\frac{18}{\sqrt{7}}+\frac{20}{\sqrt{13}}+\frac{10}{\sqrt{21}}+\frac{30}{\sqrt{3}}+\frac{70}{\sqrt{19}}+\frac{14}{\sqrt{37}}.$$

Proof: By using the definition and edge partition of R, we deduce

$$RDEU(R)=\sum_{uv\in E(R)}\frac{d_R(u)+d_R(v)}{\sqrt{d_R(u)^2+d_R(v)^2+d_R(u)d_R(v)}}$$

$$=2\frac{1+2}{\sqrt{1^2+2^2+1\times 2}}+5\frac{1+3}{\sqrt{1^2+3^2+1\times 3}}$$
$$+2\frac{1+4}{\sqrt{1^2+4^2+1\times 4}}+9\frac{2+2}{\sqrt{2^2+2^2+2\times 2}}$$
$$+14\frac{2+3}{\sqrt{2^2+3^2+2\times 3}}+4\frac{2+4}{\sqrt{2^2+4^2+2\times 4}}$$
$$+6\frac{3+3}{\sqrt{3^2+3^2+3\times 3}}+2\frac{3+4}{\sqrt{3^2+4^2+3\times 4}}$$
$$=\frac{18}{\sqrt{7}}+\frac{20}{\sqrt{13}}+\frac{10}{\sqrt{21}}+\frac{30}{\sqrt{3}}+\frac{70}{\sqrt{19}}+\frac{14}{\sqrt{37}}.$$

Theorem 18. Let R be the chemical structure of remdesivir. Then

$$RDEU(R,x)=2x^{\frac{3}{\sqrt{7}}}+2x^{\frac{4}{\sqrt{13}}}+8x^{\frac{2}{\sqrt{3}}}+12x^{\frac{5}{\sqrt{19}}}.$$

Proof: By using the definition and edge partition of R, we deduce

$$RDEU(R,x)=\sum_{uv\in E(R)}x^{\frac{d_R(u)+d_R(v)}{\sqrt{d_R(u)^2+d_R(v)^2+d_R(u)d_R(v)}}}$$

$$=2x^{\frac{1+2}{\sqrt{1^2+2^2+1\times 2}}}+5x^{\frac{1+3}{\sqrt{1^2+3^2+1\times 3}}}+2x^{\frac{1+4}{\sqrt{1^2+4^2+1\times 4}}}$$
$$+9x^{\frac{2+2}{\sqrt{2^2+2^2+2\times 2}}}+14x^{\frac{2+3}{\sqrt{2^2+3^2+2\times 3}}}+4x^{\frac{2+4}{\sqrt{2^2+4^2+2\times 4}}}$$
$$+6x^{\frac{3+3}{\sqrt{3^2+3^2+3\times 3}}}+2x^{\frac{3+4}{\sqrt{3^2+4^2+3\times 4}}}$$
$$=2x^{\frac{3}{\sqrt{7}}}+5x^{\frac{4}{\sqrt{13}}}+2x^{\frac{5}{\sqrt{21}}}$$
$$+9x^{\frac{2}{\sqrt{3}}}+14x^{\frac{5}{\sqrt{19}}}+4x^{\frac{3}{\sqrt{7}}}$$
$$+6x^{\frac{2}{\sqrt{3}}}+2x^{\frac{7}{\sqrt{37}}}$$

5. Conclusion

In this research, the diminished, diminished Sombor, diminished Euler Sombor and reciprocal diminished Euler Sombor indices and their exponentials for certain chemical structures are determined.

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