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Estimating Fractal Dimension of Indian Coastlines Using Box-Counting and Divider Methods

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Abstract: ***Fractal dimension provides a quantitative way to describe coastline roughness and motivates the classic coastline paradox. In this work, five Indian coastline regions (Gulf of Kutch, Sundarbans, Konkan, Chilika, and Palk Strait) are analyzed using box-counting and Richardson divider methods. Coastline vectors from Natural Earth (1:10m) are clipped to fixed bounding boxes, projected to Web Mercator (selected for uniform pixel spacing within each bounded region, with the caveat that inter-regional distance comparisons are not performed), rasterized at 512–2048 px, and converted to edge masks. Box-counting dimension is estimated from log–log slopes with 128 random grid-origin bootstraps to obtain 95% confidence intervals, and a two-segment piecewise fit is used to identify the best-fit linear scaling region. Segmented box-counting dimensions range from 1.084 to 1.467, with scaling bands of 2.9–4.3 octaves. Divider-based dimensions are consistently lower than segmented box-counting estimates. The resulting pipeline provides a reproducible, uncertainty-quantified approach for comparing coastline roughness across regions and measurement scales.***

Keywords: Fractal dimension, Fractal geometry, Coastline paradox, Box-counting method, Richardson divider method, Indian coastline, Geospatial analysis

1. Introduction

Fractals are complex, self-similar patterns generated by recursion or self-repetition. They are ubiquitous in chaos theory, where deterministic systems produce outcomes whose long-term behavior is effectively unpredictable [1]. Fractal mathematics provides a way to describe the geometric complexity of such systems in a compact, quantitative form [2].

A central tool for this description is the fractal dimension, a non-integer dimension that captures how a geometric object fills space at progressively finer scales. Natural coastlines are a canonical example: as the ruler used to measure a coastline shrinks, the measured length grows without bound- a phenomenon known as the coastline paradox [8]. Mandelbrot (1967) first quantified this scaling behavior for the coastline of Britain, reporting a fractal dimension of approximately 1.25 [8].

India has 7,500 km of coastline spanning geomorphologies as varied as the mangrove-dominated Sundarbans delta, the barrier-lagoon Chilika system, the rocky Konkan shore, the tidal inlet of the Gulf of Kutch, and the shallow-reef Palk Strait. Despite this diversity, fractal-dimension estimates for Indian coastlines remain comparatively sparse in the published literature [18]. The present study estimates and compares fractal dimensions across five Indian coastline regions using two complementary estimators- box-counting and walking-divider (Richardson)- with explicit uncertainty quantification.

2. Literature Survey

The mathematical foundation for fractal analysis of natural objects was laid by Mandelbrot, who demonstrated that the apparent length of the British coastline follows a power-law relationship with the ruler scale and proposed fractal dimension as a representative descriptor of coastline roughness [8, 20]. Falconer subsequently formalized the relationship between topological and fractal dimensions: an object is fractal when its measured dimension exceeds its topological dimension, i.e., $D > 1$ for a coastline [17]. Barnsley provided a general treatment of fractal geometry across mathematical and natural systems [21].

Two estimators dominate the literature. The walking-divider (Richardson) method approximates coastline length at a fixed divider step δ and fits a power law $L(\delta) \propto \delta^{(1-D)}$ [8]. The box-counting method tiles the coastline image with grids of size ε and counts occupied boxes $N(\varepsilon)$; the slope of $\log N(\varepsilon)$ versus $\log(1/\varepsilon)$ gives D [6, 7]. Wu et al. presented a computational implementation of the box-counting method using interval-based scaling and reported its sensitivity to grid placement and scaling-band selection [6]. Piecewise-linear segmented regression, as implemented in the pwlf library [12], has become a standard approach for isolating the best-fit linear scaling region when the log–log plot shows scale breaks.

Region-specific work on Indian coastlines includes Rai et al., who applied fractal-geometry methods to several Indian coastal segments and reported dimensions in the range 1.1–1.4 depending on geomorphic setting [18]. Comparable analyses for United States coastlines by Jiang and Plotnick showed similar inter-regional variability and emphasized the need for standardized scaling-band reporting [19]. Open-source coastline vector products such as Natural Earth [9], the GSHHG shoreline database [11], and OpenStreetMap coastlines [10] now make systematic, reproducible analyses across multiple regions feasible without bespoke digitization.

