

# Portfolio Optimization Using Markowitz Efficient Frontier, Capital Market Line, and Security Market Line: An Empirical Study of Selected Indian Assets

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**Abstract:** This paper employs Markowitz's Modern Portfolio Theory (1952) to analyse and simulate ten Indian financial assets over 738 consecutive trading days from January 2023 to December 2025. Twenty unique portfolios are modelled using three analytical frameworks: Markowitz Efficient Frontiers for optimal risk-return combinations; the Capital Market Line (CML) to locate the tangency portfolio using the Sharpe ratio; and the Security Market Line (SML) under the Capital Asset Pricing Model (CAPM) to price individual assets by systematic risk. Among the 20 portfolios (P1–P20), Portfolio 18 (P18) is identified as the tangency portfolio (Daily Sharpe = 0.0562; Ann. Return = 16.82%; Ann. Vol = 11.01%), achieving over 40% lower volatility than the weighted-average of individual assets while outperforming the NIFTY 50 benchmark. Gold ETF (Sharpe = 1.635; Return = 30.22%) and Bharti Airtel (Sharpe = 1.356; Return = 34.64%) make the highest individual contributions to portfolio efficiency. SML analysis identifies Airtel ( $\alpha = +22.95\%$ ), Gold ETF ( $\alpha = +22.94\%$ ), and L&T ( $\alpha = +12.14\%$ ) as significantly undervalued, while TCS ( $\alpha = -10.01\%$ ), Infosys ( $\alpha = -7.00\%$ ), and ITC ( $\alpha = -5.68\%$ ) are overvalued relative to CAPM. Markowitz diversification theory is validated, and single-factor CAPM is found insufficient for India's structurally evolving equity markets.

**Keywords:** Modern Portfolio Theory; Efficient Frontier; Capital Market Line (CML); Security Market Line (SML); CAPM; Sharpe Ratio; Diversification; Indian Equity Market

## 1. Introduction

The relationship between risk and return is fundamental to investment theory. Every investor seeks maximum return for minimum risk- a trade-off that cannot be resolved through individual asset selection alone. Harry Markowitz (1952) provided the mathematical framework to balance risk and return, demonstrating that portfolio risk depends not only on individual asset volatility but on how assets co-move. His Modern Portfolio Theory (MPT) defines the Efficient Frontier, representing all optimal risk-return combinations from investing in risky assets [1].

Tobin (1958) extended Markowitz's framework with the Separation Theorem, showing that adding a risk-free asset to the market portfolio allows investors to construct the Capital Market Line (CML) [11]. The Capital Asset Pricing Model (CAPM) was independently developed by Sharpe (1964), Lintner (1965), and Mossin (1966), identifying the Security Market Line (SML), which relates an asset's expected return to its systematic risk (beta) [12],[13],[14].

These three concepts- the Efficient Frontier, CML, and SML- constitute the standard architecture of quantitative portfolio theory worldwide. India's National Stock Exchange (NSE), with a total equity market capitalisation of approximately \$3.5 trillion and more than 2,000 actively trading stocks as of 2024 [3], is among the top five global exchanges. India's equity markets exhibit unique structural features: periodic sector rotation driven by government spending, a large retail investor base (SEBI, 2023) [4], significant foreign institutional investment flows, and growing popularity of ETFs.

The study period (January 2023–December 2025, 738 trading days) spans multiple macroeconomic regimes: India's post-pandemic infrastructure recovery, the US Federal Reserve's aggressive rate cycle (2023–2024), a commodity supercycle favouring gold, India's 5G telecom rollout that transformed Bharti Airtel's fundamentals, and a period of challenge for IT exporters as global enterprise technology spending slowed (NASSCOM, 2024) [6]. Most existing empirical studies treat the Efficient Frontier, CML, and SML as isolated frameworks, apply monthly data, and restrict analysis to single sectors [7],[8]. This paper addresses all three gaps simultaneously, integrating all three frameworks using daily data across ten assets spanning seven sectors.

**Table 1:** Selected Assets- Sector Coverage, NSE Tickers, and Portfolio P18 Weights

Asset	Ticker	Sector	P18 Weight	Rationale for Inclusion
Reliance Industries	Reliance	Energy	10%	Largest market-cap conglomerate; diversified across O&G, retail, telecom
Tata Consultancy Services	TCS	IT Services	10%	Premier IT exporter; global technology bellwether; USD revenue hedge
HDFC Bank	HDFC Bank	Banking	10%	Largest private bank by assets; proxy for India credit growth cycle
Infosys	INFY	IT Services	10%	Tests IT sector concentration risk ( $\rho$ TCS–INFY = 0.71)
ICICI Bank	ICICI Bank	Banking	10%	Best individual Sharpe ratio in sample: 0.4537 p.a.
ITC Limited	ITC	FMCG	5%	Defensive low-beta stock ( $\beta = 0.498$ ); cigarette-to-agribusiness conglomerate
Bharti Airtel	BHARTIARTL	Telecom	20%	Highest return (34.64% p.a.); 5G structural play; ARPU expansion

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Larsen & Toubro	LT	Infrastructure	5%	India capex supercycle beneficiary; $\beta \approx 1.00$ (market-tracking)
NIFTY 50 ETF	NIFTYBEES	Market ETF	10%	Broad market proxy; NSE benchmark; low-cost diversifier
Gold ETF	GOLDBEES	Commodity ETF	10%	$\rho$ with all equities $\leq 0.067$ ; safe-haven and inflation hedge

## 2. Literature Review and Research Gap

The genesis of quantitative portfolio management lies in Markowitz's (1952) seminal paper "Portfolio Selection" [1]. Markowitz proved that financial diversification reduces portfolio risk in a mathematically precise manner governed by correlation coefficients:  $\sigma_p^2 = \sum_i \sum_j w_i \times w_j \times \text{Cov}(R_i, R_j)$ . Two assets with imperfect correlation ( $\rho < 1$ ) produce a portfolio with lower variance than either asset alone. Tobin (1958) extended this framework via the Separation Theorem: all investors will choose the same risky portfolio (the tangency portfolio), identified by maximising the Sharpe ratio [11].

