Mathematical Modeling for the Spread and Control of Infectious Diseases by Spread of Awareness in Human Population

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Abstract: ***Now India become populous country in whole world and with the growth in population diseases are also increasing day by day. Some of these diseases have serious concern which effect wide range of area of whole country. One of such disease which quickly escalate and whole nation suffered a lot was Covid-19. If we know in advance growth rate of infection and able to predict its future trends, effective policies and preventive measures can be taken which will help in minimizing the serious effects of the diseases. Also, it is said that precaution is better than cure. Keeping in view the current situation of society, the objective of the present paper is to study the effect of spreading awareness regarding the infectious diseases in human population. A mathematical model is proposed which portray the spread and control of infectious diseases by spreading the awareness of diseases among humans. The model leverages the stability theory of differential equations to understand the system behavior in several situations. Numerical simulation is conducted to conclude the results obtained in this study.***

Keywords: Mathematical Modeling; Infectious Diseases, Control Programs; Differential Equations; Numerical Simulation; Control Measures.

1. Introduction

Over the course of the last 20 years, the world has faced several major infectious disease outbreaks, with notable examples including swine flu, SARS, Ebola, Covid-19 and the more recent emergence of the Zika virus [1]. Infectious diseases caused by microorganisms like viruses, fungi, bacteria, and parasites can be very harmful and life-threatening. These diseases spread in various ways, including direct and indirect person-to-person transmission, contamination from food and water, or through animals (zoonotic) and insects [2]. Due to the globalized travel and significant advances in social media, information about these outbreaks is spreading quite quickly, and this, in turn, can have a profound effect on the actual epidemic dynamics [3, 4, 5, 6].

The advantage of applying Mathematical models of differential equations in disease dynamics is actually an important approach to understand the disease pattern in any population [7,8].Here, we consider a mathematical model to see the effect of imparting information about a disease in human population. Human population behavior plays a very crucial role in controlling number of cases of morbidity and mortality [9,10]. People have a very usual behavior towards the disease spread but after getting more knowledge of the disease, they become aware and their behavior is changed. This effect has been observed since few decades. In the 19th century, many awareness programs have been conducted by the several health care agencies, like WHO, CDC, etc. [11,12,13].

Most of the nonlinear mathematical model has tried to show the disease dynamics and their control by introducing the control measure as a dynamical variable. However, to understand exactly the behavior of human population towards the disease in absence as well as in presence of awareness programs by media campaign, it is very important to describe the disease pattern in a basic manner. Therefore, we present a mathematical model which not only explains the certain disease dynamics but also its control [14].

The whole paper is organized as follows: In section 2, model is proposed, equilibrium analysis and stability analysis of all possible equilibria is presented in section 3. Numerical simulation is performed to validate the analytical findings in section 4. Finally, in section 5 concluding remarks has been presented with references at the end.

2. Model Proposed

Let in the region, N(t) be the total human population at any time t and is categorized into two classes; namely, susceptible human population X(t) and infected human population Y(t). Let A be the immigration rate to the class of the susceptible population. Let M(t) be the number of awareness programs. We assume that the growth rate of number of awareness programs to aware the population is to follow the logistic model with intrinsic growth rate r and carrying capacity K. Further, it is assumed that growth rate of number of awareness programs is a bilinear response between the number of infected humans and the number of awareness programs. The model involves the following nonlinear ordinary differential equations

$$\frac{dX(t)}{dt} = A - \beta\, X(t)Y(t) - \mu\, X(t) + \upsilon\, Y(t),$$
$$\frac{dY(t)}{dt} = \beta\, X(t)Y(t) - \upsilon\, Y(t) - \alpha\, Y(t) - \mu\, Y(t) \qquad (1)$$
$$\frac{dM(t)}{dt} = r\left(1 - \frac{M(t)}{K}\right)M(t) + \theta Y(t)M(t).$$

where, $X(0) = X0 > 0, Y(0) = Y0 \geq 0$, $M(0) = M0 \geq 0$.