3. Problem Definition

Despite the maturity of fractal-dimension methods, published estimates for Indian coastlines are limited, are typically reported without uncertainty quantification, and rarely include cross-method comparison between box-counting and divider estimators on the same coastline. This study addresses three specific questions:

1) What are the fractal dimensions of five geomorphologically distinct Indian coastline regions- Gulf of Kutch, Sundarbans (Indian sector), Konkan coast around Ratnagiri, Chilika Lake coast, and Palk Strait near Rameswaram- when estimated from publicly available 1:10m coastline vectors?
2) How sensitive are these estimates to grid placement, rasterization resolution, and the choice of scaling band, and what confidence intervals follow from a grid-origin bootstrap procedure?
3) How do segmented box-counting dimensions compare with walking-divider (Richardson) dimensions applied to the same coastline polyline, and what does the systematic difference between the two estimators imply about reporting practice?

4. Methodology/ Approach

4.1 Data sources and study regions

Coastline vectors were obtained from the Natural Earth 1:10m coastline dataset [9] and clipped to fixed latitude–longitude bounding boxes for five Indian coastal regions, selected to represent a broad range of geomorphological settings (tidal inlet, mangrove delta, rocky shore, barrier-lagoon, and shallow-reef strait): Gulf of Kutch, Gujarat (69.00–70.60°E, 22.10–23.60°N); Sundarbans (Indian sector), West Bengal (88.00–88.90°E, 21.50–22.30°N); Konkan coast around Ratnagiri, Maharashtra (73.00–73.80°E, 16.20–17.40°N); Chilika Lake coast, Odisha (85.00–85.60°E, 19.30–19.90°N); and Palk Strait near Rameswaram, Tamil Nadu (79.00–79.80°E, 8.90–9.70°N).

4.2 Preprocessing

For each region, coastline vectors were clipped to the bounding box and projected to Web Mercator (EPSG:3857). This projection was selected because it yields approximately uniform pixel spacing within each small bounding box; absolute distance comparisons between regions are not performed, so area distortion at these latitudes does not affect the relative fractal estimates. The projected vectors were then rasterized to binary masks at 512, 1024, and 2048 pixel widths while preserving aspect ratio. To ensure the coastline remained visible at all raster scales, the line was buffered ("stroked") before rasterization and an edge mask was extracted using a light dilation–erosion operation.

4.3 Box-counting with grid-origin bootstrap

Box-counting dimension was estimated from an edge mask by counting the number of occupied boxes $N(\varepsilon)$ on a grid of box size ε, then fitting a line to $\log N(\varepsilon)$ versus $\log(1/\varepsilon)$ [6, 8]. To reduce sensitivity to grid placement, a grid-origin bootstrap was used: for each ε, the counting grid was shifted by a random origin offset and the slope was recomputed across 128 bootstrap replicates. The bootstrap distribution was used to report a 95% confidence interval for D.

4.4 Scaling-band selection (segmented fit)

Natural coastlines often show scale breaks rather than a single straight log–log slope. To identify a robust scaling band, a continuous two-segment piecewise linear model was fit to the log–log data, and the slope of the longer segment was reported as the segmented box-counting dimension D_band [12].

4.5 Walking divider (Richardson) method

As an independent cross-check, a walking divider (Richardson) method was applied to the coastline polyline. The coastline length $L(\delta)$ was estimated by stepping along the polyline with divider length δ, and a log–log fit of $L(\delta)$ versus δ was used to estimate D via $D = 1 - m$, where m is the fitted slope [8].

4.6 Software

All analyses were performed in Python using GeoPandas and Shapely for vector processing, rasterio for rasterization, NumPy and SciPy for numerical computation, and pwlf for segmented regression [12]. The complete pipeline was implemented and executed in Google Colab.

5. Results and Discussion

Table 1 summarizes fractal-dimension estimates for the five Indian coastline regions. Segmented box-counting dimensions ranged from 1.084 to 1.467, and divider (Richardson) dimensions ranged from 1.102 to 1.342. Representative analysis outputs are shown in Figures 1 through 19. Figures 1 to 9 illustrate the conceptual framework (geometric fractals, the Mandelbrot set, and the Britain-coastline didactic example); Figures 10 to 19 present the study region and segmented log–log fit for each of the five coastlines.