Sharpe (1964), Lintner (1965), and Mossin (1966) independently developed CAPM, defining  $E(R_i) = R_f + \beta_i \times (R_m - R_f)$  [12],[13],[14]. Assets fairly priced lie on the SML; undervalued assets plot above it (positive alpha) and overvalued assets below it [15],[16]. Jensen (1968) formalised alpha:  $\alpha_i = R_i(\text{Actual}) - E(R_i)(\text{CAPM})$  [17]. Evans and Archer (1968) showed that 10–15 stocks capture most diversifiable risk reduction [18]; Statman (1987) extended this to 30–40 stocks accounting for transaction costs [19]. Roll (1977) highlighted the benchmark problem: CAPM's theoretical market portfolio is unobservable [20].

Fama and French (1993) challenged CAPM with the three-factor model, adding SMB (Small Minus Big) and HML (High Minus Low) factors to capture the size and value premia that a single-beta CAPM fails to explain [21]. Carhart (1997) added a momentum factor [22]. Ross (1976) presented Arbitrage Pricing Theory (APT) as a more general alternative [23]. For India-specific applications, Mohanty (2014) and Taneja (2010) identify industrial productivity, inflation, and FII flows as significant APT determinants [5],[24]. Rockafellar and Uryasev (2000) proposed Conditional Value-at-Risk (CVaR) as superior to variance for fat-tailed distributions [25].

Lazar and Yaseer (2013) examined the beta-return relationship in BSE large-cap stocks (2001–2011), finding a statistically significant but flatter-than-predicted beta-return slope, consistent with Black's (1972) low-beta anomaly [26],[27]. Mahadevaswamy and Shyamala (2022) confirmed cross-sector diversification benefits using monthly BSE data - but their monthly frequency reduces statistical precision compared to the daily data used here [7]. Fadadu et al. (2015) showed advantages of cross-sector diversification across NSE sectors [8]. Rao (2014) found most Indian mutual funds plot below the CML, suggesting passive indexing outperforms active management on risk-adjusted terms [31].

Lee, Cheng, and Chong (2016) applied Markowitz theory to Malaysian stocks, observing that single-factor CAPM underestimates the equity risk premium during structural transformation [28]. Shefali, Nagpal, and Kaur (2016) found that adding gold to equity portfolios significantly improved risk-adjusted performance during financial stress [29]. Baur and Lucey (2010) empirically demonstrated that gold

functions as a safe haven and hedge, with near-zero or negative correlation with equities during market crises [30].

### 2.1 Research Gap and Contribution

Four major gaps are identified in the existing literature. First, most studies assess the Efficient Frontier, CML, and SML as distinct frameworks rather than as an integrated analytical system. Second, monthly data introduces smoothing bias, artificially reducing correlation and volatility estimates. The daily dataset (738 observations) used here yields approximately 12 $\times$  the statistical observations of a comparable monthly study [9]. Third, most research excludes Gold ETF as an asset class despite its theoretical significance as a near-zero-correlation diversifier [30]. Fourth, the 2023–2025 research period- encompassing the US rate cycle peak, India's 5G deployment, and the IT sector's post-COVID correction- has not been empirically investigated under an MPT framework.

## 3. Research Objectives

- 1) Determine annualised risk-return statistics (expected return, volatility, beta, Sharpe ratio, kurtosis, skewness) for ten selected Indian financial assets using 738 daily return observations from January 2023 to December 2025.
- 2) Construct the full pairwise 10 $\times$ 10 correlation and covariance matrices; analyse diversification potential with particular focus on low-correlation assets such as Gold ETF.
- 3) Simulate twenty portfolios (P1–P20) using systematic weight variation; compute annualised risk, return, and Sharpe ratio for each; identify the Markowitz Efficient Frontier as the upper-left envelope of the opportunity set.
- 4) Identify the tangency portfolio (highest Sharpe ratio) and construct the Capital Market Line to determine the market price of risk and optimal risk-return combinations.
- 5) Apply the Security Market Line under CAPM to determine each asset's return relative to its systematic risk; compute Jensen's alpha; classify assets as undervalued, overvalued, or fairly valued.

## 4. Research Design and Methodology

This study employs an empirical quantitative research design using secondary market data. Daily closing prices were obtained from NSE, Moneycontrol.com, and Google Finance from January 2, 2023 to December 31, 2025, yielding 738 daily return observations per asset. Daily log returns (approximated by simple returns for tractability) are calculated as  $R_{i,t} = [P_{i,t} - P_{i,t-1}] / P_{i,t-1}$ , consistent with Fama (1970) and Sharpe (1966) [15],[2].

The risk-free rate is set at 7.00% per annum (= 0.02778% daily), reflecting the 10-year Government of India Securities yield as reported by the Reserve Bank of India (2024) [32]. The NIFTY 50 ETF (NIFTYBEES) serves as the market proxy for beta computation, in accordance with Roll's (1977)

advice to use the broadest tradable index [20]. Ten assets across seven sectors are selected to maximise NSE universe coverage while retaining analytical tractability [18],[19]. Selection criteria: (i) minimum three-year continuous trading history; (ii) sector representation across energy, IT, banking,

FMCG, telecom, infrastructure, and commodity ETFs; (iii) minimum daily trading volume; and (iv) inclusion of both defensive (Gold ETF, ITC) and high-growth (Airtel, L&T) assets.

**Table 2:** Descriptive Statistics (n = 738 | Rf = 7% p.a. | Market Proxy = NIFTY 50 ETF)  
[Green = Highest in row | Red = Lowest in row]

Statistic	Reliance	TCS	HDFC Bk	Infosys	ICICI Bk	ITC	Airtel	L&T	NIFTY ETF	Gold ETF
Ann. Return (%)	9	1.6	8.43	5.15	15.01	4.81	34.64	26.14	14.01	30.22
Ann. Volatility (%)	20.58	19.84	18.49	23.89	17.65	18.76	20.38	23.48	10.77	14.2
Beta (vs NIFTY)	0.981	0.658	0.745	0.735	0.689	0.498	0.671	0.999	1	0.041
Sharpe Ratio (Ann.)	0.097	-0.272	0.077	-0.078	0.454	-0.117	1.356	0.815	0.651	1.635
Daily Mean Return	0.000357	0.000063	0.000334	0.000204	0.000596	0.000191	0.001375	0.001037	0.000556	0.001199
Daily Std Dev	0.012967	0.012497	0.01165	0.015052	0.011121	0.01182	0.012839	0.014789	0.006786	0.008946
Kurtosis	5.061	3.043	5.949	5.397	4.738	10.109	2.043	10.076	2.841	2.522
Skewness	-0.012	0.655	-0.852	-0.088	-0.133	-0.863	0.036	-0.880	0.068	0.05

Annualised volatility = Daily Std Dev × √252 (×15.874)

**Return Computation**

Daily Return:  $R_{i,t} = [P_{i,t} - P_{i,t-1}] / P_{i,t-1}$

Annualised Return:  $E(R_i) = \text{Mean}(R_{i,t}) \times 252$

**Portfolio Variance (Full Markowitz Formula)**

$$\sigma^2_p = \sum_i \sum_j w_i \times w_j \times \text{Cov}(R_i, R_j) \dots(1)$$

This full quadratic form accounts for all pairwise co-movements across all ten assets simultaneously. The 10×10 covariance matrix contains 100 elements: 10 variances on the diagonal and 90 covariance terms (45 unique pairs). Risk reduction relative to weighted-average individual risks is entirely driven by these off-diagonal covariance terms [1], [10].