In the above model system (1), β is the contact rate between susceptible humans with infected humans in absence of awareness. The constants v, α and μ represent the recovery rate, disease-induced death rate and natural death rate, respectively. The constant r represents the intrinsic growth rate and K is it carrying capacity. The parameter θ represents the per capita growth rate of the awareness programs which is allotted to aware the population due to increase in infected individuals. All the above parameters are assumed to be positive.

The 3rd equation of model system (1) assumes that the per capita growth rate of awareness programs to aware the population is proportional to a number of infected individuals in the population. Using the fact that X(t) + Y (t) = N(t), the above system reduces to the following system:

$$\frac{dY(t)}{dt} = \beta\left(N(t) - Y(t)\right)Y(t) - v\,Y(t) - \alpha\,Y(t) - \mu\,Y(t),$$

$$\frac{dN(t)}{dt} = A - \mu\,N(t) - \alpha\,Y(t), \quad (2)$$

$$\frac{dM(t)}{dt} = r\left(1 - \frac{M(t)}{K}\right)M(t) + \theta Y(t)M(t). \quad (3)$$

3. Equilibrium Analysis and Stability Analysis

3.1 Equilibrium Analysis

In this section, we show the equilibrium conditions of all the possible equilibria by putting, the rate of change of the dynamical variables to zero. The model system (3) has four feasible equilibria, which are follows:

1) The disease and awareness free equilibrium $E_1(0, A/\mu, 0)$.
2) The disease-free equilibrium $E_2(0, A/\mu, K)$.
3) The awareness free endemic equilibrium $E_3(Y_3^*, N_3^*, 0)$. The equilibrium E_3 is feasible only when $\beta A - \mu(v + \alpha + \mu) > 0$.
4) The interior equilibrium $E_4, (Y*, N*, M*)$ The equilibrium E_4 is feasible if

$$\beta A - \mu(v + \alpha + \mu) > 0.$$

Let us denote, $R_0 = \frac{\beta A}{\mu(v+\alpha+\mu)}$, then the quantity R0 is known as 'basic reproduction number', the average number of secondary infection produced by an infected individual during his/ her whole infectious period in fully susceptible population [1]. For model system (3) if $R_0 > 1$ then there exists a unique awareness free endemic equilibrium E_3 i.e.,human remain infected if on average an infected individual infects more than one susceptible individual during his/ her whole infectious period.

In equilibrium E_3 $(Y_3^*, N_3^*, 0)$, the values of Y_3^* and N_3^*, are found by solving the following algebraic equations $(for\ Y \neq 0\ and\ M = 0)$:

$$\beta(N - Y) - (v + \alpha + \mu) = 0 \quad (4) \quad \text{and}$$

$$A - \mu N - \alpha Y = 0 \quad (5)$$

From equations (4) and (5), we have $Y_3^* = \beta A - \mu(v + \alpha + \mu)\beta(\alpha + \mu)$ which is positive if $R_0 > 1$ and for this positive value of Y_3^*, we get positive value of N_3^* from equilibrium equation (5).

3.2 Stability Analysis

Stability analysis of the model system (1) portrays the following statements

1) The equilibrium E_1 is unstable always.
2) The equilibrium E_2 is unstable if $R_0 > 1$ and it is locally asymptotically stable if $R_0 < 1$.
3) The equilibrium E_3 is unstable if $R_0 > 1$.
4) The equilibrium E_4 is locally asymptotically stablewithout any condition.

4. Numerical Simulation

The qualitative analysis of model (3) is presented in the previous sections. In this section our analytical findings are validated using numerical simulation to study the significance of parameters considered in the model formulation. Following is the table of the parameter values for which two plots have been made of infected humans with respect to different values of the parameter θ and the carrying capacity K.

Table: Parameter values defined in the model system (1)

Parameter	Values
A	300
β	0.002
β_1	0.012
v	0.5
α	0.02
μ	0.0001
R	0.005
θ	0.0004
K	200

For the above set of parameter values, it may be noted that all the conditions of existence of all the equilibria and stability is verified. Following are the equilibrium values obtained for the above set of parameter values: $Y^* = 797$, $N^* = 38405$, $M^* = 897$. The basic reproduction number R_0, for above set of parameter values is found to be 1.77. For the above set of the parameter value, which is given in table we plot the variation of Y with respect to time for different value of θ and we can see that as the value of θ increases the number of infected individuals' deceases, which show that the number of infected individuals decreases as the value of per capita growth rate of number of awareness programs per infected individuals increases, which is depicted in Fig.1. Now, we plot the variation of Y with respect to time for the different value of K and we can see that as the value of K increases, the number of infected individuals decreases, which is depicted in Fig.2.