Table 1: Fractal-dimension estimates for five Indian coastline regions. Bootstrap means and 95% confidence intervals are reported at three raster resolutions; segmented D is the two-slope box-counting dimension at 1024 px; Richardson D is the walking-divider estimate. Dimensions vary substantially across regions, and Richardson estimates are consistently lower than the segmented box-counting values

Region	Resolution (px)	Bootstrap D	95% CI	Segmented D (1024 px)	Richardson D
Gulf of Kutch	512 / 1024 / 2048	1.320 / 1.275 / 1.233	[1.298, 1.341] / [1.258, 1.292] / [1.219, 1.246]	1.292 ($R^2 = 0.999$)	1.132
Sundarbans	512 / 1024 / 2048	1.416 / 1.332 / 1.268	[1.405, 1.427] / [1.324, 1.340] / [1.261, 1.276]	1.467 ($R^2 = 1.000$)	1.342
Konkan	512 / 1024 / 2048	1.232 / 1.189 / 1.154	[1.218, 1.247] / [1.177, 1.199] / [1.144, 1.162]	1.084 ($R^2 = 1.000$)	1.120
Chilika	512 / 1024 / 2048	1.339 / 1.279 / 1.233	[1.320, 1.355] / [1.266, 1.296] / [1.223, 1.244]	1.413 ($R^2 = 1.000$)	1.175
Palk Strait	512 / 1024 / 2048	1.339 / 1.274 / 1.219	[1.304, 1.365] / [1.252, 1.292] / [1.201, 1.234]	1.369 ($R^2 = 1.000$)	1.102

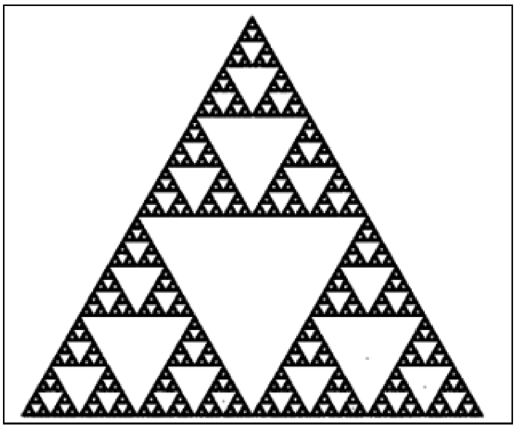

Figure 1: Sierpinski triangle, a classic geometric fractal formed by iteratively removing the central sub-triangle from each remaining triangle. The number of filled triangles triples with each iteration, illustrating exact self-similarity across scales [13].

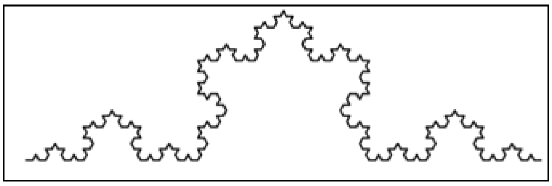

Figure 2: Koch curve, constructed by replacing each line segment with four segments each one-third the original length. The total length grows without bound with each iteration while the curve remains bounded- an intuitive analog of the coastline paradox [14].

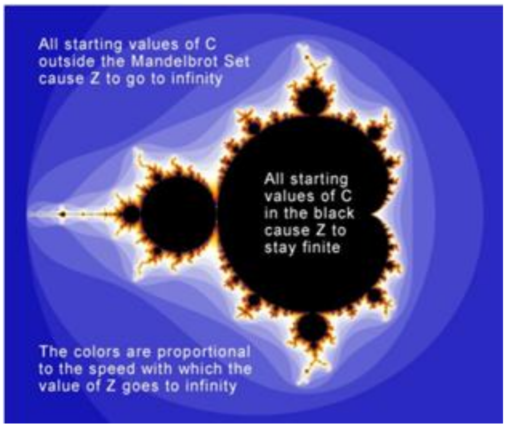

Figure 3: Mandelbrot set, the set of complex numbers c for which the iteration $z \to z^2 + c$ remains bounded starting from $z = 0$. Its boundary is an infinitely complex fractal exhibiting self-similar structure at every scale [15].