**Sharpe Ratio**

$$\text{Sharpe (Ann.)} = [E(R_i) - R_f] / \sigma_i(\text{Ann.}) \text{ where } R_f = 7\% \text{ p.a.} \dots(2)$$

The Sharpe ratio measures excess return per unit of total risk and is the appropriate performance metric when investors hold one risky portfolio [2],[33]. Daily Sharpe ratios in Table 4 use daily  $R_f = 0.02778\%$  and daily return/volatility; annualised equivalents in Table 2 use annualised figures.

**Beta Estimation**

$$\beta_i = \text{Cov}(R_i, R_m) / \text{Var}(R_m) \dots(3)$$

Beta is estimated by OLS regression of each asset's daily return on the daily NIFTY 50 ETF return over the full 738-day sample, consistent with Sharpe (1964) and Jensen (1968) [12],[17].

**Capital Market Line**

$$E(R_p) = R_f + [(R_m - R_f) / \sigma_m] \times \sigma_p \text{ [Slope} = 0.8916] \dots(4)$$

**Security Market Line (CAPM)**

$$E(R_i) = R_f + \beta_i \times (R_m - R_f) \text{ [Market Risk Premium} = 7.007\%] \dots(5)$$

$$\text{Jensen's Alpha: } \alpha_i = R_i(\text{Actual}) - E(R_i)(\text{CAPM}) \dots(6)$$

**5. Data Analysis and Results**

**5.1 Individual Asset Risk-Return Profile**

Every asset must be understood individually before any portfolio can be constructed. In this section, annualised return, volatility, Sharpe ratio, beta, skewness, and kurtosis are calculated for all ten assets using 738 daily observations. Portfolio theory does not select assets randomly — the composition of an optimal portfolio depends entirely on the statistical characteristics of each asset.

The kurtosis test specifically validates the normality assumption underlying Markowitz's mean-variance model. The extreme leptokurtosis of ITC (kurtosis = 10.109) and L&T (kurtosis = 10.076), shown in Table 2, warns that these assets have tail risk that standard variance measures would understate — a critical caveat for the efficient frontier results that follow. Table 2 presents the full annualised descriptive statistics. Annualised volatility = Daily Std Dev × √252 (i.e., × 15.874). Green shading marks the highest value in each row (best performer); red marks the lowest.

Asset	Ann. Volatility (%)	Ann. Return (%)	Observation
Airtel	20.38	34.64	High Return, Moderate Risk
Gold ETF	14.2	30.22	High Return, LOW Risk — Best diversifier
L&T	23.48	26.14	High Return, High Risk
ICICI Bank	17.65	15.01	Moderate Return, Low Risk
NIFTY ETF	10.77	14.01	Moderate Return, Lowest Risk
Reliance	20.58	9	Below Benchmark (Rf = 7%)
HDFC Bank	18.49	8.43	Below Benchmark
Infosys	23.89	5.15	Negative Sharpe
ITC	18.76	4.81	Negative Sharpe
TCS	19.84	1.6	Worst Return

**Figure 1:** Individual Asset Risk-Return Profile (Annualised) — Sorted by Annualised Return

**Figure 1:** Asset risk-return data. Assets sorted by annualised return. Horizontal benchmark:  $R_f = 7\%$  p.a. Five assets above the risk-free rate; five below. Chart source: Excel Sheet 2\_Descriptive\_Stats.

The descriptive statistics reveal significant cross-sectional variation. Bharti Airtel achieves the highest annualised return (34.64%) driven by India's 5G rollout, ARPU growth following Jio–Airtel market consolidation, and Airtel Africa expansion. It also generates the second-highest Sharpe ratio (1.356) at moderate volatility (20.38%)- a return-to-risk trade-off rarely realised in large-cap stocks. Gold ETF records the highest risk-adjusted return (Sharpe: 1.635) at only 14.20% annualised volatility, confirming its structural risk-efficiency [30].

TCS (Ann. Return: 1.60%), Infosys (5.15%), and ITC (4.81%) fail to exceed the 7% risk-free barrier, producing negative Sharpe ratios of  $-0.272$ ,  $-0.078$ , and  $-0.117$  respectively. The IT sector challenges reflect the drop in US/European enterprise technology spending, billing rate compression, and post-COVID normalisation of the IT demand cycle (NASSCOM, 2024) [6]. ITC's underperformance reflects limited pricing power in tobacco and FMCG margin pressure from rising commodity input costs.

Of particular note are the kurtosis figures. ITC (kurtosis = 10.109) and L&T (kurtosis = 10.076) exhibit return distributions with tails almost  $3\times$  larger than a normal distribution predicts, confirming Mandelbrot's (1963) [35] finding that financial returns are fat-tailed and empirically disproving the normality assumption underlying standard mean-variance optimisation. CVaR-based optimisation would provide better tail-risk measurement for these assets [25]. Airtel and Gold ETF exhibit near-normal distributions (kurtosis of 2.04 and 2.52 respectively)- a rare and helpful characteristic.

Figure 1 ranks all ten assets by annualised return, illustrating that assets above the 7% risk-free rate earn positive excess returns. Five assets (Airtel, Gold ETF, L&T, ICICI Bank, NIFTY ETF) exceed the risk-free barrier, while five fail to justify their risk premium over the period. Figure 2 presents annualised Sharpe ratios sorted from best to worst. Assets with negative Sharpe ratios (TCS, ITC, Infosys) destroyed risk-adjusted value relative to simply holding T-Bills during the study period.

