Degree Ratio Inverse Sum Indeg Index of Certain Nanotubes and Networks

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Abstract: ***In this paper, we introduce the degree ratio inverse sum indeg and the modified degree ratio inverse sum indeg indices of a graph. We compute these newly defined degree ratio inverse sum indeg indices for armchair polyhex nanotubes, zigzag polyhex nanotubes and carbon nanocone networks.***

Keywords: degree ratio inverse sum indeg index, modified degree ratio inverse sum indeg index, nanotubes and networks.

1. Introduction

The simple graphs which are finite, undirected, connected graphs without loops and multiple edges are considered. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u.

In [1], the degree ratio Nirmala index of a graph is defined as

$$DRN(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}}.$$

Recently, some degree ratio indices were studied in [2, 3, 4].

The inverse sum indeg index [5] of a graph is defined as

$$ISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)}.$$

Recently, some inverse sum indeg indices were studied in [6, 7, 8].

We define the degree ratio inverse sum indeg index of a graph G as

$$DRISI(G)=\sum_{uv\in E(G)}\frac{\frac{d_G(u)}{d_G(v)}\frac{d_G(v)}{d_G(u)}}{\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}}$$

$$=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)^2+d_G(v)^2}.$$

Considering the degree ratio inverse sum indeg index, we introduce the degree ratio inverse sum indeg polynomial of a graph G and defined it as

$$DRISI(G,x)=\sum_{uv\in E(G)}x^{\frac{d_G(u)d_G(v)}{d_G(u)^2+d_G(v)^2}}.$$

We define the modified degree ratio inverse sum indeg index of a graph G as

$$^{m}DRISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)^2+d_G(v)^2}{d_G(u)d_G(v)}.$$

Considering the modified degree ratio inverse sum indeg index, we introduce the modified degree ratio inverse sum indeg polynomial of a graph G and defined it as

$$^{m}DRISI(G,x)=\sum_{uv\in E(G)}x^{\frac{d_G(u)^2+d_G(v)^2}{d_G(u)d_G(v)}}.$$

In this research, we compute the degree ratio inverse sum indeg and modified degree ratio inverse sum indeg indices and their polynomials for armchair polyhex nanotubes, zigzag polyhex nanotubes and carbon nanocone networks.

2. Armchair Polyhex Nanotubes

Carbon polyhex nanotubes exist in nature with remarkable stability and possess very interesting electrical, thermal and mechanical properties. The molecular graph of armchair polyhex nanotube $TUAC_6\,[p, q]$ is shown in the below graph.

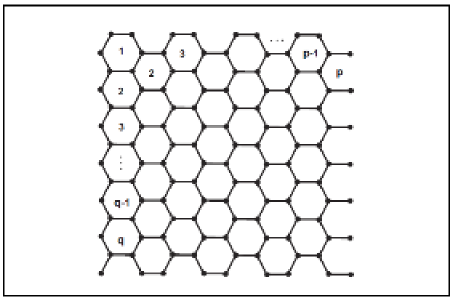

Figure 1

The graphs of armchair polyhex nanotubes have $2p(q+1)$ vertices and $3pq + 2p$ edges are shown in the above graph. Let $G= TUAC_6[p, q]$.

We obtain that $\{d(u), d(v): uv \in E(G)\}$ has three edge set partitions.

$d(u), d(v) \backslash uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	p	$2p$	$3pq - p$

Theorem 1. The degree ratio Nirmala index of $TUAC_6$ [*p*, *q*] is given by

$$DRN(G)=3\sqrt{2}pq+2\sqrt{\frac{13}{6}}p.$$

Proof: Applying definition and edge partition of $TUAC_6$ [*p*, *q*], we conclude

$$DRN(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}}$$
$$=p\left(\sqrt{\frac{2}{2}+\frac{2}{2}}\right)+2p\left(\sqrt{\frac{2}{3}+\frac{3}{2}}\right)$$
$$+(3pq-p)\left(\sqrt{\frac{3}{3}+\frac{3}{3}}\right)$$

By solving the above equation, we get the desired result.

Theorem 2. The inverse sum indeg index of $TUAC_6$ [*p*, *q*] is

$$ISI(G)=\frac{9}{2}pq+\frac{19}{10}p.$$

Proof: Applying definition and edge partition of $TUAC_6$ [*p*, *q*], we conclude

$$ISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)}$$
$$=p\left(\frac{2\times2}{2+2}\right)+2p\left(\frac{2\times3}{2+3}\right)+(3pq-p)\left(\frac{3\times3}{3+3}\right).$$

By solving the above equation, we get the desired result.