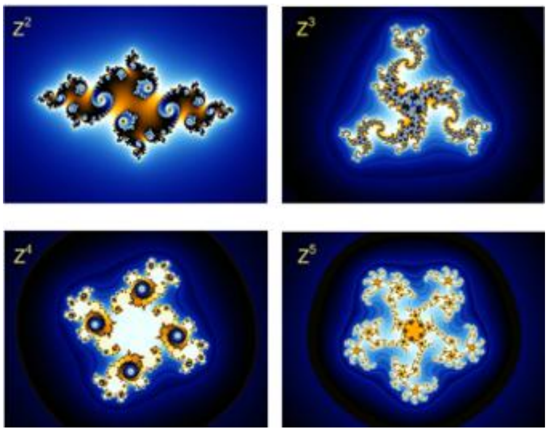

Figure 4: Successive zooms into the Mandelbrot set, illustrating self-similarity across scales. Each panel zooms into a region of the previous, revealing branched structure at every magnification.

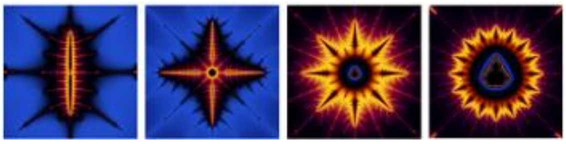

Figure 5: Mandelbrot-type sets generated by replacing z^2 with higher powers ($z \to z^n + c$, $n = 3, 4, 5, 6$). Raising the exponent increases the order of rotational symmetry and the degree of branching

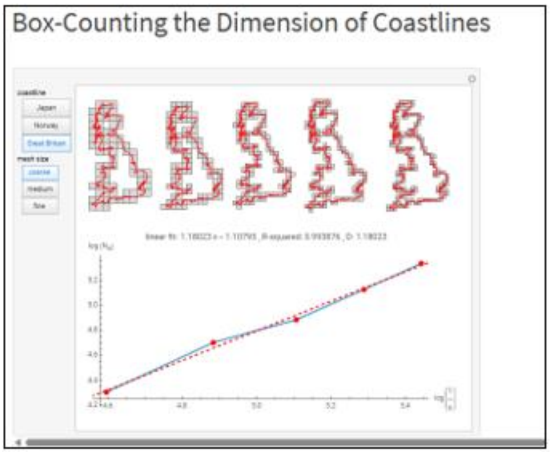

Figure 6: Box-counting visualization of the Great Britain coastline at three mesh sizes (coarse, medium, fine) from the Wolfram Demonstrations Project, with the corresponding log(N) vs log(1/s) plot yielding $D \approx 1.18$ [7]

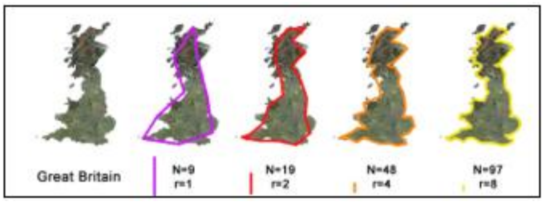

Figure 7: The Britain coastline measured with rulers of length $r = 1, 2, 4$, and 8, yielding perimeter counts $N = 9, 19, 48$, and 97 respectively. As the ruler shrinks, the measured perimeter grows- the empirical signature of the coastline paradox.

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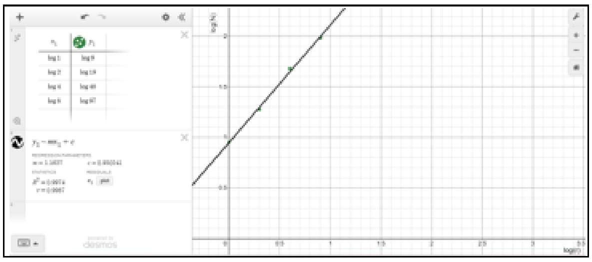

Figure 8: Linear plot of N versus r for the Britain coastline example, reproduced in Desmos. The fitted slope m ≈ 1.1627 with $R^2 = 0.997$ confirms the power-law relationship between coastline length and ruler scale