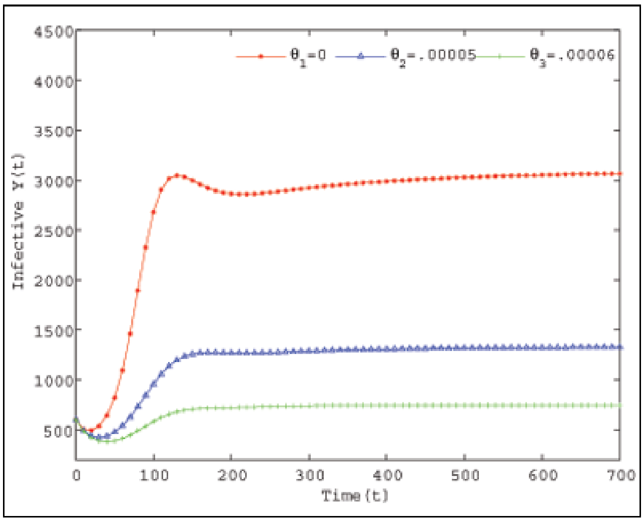

Figure 1: The Variation of Y with Respect to Time for Different Value of θ

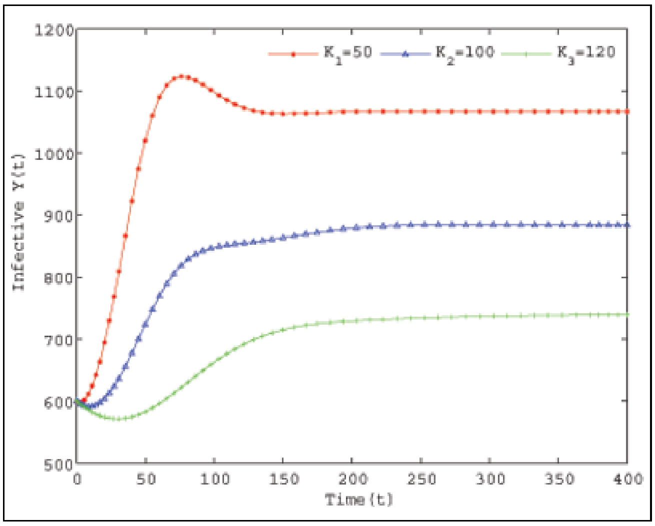

Figure 2: The Variation of Y with Respect to Time for the Different Value of K

5. Conclusion

In this paper, a nonlinear SIS mathematical model has been proposed and analyzed to study the effect of introducing awareness programs on the human population on the emergence of any new infectious disease, in human population with immigration. The proposed model has four non-negative equilibria; E_1, E_2, E_3 and E_4. The equilibrium E_1 is always feasible and unstable without any condition, whereas the equilibrium E_2 is always feasible and it locally asymptotically stable if R_0< 1 and is unstable if R_0> 1. The equilibrium E_3 is feasible if R_0> 1 and is unstable if R_0> 1, whereas the interior equilibrium E_4 is feasible if R_0> 1 and is locally asymptotically stable without any condition. The analysis of the model system has shown that interior equilibrium E_4 is feasible if R_0> 1, whereas the disease-free equilibrium E_2 is unstable if $R_0 > 1$ i.e., E_2 is unstable whenever E_4 is feasible.

From the analysis of the model system (1), it may be concluded the prevalence of the disease can be minimized positively by increasing the number of awareness programs in the region under consideration. The number of awareness programs can be increased as the number of infected individual increases. However, the health care agencies as well as the government of any country may not increase awareness programs in same proportion of number of infected individuals. The reason behind this is that as the time passes humans change their habit of keeping themselves away from the infection. It may be a challenging task about the introducing awareness programs at the appropriate time. The awareness programs should be introduced in large number at the time of prevalence of disease.

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