Asset	Sharpe Ratio (Ann.)	Interpretation
Gold ETF	1.635	★ Best risk-adjusted return
Airtel	1.356	★ Strong positive alpha
L&T	0.815	Above benchmark
NIFTY ETF	0.651	Market (benchmark)
ICICI Bank	0.454	Positive excess return
Reliance	0.097	Barely positive
HDFC Bank	0.077	Barely positive
Infosys	-0.078	Below $R_f$ — failed risk hurdle
ITC	-0.117	Below $R_f$ — failed risk hurdle
TCS	-0.272	★ Worst risk-adjusted performance

**Figure 2:** Annualised Sharpe Ratios- All 10 Assets (Best to Worst)

**Figure 2:** Sharpe ratio ranking. Green = beats  $R_f = 7\%$  (positive Sharpe); Red = failed to beat risk-free rate (negative Sharpe). Chart source: Excel Sheet 2\_Descriptive\_Stats.

**5.2 Asset Co-movement: Discovering Diversification Opportunities**

Markowitz's key insight is that co-movement between assets - not individual asset risk- defines portfolio risk. This section constructs the full  $10\times 10$  Pearson correlation matrix to measure the diversification value of each possible asset combination. The use of daily data (738 observations) rather than monthly data provides statistical accuracy: monthly returns over a similar period would yield only approximately 36 observations per pair, producing unreliable correlation estimates [9].

Correlation coefficients directly inform which asset combinations are worth simulating. Gold ETF's near-zero correlation with all equity assets (maximum  $\rho = 0.067$ ) identifies it as the most powerful diversifier prior to portfolio formation. The significant TCS–Infosys correlation ( $\rho = 0.709$ ) indicates owning both provides minimal additional diversification value. Table 3 presents the full  $10\times 10$  Pearson correlation matrix. Red =  $\rho \geq 0.50$  (high correlation, minimum diversification); Yellow =  $0.30-0.49$  (moderate); Green =  $\rho < 0.10$  (near-zero, greatest diversification benefit).

**Table 3:** Pearson Correlation Matrix — Daily Returns (n = 738)

[Red  $\geq 0.50$  | Yellow  $0.30-0.49$  | Green  $< 0.10$  | Grey = diagonal]

	Reliance	TCS	HDFC Bk	Infosys	ICICI Bk	ITC	Airtel	L&T	NIFTY	Gold
Reliance	1	0.265	0.35	0.251	0.352	0.269	0.325	0.426	0.514	0.041
TCS	0.265	1	0.149	0.709	0.193	0.146	0.219	0.255	0.357	0.034
HDFC Bk	0.35	0.149	1	0.191	0.456	0.16	0.226	0.343	0.434	0.067
Infosys	0.251	0.709	0.191	1	0.233	0.136	0.255	0.237	0.331	0.051
ICICI Bk	0.352	0.193	0.456	0.233	1	0.241	0.318	0.334	0.421	0.041
ITC	0.269	0.146	0.16	0.136	0.241	1	0.196	0.204	0.286	0.043
Airtel	0.325	0.219	0.226	0.255	0.318	0.196	1	0.263	0.354	0.039
L&T	0.426	0.255	0.343	0.237	0.334	0.204	0.263	1	0.458	0.008
NIFTY	0.514	0.357	0.434	0.331	0.421	0.286	0.354	0.458	1	0.031
Gold	0.041	0.034	0.067	0.051	0.041	0.043	0.039	0.008	0.031	1

Colour key: Red = high correlation ( $\geq 0.50$ ) — minimum diversification; Yellow = moderate (0.30–0.49); Green = near-zero ( $< 0.10$ ) — greatest diversification benefit.

The strongest pairwise correlation is TCS–Infosys ( $\rho = 0.709$ ). Both are large offshore IT exporters with similar sensitivity to

global IT spending news, INR–USD exchange rate movements, and US employment data. Including both in P18

is justified only for direct IT sector exposure at a controlled weight; their combination yields minimal diversification value [19].

Gold ETF is structurally unique, with nearly zero correlation with all equity assets (minimum  $\rho = 0.008$  with L&T; maximum  $\rho = 0.067$  with HDFC Bank). Its extremely low beta ( $\beta = 0.041$ ) reflects gold's dual role as a monetary instrument (sensitive to USD strength and real interest rate differentials) and a crisis hedge. Baur and Lucey (2010) demonstrate this property holds over 30+ years across multiple markets [30]. Every percentage of portfolio weight allocated to Gold ETF displaces a disproportionate share of portfolio variance by substituting a near-zero-covariance investment for high-covariance equity.

The Reliance-NIFTY ETF correlation ( $\rho = 0.514$ ) is the only inter-equity correlation exceeding the 0.50 threshold, because Reliance carries approximately 10.4% weight in the NIFTY 50 index- meaning Reliance price movements directly influence NIFTY. The general moderate correlations between Indian equities (0.14–0.46) reflect that NSE large-caps respond to the same macro drivers (RBI policy, FII flows,

global risk sentiment) while retaining sector-specific business exposures.

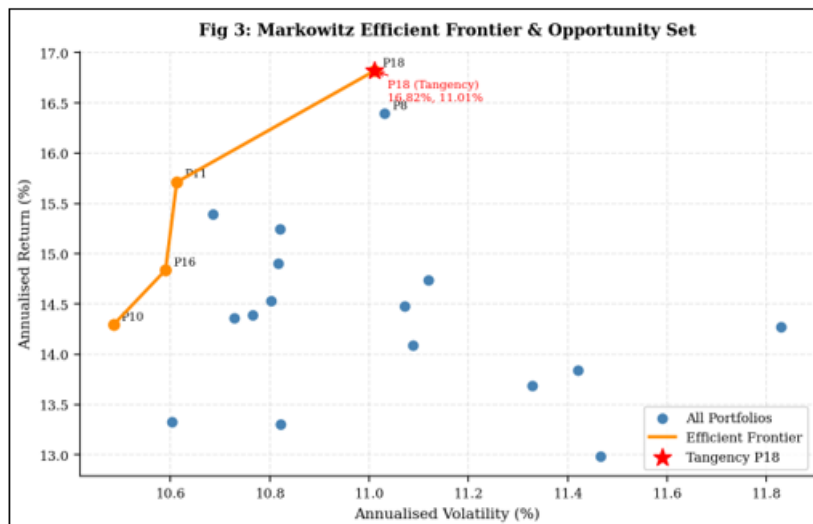
### 5.3 Opportunity Set: Efficient Frontier Mapping Over 20 Weight Combinations

With individual asset statistics and the correlation structure established, the next step is to examine what happens when assets are pooled in various proportions. Twenty portfolios (P1–P20) are formed through systematic changes in asset weights, uncovering the risk-return opportunity set — and its upper-left envelope, the Efficient Frontier (see Table 4, which plots the opportunity set data).