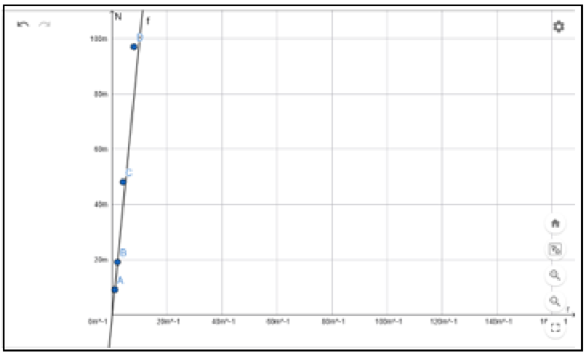

Figure 9: The same example reproduced in GeoGebra, plotting the four (r, N) points A–D. The consistent linear fit across both graphing platforms establishes the reliability of the fractal-dimension estimate for this didactic example

5.1 Gulf of Kutch

Segmented box-counting dimension at 1024 px: D = 1.292 [1.280, 1.310]; scaling-band width ≈ 4.3 octaves ($R^2 = 0.999$). Divider (Richardson) dimension D ≈ 1.132. Bootstrap means at the three resolutions were 1.320, 1.275, and 1.233. Figure 10 shows the study region and Figure 11 the segmented log–log fit.

Figure 10: Gulf of Kutch coastline study region. The narrow tidal inlet between Kachchh and Saurashtra produces moderate coastline complexity.

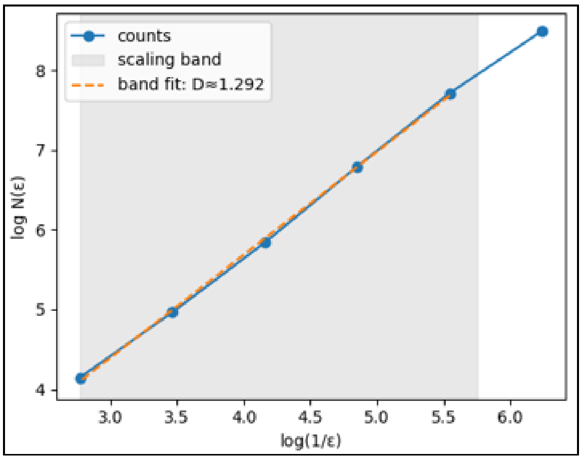

Figure 11: Segmented two-slope log–log box-counting fit for the Gulf of Kutch at 1024 px, yielding D_band = 1.292 with a 4.3-octave scaling band ($R^2 = 0.999$).

5.2 Sundarbans (Indian sector)

Segmented box-counting dimension at 1024 px: D = 1.467 [1.442, 1.488]; scaling-band width ≈ 2.9 octaves ($R^2 = 1.000$). Divider (Richardson) dimension D ≈ 1.342. The dense network of tidal channels and mangrove creeks produces the highest dimension of all five regions (Figures 12, 13).

Figure 12: Sundarbans (Indian sector) coastline study region. The mangrove-delta channel network is visible as a dense network of distributaries

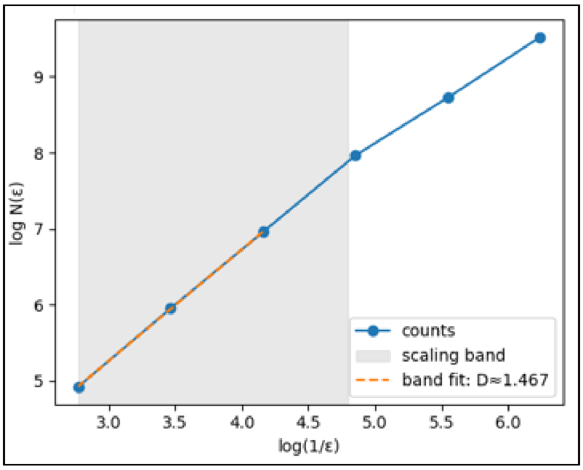

Figure 13: Segmented two-slope log–log fit for the Sundarbans at 1024 px, yielding D_band = 1.467 with a 2.9-octave scaling band ($R^2 = 1.000$)- the highest dimension among the five study regions.

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5.3 Konkan coast around Ratnagiri

Segmented box-counting dimension at 1024 px: D = 1.084 [1.077, 1.091]; scaling-band width ≈ 3.7 octaves (R^2 = 1.000). Divider (Richardson) dimension D ≈ 1.120. The Konkan coast is a relatively straight, tectonically stable rocky shore, producing the lowest dimension of the five regions (Figures 14, 15).