Theorem 3. The degree ratio inverse sum indeg index of $TUAC_6$ [*p*, *q*] is

$$DRISI(G)=\frac{3}{2}pq+\frac{12}{13}p.$$

Proof: Applying definition and edge partition of $TUAC_6$ [*p*, *q*], we conclude

$$DRISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)^2+d_G(v)^2}$$
$$=p\left(\frac{2\times2}{2^2+2^2}\right)+2p\left(\frac{2\times3}{2^2+3^2}\right)$$
$$+(3pq-p)\left(\frac{3\times3}{3^2+3^2}\right).$$

By solving the above equation, we get the desired result.

Theorem 4. The degree ratio inverse sum indeg polynomial of $TUAC_6$ [*p*, *q*] is

$$DRISI(G,x)=3pqx^{\frac{1}{2}}+2px^{\frac{6}{13}}.$$

Proof: Applying definition and edge partition of $TUAC_6$ [*p*, *q*], we conclude

$$DRISI(G,x)=\sum_{uv\in E(G)}x^{\frac{d_G(u)d_G(v)}{d_G(u)^2+d_G(v)^2}}$$
$$=px^{\frac{2\times2}{2^2+2^2}}+2px^{\frac{2\times3}{2^2+3^2}}+(3pq-p)x^{\frac{3\times3}{3^2+3^2}}.$$

By solving the above equation, we get the desired result.

Theorem 5. The modified degree ratio inverse sum indeg index of $TUAC_6$ [*p*, *q*] is

$$^{m}DRISI(G)=6pq+\frac{13}{3}p.$$

Proof: Applying definition and edge partition of $TUAC_6$ [*p*, *q*], we conclude

$$^{m}DRISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)^2+d_G(v)^2}{d_G(u)d_G(v)}$$
$$=p\left(\frac{2^2+2^2}{2\times2}\right)+2p\left(\frac{2^2+3^2}{2\times3}\right)$$
$$+(3pq-p)\left(\frac{3^2+3^2}{3\times3}\right).$$

By solving the above equation, we get the desired result.

Theorem 6. The modified degree ratio inverse sum indeg polynomial of $TUAC_6$ [*p*, *q*] is

$$^{m}DRISI(G,x)=3pqx^2+2px^{\frac{13}{6}}.$$

Proof: Applying definition and edge partition of $TUAC_6$ [*p*, *q*], we conclude

$$^{m}DRISI(G,x)=\sum_{uv\in E(G)}x^{\frac{d_G(u)^2+d_G(v)^2}{d_G(u)d_G(v)}}$$
$$=px^{\frac{2^2+2^2}{2\times2}}+2px^{\frac{2^2+3^2}{2\times3}}+(3pq-p)x^{\frac{3^2+3^2}{3\times3}}.$$

By solving the above equation, we get the desired result.

3. ZigZag Polyhex Nanotubes

The molecular graph of zigzag polyhex nanotube $TUZC_6$ [*p*, *q*] is depicted in below graph.

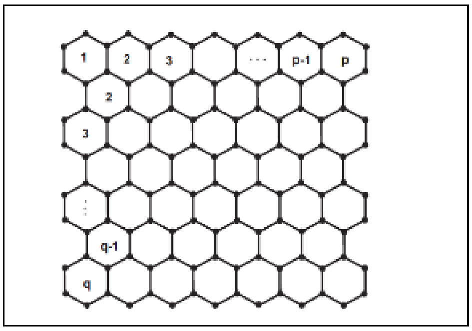

Figure 2

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The graphs of zigzag polyhex nanotubes have $2p(q+1)$ vertices and $3pq + 2p$ edges are shown in the above graph. Let $B = TUZC_6[p, q]$.

We obtain that $\{d(u), d(v): uv \in E(B)\}$ has three edge set partitions.

$d(u), d(v) \setminus uv \in E(B)$	(2, 3)	(3, 3)
Number of edges	$4p$	$3pq - 2p$

Theorem 7. The degree ratio Nirmala index of $TUZC_6$ [p, q] is given by

$$DRN(G) = 6pq + \left(4\sqrt{\frac{13}{6}} - 4\right)p.$$

Proof: Applying definition and edge partition of $TUZC_6$ [p, q], we conclude

$$DRN(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u)}{d_G(v)} + \frac{d_G(v)}{d_G(u)}}$$

$$= 4p\left(\sqrt{\frac{2}{3} + \frac{3}{2}}\right) + (3pq - 2p)\left(\sqrt{\frac{3}{3} + \frac{3}{3}}\right).$$

By solving the above equation, we get the desired result.