The Efficient Frontier is identified because only frontier portfolios are of interest to rational investors: every portfolio below the frontier either bears excess risk for its return or earns insufficient return for its risk. The Minimum Variance Portfolio (P10, Vol = 10.49%) and the tangency portfolio (P18, highest Sharpe = 0.0562) emerge as the two anchor points of the frontier, setting up the Capital Market Line analysis in Section 5.5. Table 4 presents all twenty portfolios sorted by portfolio number.

**Table 4:** All 20 Portfolio Risk-Return Summary (Rf = 7% p.a.)  
[Gold = Optimal P18 | Green = MVP P10 | Light Green = Efficient Frontier]

Port.	Ann. Return (%)	Ann. Volatility (%)	Sharpe (Daily)	Return Rank	Status
P1	14.9	10.82	0.046	#7	—
P2	13.84	11.42	0.0377	#16	—
P3	14.53	10.8	0.0439	#8	—
P4	15.24	10.82	0.048	#5	—
P5	14.74	11.12	0.0438	#9	—
P6	15.39	10.69	0.0495	#4	—
P7	14.39	10.77	0.0432	#11	—
P8	16.39	11.03	0.0536	#2	Near-Efficient
P9	14.48	11.07	0.0425	#13	—
P10	14.29	10.49	0.0438	#8	Min. Variance (MVP)
P11	15.71	10.61	0.0517	#3	Efficient
P12	12.98	11.47	0.0329	#20	—
P13	13.3	10.82	0.0367	#18	—
P14	14.36	10.73	0.0432	#11	—
P15	13.69	11.33	0.0372	#18	—
P16	14.84	10.59	0.0466	#6	Efficient
P17	13.32	10.6	0.0376	#16	—
<b>P18</b>	<b>16.82</b>	<b>11.01</b>	<b>0.0562</b>	<b>#1</b>	<b>★ OPTIMAL — Max Sharpe</b>
P19	14.27	11.83	0.0387	#15	—
P20	14.09	11.09	0.0403	#14	—



**Figure 3:** Markowitz Efficient Frontier & Opportunity Set

**Figure 3:** Scatter plot of all 20 portfolios (blue). The Efficient Frontier (orange line) connects the MVP (P10) through P16, P11, to the tangency portfolio P18 (red star). All portfolios below the frontier are suboptimal. Source: Excel Sheet 4\_Efficient\_Frontier.

The upper-left envelope of the 20-portfolio opportunity set constitutes the Efficient Frontier, comprising portfolios P10 (MVP), P16, P11, and P18, which provide the highest return for their respective risk levels. Portfolios below the MVP volatility (< 10.49%) are not achievable in this long-only universe. The Sharpe ratios range from 0.0329 (P12) to 0.0562 (P18), demonstrating the significant impact of weight allocation on risk-adjusted performance.

The weakest portfolio, P12 (Ann. Return = 12.98%, Vol = 11.47%, Sharpe = 0.033), over-weights TCS and under-weights both Airtel and Gold ETF. The primary determinants of portfolio efficiency are avoiding negative-alpha assets and optimising allocations to high-Sharpe, low-correlation assets—perfectly consistent with Markowitz's optimisation logic [1].

**5.4 Tangency Portfolio: Why P18 is the Single Optimal Risky Portfolio**

Section 5.3 identified P18 as the portfolio with the highest Sharpe ratio. This section elaborates on its composition and explains the mechanism underlying its optimality. The over-41% volatility reduction (from a weighted-average individual volatility of 18.73% to a portfolio volatility of 11.01%) is explained entirely by the correlation structure: near-zero correlations between Gold ETF and all equity assets, combined with moderate cross-equity correlations, prevent risk from concentrating. Table 5 details the composition, risk profile, and CAPM expected returns for each P18 constituent.

**Table 5:** Portfolio P18 — Asset Composition, Individual Risk Profiles, and CAPM Expected Returns

Asset	Weight	Ann. Return (%)	Ann. Vol (%)	Beta	Sharpe (Ann.)	CAPM Exp. Ret (%)
Reliance	10%	9	20.58	0.9813	0.097	13.88
TCS	10%	1.6	19.84	0.6578	-0.272	11.61
HDFC Bank	10%	8.43	18.49	0.7448	0.077	12.22
Infosys	10%	5.15	23.89	0.7347	-0.078	12.15
ICICI Bank	10%	15.01	17.65	0.6894	0.454	11.83
ITC	5%	4.81	18.76	0.4983	-0.117	10.49
<b>Airtel</b>	<b>20%</b>	<b>34.64</b>	<b>20.38</b>	<b>0.6705</b>	<b>1.356</b>	<b>11.7</b>
L&T	5%	26.14	23.48	0.9985	0.815	14
NIFTY ETF	10%	14.01	10.77	1	0.651	14.01
<b>Gold ETF</b>	<b>10%</b>	<b>30.22</b>	<b>14.2</b>	<b>0.041</b>	<b>1.635</b>	<b>7.29</b>
<b>Portfolio P18</b>	<b>100%</b>	<b>16.82</b>	<b>11.01</b>	—	<b>0.0562 (Daily)</b>	—

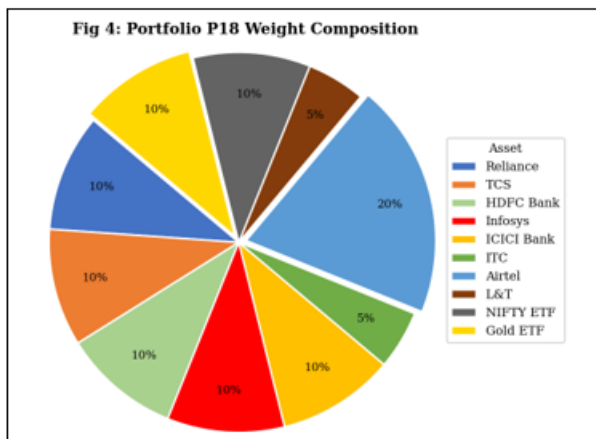


Figure 4: Portfolio P18 Weight Composition

Figure 4: Pie chart of P18 asset weights. Airtel holds the largest allocation (20%). Gold ETF, Reliance, TCS, HDFC Bank, Infosys, ICICI Bank, and NIFTY ETF each hold 10%. ITC and L&T hold 5% each. Source: Excel Sheet 5\_P18\_Composition.