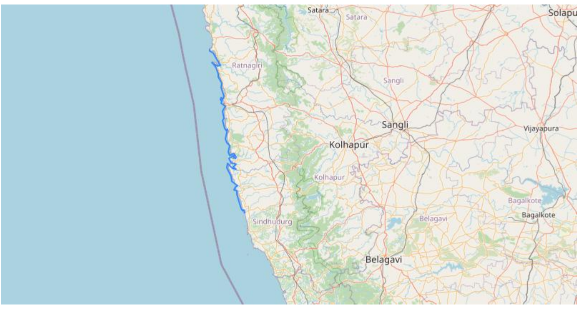

Figure 14: Konkan coast study region around Ratnagiri, Maharashtra. The relatively straight rocky shore lacks major embayments

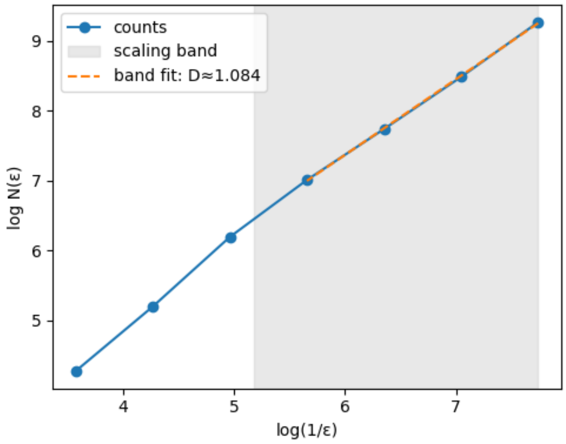

Figure 15: Segmented log–log fit for the Konkan coast at 1024 px, yielding D_band = 1.084 with a 3.7-octave scaling band (R^2 = 1.000). The near-1D dimension confirms the smooth, near-linear shoreline geometry.

5.4 Chilika Lake coast

Segmented box-counting dimension at 1024 px: D = 1.413 [1.383, 1.450]; scaling-band width ≈ 3.4 octaves (R^2 = 1.000). Divider (Richardson) dimension D ≈ 1.175. The barrier-lagoon morphology yields a high segmented dimension but a much lower Richardson estimate, reflecting estimator sensitivity to mixed shoreline types (Figures 16, 17).

Figure 16: Chilika Lake coast study region. The large coastal lagoon is separated from the Bay of Bengal by a sand barrier; both the lagoon margin and the barrier coast contribute to the analysis.

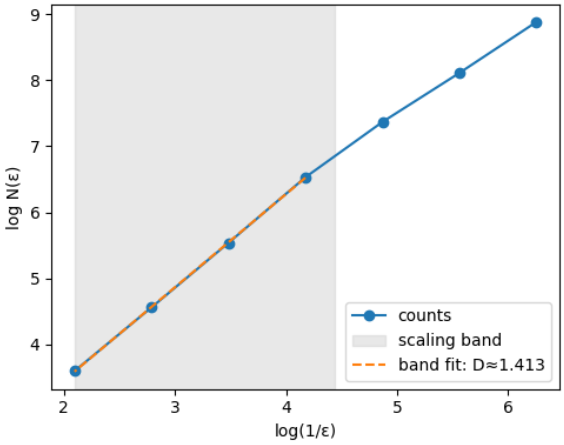

Figure 17: Segmented log–log fit for the Chilika Lake coast at 1024 px, yielding D_band = 1.413 with a 3.4-octave scaling band (R^2 = 1.000). The convoluted lagoon margin drives the high dimension.

5.5 Palk Strait near Rameswaram

Segmented box-counting dimension at 1024 px: D = 1.369 [1.306, 1.438]; scaling-band width ≈ 3.0 octaves (R^2 = 1.000). Divider (Richardson) dimension D ≈ 1.102- the lowest Richardson estimate among the five regions. The wide bootstrap confidence interval reflects the shallow-reef and sand-bar morphology, which is highly sensitive to grid orientation (Figures 18, 19).