Theorem 8. The inverse sum indeg index of $TUZC_6$ [p, q] is

$$ISI(G) = \frac{9}{2}pq + \frac{9}{5}p.$$

Proof: Applying definition and edge partition of $TUZC_6$ [p, q], we conclude

$$ISI(G) = \sum_{uv \in E(G)} \frac{d_G(u) d_G(v)}{d_G(u) + d_G(v)}$$

$$= 4p\left(\frac{2 \times 3}{2 + 3}\right) + (3pq - 2p)\left(\frac{3 \times 3}{3 + 3}\right).$$

By solving the above equation, we get the desired result.

Theorem 9. The degree ratio inverse sum indeg index of $TUZC_6$ [p, q] is

$$DRISI(G) = \frac{3}{2}pq + \frac{11}{13}p.$$

Proof: Applying definition and edge partition of $TUZC_6$ [p, q], we conclude

$$DRISI(G) = \sum_{uv \in E(G)} \frac{d_G(u) d_G(v)}{d_G(u)^2 + d_G(v)^2}$$

$$= 4p\left(\frac{2 \times 3}{2^2 + 3^2}\right) + (3pq - 2p)\left(\frac{3 \times 3}{3^2 + 3^2}\right).$$

By solving the above equation, we get the desired result.

Theorem 10. The degree ratio inverse sum indeg polynomial of $TUZC_6$ [p, q] is

$$DRISI(G, x) = 3pqx^{\frac{1}{2}} + 4px^{\frac{6}{13}} - 2px^{\frac{1}{2}}.$$

Proof: Applying definition and edge partition of $TUZC_6$ [p, q], we conclude

$$DRISI(G, x) = \sum_{uv \in E(G)} x^{\frac{d_G(u) d_G(v)}{d_G(u)^2 + d_G(v)^2}}$$

$$= 4px^{\frac{2 \times 3}{2^2 + 3^2}} + (3pq - 2p)x^{\frac{3 \times 3}{3^2 + 3^2}}.$$

By solving the above equation, we get the desired result.

Theorem 11. The modified degree ratio inverse sum indeg index of $TUZC_6$ [p, q] is

$${}^{m}DRISI(G) = 6pq + \frac{14}{3}p.$$

Proof: Applying definition and edge partition of $TUZC_6$ [p, q], we conclude

$${}^{m}DRISI(G) = \sum_{uv \in E(G)} \frac{d_G(u)^2 + d_G(v)^2}{d_G(u) d_G(v)}$$

$$= 4p\left(\frac{2^2 + 3^2}{2 \times 3}\right) + (3pq - 2p)\left(\frac{3^2 + 3^2}{3 \times 3}\right).$$

By solving the above equation, we get the desired result.

Theorem 12. The modified degree ratio inverse sum indeg polynomial of $TUZC_6$ [p, q] is

$${}^{m}DRISI(G, x) = 3pqx^2 + 4px^{\frac{13}{6}} - 2px^2.$$

Proof: Applying definition and edge partition of $TUZC_6$ [p, q], we conclude

$${}^{m}DRISI(G, x) = \sum_{uv \in E(G)} x^{\frac{d_G(u)^2 + d_G(v)^2}{d_G(u) d_G(v)}}$$

$$= 4px^{\frac{2^2 + 3^2}{2 \times 3}} + (3pq - 2p)x^{\frac{3^2 + 3^2}{3 \times 3}}.$$

By solving the above equation, we get the desired result.

4. Carbon Nanocone Networks

The molecular graph of pentagonal nanocone network CNC_5 [n] is depicted in below graph.

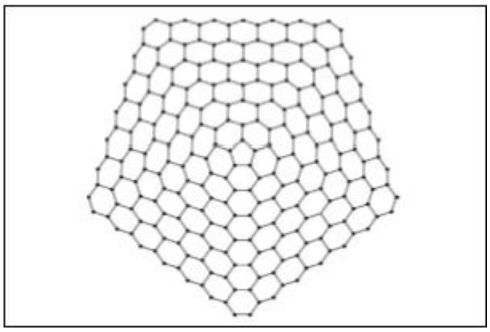

Figure 3

The graphs of pentagonal nanocone networks have $5(n+1)^2$ vertices and $\frac{15}{2}n^2 + \frac{25}{2}n + 5$ edges. Let $G = CNC_5[n]$.

We obtain that $\{d(u), d(v): uv \in E(G)\}$ has three edge set partitions.