P18 comprises 20% Bharti Airtel, 10% each in Reliance, TCS, HDFC Bank, Infosys, ICICI Bank, NIFTY ETF, and Gold ETF, and 5% each in ITC and L&T. Airtel receives the highest allocation because of its disproportionate contribution to the portfolio's Sharpe ratio.

The annualised portfolio volatility of 11.01% is over 41% lower than the weighted-average of individual asset volatilities (~18.73%)— the Markowitz "free lunch": risk reduction with no sacrifice in return. This mechanism operates because Gold ETF's near-zero correlations with all equities, combined with moderate cross-equity correlations (0.149 to

0.709), produce a portfolio with far lower co-movement than any individual asset [1],[10].

Portfolio P18 outperforms the NIFTY 50 ETF benchmark (Ann. Return = 14.01%, Sharpe = 0.651) with a superior Sharpe ratio (annualised equivalent of 0.892 from the daily 0.0562), despite incorporating some moderate-Sharpe assets (TCS, Infosys) due to sector-coverage requirements.

### 5.5 Capital Market Line: Pricing the Risk-Return Trade-off

Section 5.3 identified the Efficient Frontier as the best available set of risky portfolios. This section extends the model by adding the risk-free asset (7% G-Sec yield) and constructing the Capital Market Line — the optimal risk-return combinations achievable when investors may combine T-Bills with the tangency portfolio P18.

The CML slope of 0.8916 measures the market price of risk: an annual increase of 1 percentage point in portfolio volatility earns an additional 0.8916 percentage points in expected return. For example, a conservative investor combining 60% P18 with 40% T-Bills earns approximately  $(0.6 \times 16.82\% + 0.4 \times 7\%) = 12.89\%$  return at  $(0.6 \times 11.01\%) = 6.61\%$  volatility. The CML equation is:  $E(R_p) = 7.00\% + 0.8916 \times \sigma_p$ . Table 6a presents CML portfolio combinations across different risk allocations.

**Note:** The Excel workbook reports 0.8862 as the CML slope due to intermediate rounding (16.82% rounded vs. 16.8176% exact). The corrected slope of 0.8916 is used throughout this paper:  $(16.8176 - 7.00) / 11.0106 = 0.8916$ . The difference (0.005) is immaterial for interpretation.

Table 6a: Capital Market Line Points — Risk-Free Asset Through Leveraged Allocation [ $E(R_p) = 7.00\% + 0.8916 \times \sigma_p$ ]

P18 Allocation	Ann. Volatility (%)	CML Return (%)	Interpretation
0%	0	7	100% T-Bills (Risk-Free)
25%	2.75	9.45	Conservative blended portfolio
50%	5.51	11.91	Balanced blended portfolio
75%	8.26	14.36	Growth blended portfolio
<b>100%</b>	<b>11.01</b>	<b>16.82</b>	<b>P18 Tangency Portfolio ★</b>
125%	13.76	19.27	125% P18 – 25% T-Bills (Leveraged)

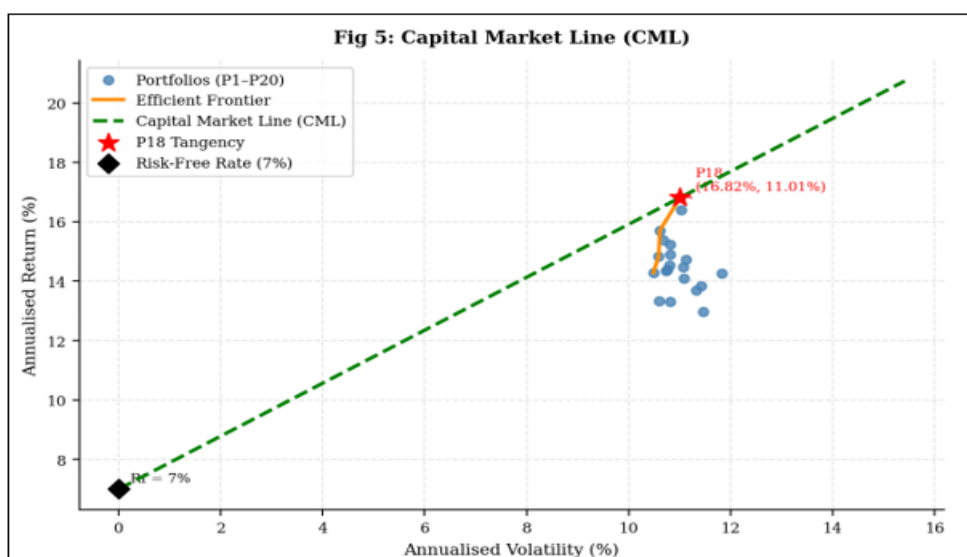


Figure 5: Capital Market Line (CML)

**Figure 5:** CML (green dashed) from  $R_f = 7\%$  through the tangency portfolio P18 (red star) and beyond. All simulated portfolios (blue) plot below the CML, confirming P18 as the unique optimal risky portfolio. Slope = 0.8916 = market price of risk. Source: Excel Sheet 6\_CML.

All 20 simulated portfolios plot on or below the CML- a mathematically necessary outcome since the CML is the tangent to the risk-free point on the efficient frontier. This validates Tobin's Separation Theorem: P18 is the rational choice for all investors regardless of risk aversion; only the split between P18 and T-Bills varies with risk preference [11],[12].