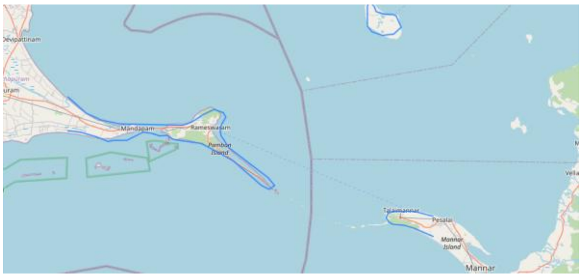

Figure 18: Palk Strait study region near Rameswaram, Tamil Nadu. The shallow-reef and sand-bar morphology between India and Sri Lanka produces the widest bootstrap confidence intervals of the five regions

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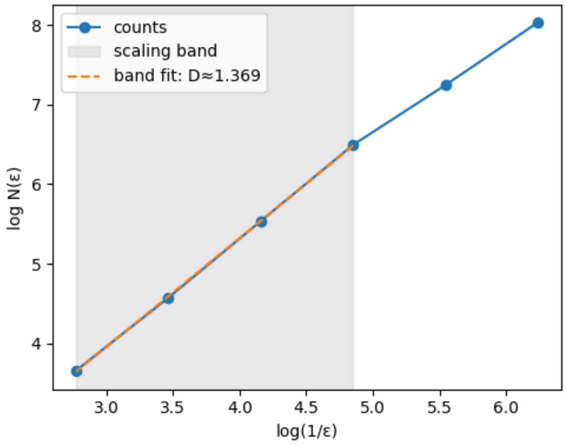

Figure 19: Segmented log–log fit for the Palk Strait at 1024 px, yielding D_band = 1.369 with a 3.0-octave scaling band (R^2 = 1.000). The narrow scaling band reflects sensitivity to grid orientation.

5.6 Discussion

A geometric object is classified as fractal when its measured dimension exceeds its topological dimension [17]. For a coastline, whose topological dimension is 1, any D > 1 indicates fractal character [18]. All five study regions yielded segmented box-counting dimensions strictly greater than 1 (range: 1.084–1.467), and even the smoothest region (the Konkan coast, D = 1.084) exceeds the topological threshold. Values closer to 1.0 indicate smoother shorelines, while values approaching 2.0 correspond to highly convoluted, near-space-filling boundaries [8, 18]. By international comparison with Mandelbrot's canonical Britain dimension of D ≈ 1.25 [8], the Sundarbans (1.467) substantially exceeds and the Konkan coast (1.084) is well below the reference, while the Gulf of Kutch, Palk Strait, and Chilika Lake coast bracket or modestly exceed it.

The wide range of values reflects genuine geomorphological differences. The Sundarbans delta, shaped by active sediment deposition and tidal channels, produces an exceptionally high fractal dimension consistent with previous fractal analyses of delta and estuarine coastlines [18, 19]. Methodological factors- rasterization resolution, choice between box-counting and divider estimators, and scaling-band width, and projection distortion from Web Mercator- also influence the reported dimension [6, 8, 12], underscoring the need for standardized methodology and explicit scaling-band reporting. Because all five regions are analyzed using the same projection and bounding-box approach, relative comparisons remain valid even though Web Mercator introduces area distortion at the geographic scales considered.

6. Conclusion

This study demonstrates that fractal dimension estimation can meaningfully distinguish geometric complexity among selected Indian coastlines, while also showing that estimated values depend strongly on measurement methodology and scale selection. The box-counting framework with uncertainty quantification provides a reproducible comparative approach, whereas divider-based estimates offer complementary scale-sensitive interpretation. Although projection choice, raster resolution, and regional selection impose limitations, the methodology provides a useful foundation for future coastline complexity studies using higher-resolution datasets and broader geographic coverage.

7. Future Scope

Several extensions follow naturally from this work. First, finer-resolution shoreline sources such as GSHHG full-resolution data [11] or OpenStreetMap coastlines [10] could be used to test whether the reported dimensions are stable at sub-decametre scales. Second, the present analysis is limited to five regions; a systematic survey of the entire Indian coastline would enable morphology-class stratification (deltaic, rocky, barrier-lagoon, reef) and statistical comparison with global benchmarks.

Third, multifractal analysis could extend the single-exponent estimator to capture local variations in scaling behaviour along a single coastline. Fourth, the same pipeline could be applied to multi-decadal coastline-vector snapshots to detect temporal changes driven by erosion, accretion, or anthropogenic interventions. Finally, direct comparison with field-surveyed coastline perimeters would help calibrate vector-derived dimensions against ground truth.

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