$d(u), d(v) \backslash uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	5	$10n$	$\frac{15}{2}n^2+\frac{5}{2}n$

Theorem 13. The degree ratio Nirmala index of $CNC_5[n]$ is given by

$$DRN(G)=\frac{15}{\sqrt{2}}n^2+10\sqrt{\frac{13}{6}}n+\frac{5}{\sqrt{2}}n+5\sqrt{2}.$$

Proof: Applying definition and edge partition of $CNC_5[n]$, we conclude

$$DRN(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}}$$

$$=5\left(\sqrt{\frac{2}{2}+\frac{2}{2}}\right)+10n\left(\sqrt{\frac{2}{3}+\frac{3}{2}}\right)$$

$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\left(\sqrt{\frac{3}{3}+\frac{3}{3}}\right).$$

By solving the above equation, we get the desired result.

Theorem 14. The inverse sum indeg index of $CNC_5[n]$ is

$$ISI(G)=\frac{45}{4}n^2+12n+\frac{15}{4}n+5.$$

Proof: Applying definition and edge partition of $CNC_5[n]$, we conclude

$$ISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)}$$

$$=5\left(\frac{2\times2}{2+2}\right)+10n\left(\frac{2\times3}{2+3}\right)$$

$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\left(\frac{3\times3}{3+3}\right).$$

By solving the above equation, we get the desired result.

Theorem 15. The degree ratio inverse sum indeg index of $CNC_5[n]$ is

$$DRISI(G)=\frac{15}{4}n^2+\frac{60}{13}n+\frac{5}{4}n+\frac{5}{2}.$$

Proof: Applying definition and edge partition of $CNC_5[n]$, we conclude

$$DRISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)^2+d_G(v)^2}$$

$$=5\left(\frac{2\times2}{2^2+2^2}\right)+10n\left(\frac{2\times3}{2^2+3^2}\right)$$

$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\left(\frac{3\times3}{3^2+3^2}\right).$$

By solving the above equation, we get the desired result.

Theorem 16. The degree ratio inverse sum indeg polynomial of $CNC_5[n]$ is

$$DRISI(G,x)=\frac{15}{2}n^2x^{\frac{1}{2}}+10nx^{\frac{6}{13}}+\frac{5}{2}nx^{\frac{1}{2}}+5x^{\frac{1}{2}}.$$

Proof: Applying definition and edge partition of $CNC_5[n]$, we conclude

$$DRISI(G,x)=\sum_{uv\in E(G)}x^{\frac{d_G(u)d_G(v)}{d_G(u)^2+d_G(v)^2}}$$

$$=5x^{\frac{2\times2}{2^2+2^2}}+10nx^{\frac{2\times3}{2^2+3^2}}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{\frac{3\times3}{3^2+3^2}}.$$

By solving the above equation, we get the desired result.

Theorem 17. The modified degree ratio inverse sum indeg index of $CNC_5[n]$ is

$$^{m}DRISI(G)=15n^2+\frac{65}{3}n+5n+10.$$

Proof: Applying definition and edge partition of $CNC_5[n]$, we conclude

$$^{m}DRISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)^2+d_G(v)^2}{d_G(u)d_G(v)}$$

$$=5\left(\frac{2^2+2^2}{2\times2}\right)+10n\left(\frac{2^2+3^2}{2\times3}\right)$$

$$+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)\left(\frac{3^2+3^2}{3\times3}\right).$$

By solving the above equation, we get the desired result.

Theorem 18. The modified degree ratio inverse sum indeg polynomial of $CNC_5[n]$ is

$$^{m}DRISI(G,x)=\frac{15}{2}n^2x^2+10nx^{\frac{13}{6}}+\frac{5}{2}nx^2+5x^2.$$

Proof: Applying definition and edge partition of $CNC_5[n]$, we conclude

$$^{m}DRISI(G,x)=\sum_{uv\in E(G)}x^{\frac{d_G(u)^2+d_G(v)^2}{d_G(u)d_G(v)}}$$

$$=5x^{\frac{2^2+2^2}{2\times2}}+10nx^{\frac{2^2+3^2}{2\times3}}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{\frac{3^2+3^2}{3\times3}}.$$

By solving the above equation, we get the desired result.

5. Conclusion

We have introduced the degree ratio inverse sum indeg and modified degree ratio inverse sum indeg indices of a graph. Also we have determined these newly defined the degree ratio inverse sum indeg indices of armchair polyhex

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nanotubes, zigzag polyhex nanotubes and carbon nanocone networks.

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