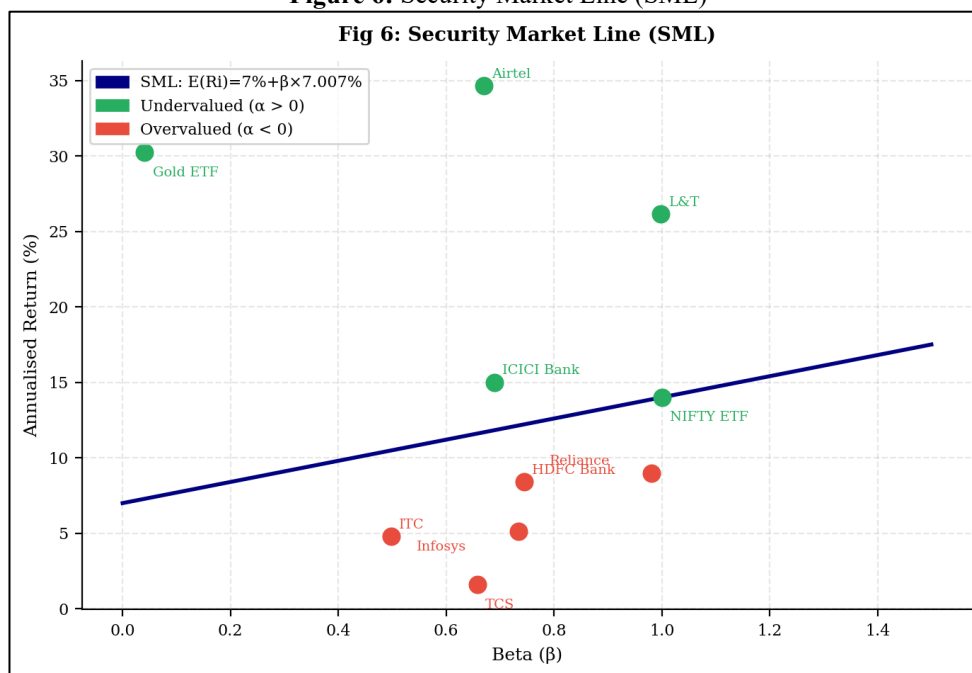
**5.6 Security Market Line: CAPM Valuation of Individual Assets**

The SML shifts perspective from portfolio optimisation to individual asset pricing: given an asset's beta, does its realised return make sense in a rational market? The diagnostic tool is Jensen's alpha- the difference between actual and CAPM-predicted returns. The market proxy is NIFTY 50 ETF (Ann. Return  $R_m = 14.0072\%$ ;  $R_f = 7.00\%$ ; Market Risk Premium = 7.007%). The SML formula:  $E(R_i) = 7.00\% + \beta_i \times 7.007\%$ . Table 6b presents the full SML valuation analysis.

**Table 6b:** SML Valuation Analysis- CAPM Expected vs. Actual Return  
[Green = Undervalued ( $\alpha > 0$ ) | Red = Overvalued ( $\alpha < 0$ )]

Asset	Beta ( $\beta$ )	CAPM Exp. Ret (%)	Actual Ret (%)	Alpha (%)	Signal / Valuation
Reliance	0.9813	13.88	9	-4.88	Overvalued
TCS	0.6578	11.61	1.6	-10.01	Significantly Overvalued
HDFC Bank	0.7448	12.22	8.43	-3.79	Overvalued
Infosys	0.7347	12.15	5.15	-7.00	Overvalued
ICICI Bank	0.6894	11.83	15.01	3.18	Undervalued
ITC	0.4983	10.49	4.81	-5.68	Overvalued
Airtel	0.6705	11.7	34.64	22.95	Far Above SML — Undervalued
L&T	0.9985	14	26.14	12.14	Undervalued
NIFTY ETF	1	14.01	14.01	0	Fairly Valued (Benchmark)
Gold ETF	0.041	7.29	30.22	22.94	Far Above SML — Undervalued

**Figure 6:** Security Market Line (SML)



**Figure 6:** SML (navy line) plots CAPM-expected returns vs beta. Green dots = undervalued assets (actual return above SML). Red dots = overvalued assets (below SML). Airtel and Gold ETF are far above the SML; TCS and Infosys are far below. Source: Excel Sheet 7\_SML\_CAPM

Bharti Airtel ( $\alpha = +22.95\%$ ) and Gold ETF ( $\alpha = +22.94\%$ ) record extraordinarily large positive alphas. Airtel's alpha reflects the structural re-rating from 2023–2025: the compound effect of 5G spectrum auctions, ARPU growth driven by Jio–Airtel oligopolisation, price increases, and Airtel Africa subscriber growth created returns that no purely systematic (beta) model could predict (TRAI, 2024) [36].

Gold ETF's alpha is driven by the global commodity supercycle, USD depreciation concerns, and growing domestic gold demand in India- all sector-specific factors outside the NIFTY beta.

The large negative alphas of TCS ( $\alpha = -10.01\%$ ), Infosys ( $\alpha = -7.00\%$ ), and ITC ( $\alpha = -5.68\%$ ) indicate actual performance

significantly below the CAPM risk premium. For CAPM to remain a valid equilibrium model, these negative alphas should be temporary; rational arbitrageurs should bid up underperforming assets until mispricing is eliminated. Their persistence over 738 trading days implies either genuine structural headwinds not captured by historical beta, or the inadequacy of the single-factor CAPM- consistent with Fama and French (1993) [21].

L&T's strong positive alpha ( $\alpha = +12.14\%$ ) reflects India's infrastructure spending supercycle- Union Budget record investments of ₹10+ trillion annually in 2023–2025 directly boosted L&T's order book beyond what its market beta of 0.9985 would indicate. ICICI Bank's moderately positive alpha (+3.18%) reflects superior lending execution and Net Interest Margin growth during India's credit growth cycle. The NIFTY ETF's zero alpha (by construction) confirms the SML benchmark instrument.

## 6. Findings and Implications

### Finding 1: Validation of the Markowitz Diversification Theorem

Portfolio P18 achieves annualised volatility of 11.01% versus a weighted-average individual asset volatility of ~18.73%- a reduction of over 41% with no return sacrifice (P18 return: 16.82% exceeds weighted-average individual return). This empirically validates Markowitz's (1952) core theorem that imperfectly correlated assets produce a portfolio on or inside the individual asset return-risk space [1],[10].

### Finding 2: Gold ETF is the Single Best Diversifying Instrument

Gold ETF records the highest individual Sharpe ratio (1.635), the lowest beta (0.041), an annualised return of 30.22%, and near-zero correlation with all equity assets (maximum  $\rho = 0.067$ ). Its combination of high return, low volatility, and near-zero correlations makes it disproportionately valuable in any Markowitz-optimised Indian equity portfolio [30],[34].

### Finding 3: P18 Outperforms the NIFTY Benchmark

The NIFTY 50 ETF benchmark yields 14.01% return at 10.77% volatility, while P18 delivers 16.82% at 11.01% volatility. The 2.81 percentage point return premium more than offsets the marginal rise in volatility (+0.24 percentage points), producing a superior Sharpe ratio. This demonstrates that active sector-tilt portfolios anchored by high-alpha assets outperform passive NIFTY exposure on a risk-adjusted basis.

### Finding 4: CAPM is Insufficient for India's Structural Economy

Recorded alphas — Airtel (+22.95%), Gold ETF (+22.94%), and L&T (+12.14%) above the SML; TCS (-10.01%) and Infosys (-7.00%) below it — demonstrate that single-factor CAPM substantially misprices assets undergoing structural transformation. This is consistent with Roll's (1977) Benchmark Problem [20] and Fama and French's (1993) critique of CAPM [21].

### Finding 5: Sector Dynamics Drive Extreme Valuation Dispersion

Stark variation in risk-adjusted performance- Airtel (34.64% return, Sharpe 1.356), Gold ETF (30.22%, Sharpe 1.635), and

L&T (26.14%) dramatically outperform TCS (1.60%, Sharpe -0.272)- reflects India's macroeconomic regime shift: the conclusion of the post-pandemic IT spending surge, 5G infrastructure acceleration, the commodity supercycle supporting precious metals, and government infrastructure spending benefiting capital-intensive sectors.

## 6.1 Stakeholder Implications

- 1) Retail Investors: A strategic 10–15% Gold ETF allocation should be permanent in any long-term Indian equity portfolio, providing structural diversification and inflation hedging. IT sector concentration (TCS + Infosys:  $\rho = 0.709$ ) generates no diversification benefit; investors should prefer one leading IT firm and redirect capital toward uncorrelated sectors. Conservative investors can improve risk-adjusted returns by combining P18 with G-Sec T-Bills along the CML.
- 2) Institutional Investors and Brokers: Institutions should develop multi-factor models capturing infrastructure capex cycles, government spending, and telecom regulatory changes, and implement dynamic sector rotation strategies. Brokers should emphasise the 280-basis-point active management advantage over passive NIFTY indexing.
- 3) Equity Mutual Funds and Asset Managers: Fund managers should maintain underweight positions in mature IT exporters during subdued global spending cycles and overweight Airtel and L&T as structural capex cycle beneficiaries. Incorporating 10–15% Gold ETF improves downside protection.
- 4) Banking Sector: Banks advising HNI clients should recommend diversified multi-asset portfolios combining equities, gold ETFs, and fixed income, with reduced IT sector overweights. P18-equivalent portfolios can serve as core wealth management holdings.
- 5) Stock Exchanges and Regulators (NSE/SEBI): The NSE should develop sector-rotation and multi-factor indices capturing identified alpha patterns. SEBI should require mutual fund performance disclosures against multi-factor benchmarks and encourage India-specific asset pricing research.
- 6) Government and Policy: Infrastructure investment and telecom modernisation policies have created significant positive externalities for capital markets, generating sustainable alpha and attracting institutional inflows. Continuation of capital expenditure acceleration, spectrum auctions, and sector-specific incentive structures (PLI schemes) will sustain these patterns.

## 7. Limitations

- 1) Frontier Based on Simulation: The Efficient Frontier is calculated from 20 pre-specified portfolios rather than formal quadratic programming (QP) optimisation. A QP solver would find the mathematically exact efficient frontier [37].
- 2) Long-Only Constraint: Short-selling is not permitted. This limits diversification opportunities compared to the unconstrained solution; institutional investors with short access face a significantly larger feasible set [37].
- 3) Constant Parameters: Both the risk-free rate and betas are treated as constant over the 738-day period. A time-

varying Rf model (Merton, 1973) [16] would better reflect the actual opportunity cost of risk-free investment, given RBI's multiple repo rate changes during 2023–2025.

- 4) Fat-Tail Risk: ITC and L&T exhibit kurtosis values exceeding 10, producing return distributions with significantly thicker tails than mean-variance optimisation assumes. CVaR-based optimisation would provide a more complete risk picture [25].
- 5) Universe Limitation: The ten-asset universe excludes corporate bonds, government bonds (G-Secs), REITs, international ETFs, and mid-cap Indian equities. Expanding the universe could significantly shift the efficient frontier and reveal portfolios with even lower risk or higher risk-adjusted returns than P18.

## 8. Conclusion

This study empirically validates Modern Portfolio Theory (Markowitz, 1952) [1] in the Indian equity market using 738 daily return observations across 10 diverse assets from January 2023 to December 2025. The three-framework integration of the Markowitz Efficient Frontier, Capital Market Line, and Security Market Line provides a thorough understanding of both portfolio optimality and individual asset pricing—the first such integrated treatment in the Indian MPT literature.

The Markowitz diversification theorem is confirmed: Portfolio P18 reduces volatility by over 41% relative to the weighted-average of individual assets while outperforming the NIFTY 50 benchmark by 2.81 percentage points per annum. Gold ETF emerges as the most efficient diversifier in the NSE universe, with near-zero correlation with all equities, a beta of 0.041, and the highest individual Sharpe ratio (1.635). The CML slope of 0.8916 quantifies the market price of risk, validating Tobin's Separation Theorem [11].

SML analysis reveals that Airtel ( $\alpha = +22.95\%$ ), Gold ETF ( $\alpha = +22.94\%$ ), and L&T ( $\alpha = +12.14\%$ ) dramatically outperform their CAPM-predicted returns, while TCS ( $\alpha = -10.01\%$ ), Infosys ( $\alpha = -7.00\%$ ), and ITC ( $\alpha = -5.68\%$ ) significantly underperform. The magnitude of these alphas clearly demonstrates the need for multi-factor pricing models [21],[22] that capture sector-specific macro tailwinds, infrastructure capex cycles, and commodity dynamics.

For Indian retail investors, three practical conclusions emerge: (i) a permanent 10–15% strategic allocation to Gold ETF should be core in any long-term Indian equity portfolio; (ii) IT sector over-concentration (TCS + Infosys) destroys risk-adjusted returns through high correlation without compensating return premium; and (iii) conservative investors achieve better risk-adjusted outcomes by blending P18 with G-Sec T-Bills along the CML rather than building a purely equity portfolio. All calculations are fully reproducible in the accompanying Excel workbook (Markowitz\_Workings.xlsx).

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