Simulation-Based Comparison of LQR and PID for Attitude Stabilization of Sub-2-kg UAVs

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Abstract: ***This study compares PID and LQR controllers for attitude stabilization of sub-2-kg quadrotor UAVs using a unified linearized simulation model. Performance is evaluated in terms of transient response, steady-state accuracy, disturbance rejection, and control energy under both nominal and noisy conditions. The results show that PID offers faster response but exhibits higher sensitivity to disturbances and larger control effort, whereas LQR provides smoother responses, stronger robustness, and improved energy efficiency at the cost of slower transient behavior. Overall, the study highlights the performance trade-offs and indicates that controller selection should depend on mission requirements and the operating disturbance environment.***

Keywords: UAV attitude control; quadrotor stabilization; linear quadratic regulator (LQR); Proportional Integral Derivative (PID); disturbance rejection; robust flight control.

1. Introduction

Multirotor Unmanned Aerial Vehicles (UAVs) have gained widespread adoption due to their simple structure, high maneuverability, and ability to operate in complex environments; however, their nonlinear, tightly coupled, and disturbance-sensitive dynamics make attitude stabilization a challenging control problem [1], [2], [18]. Small-scale UAVs with a mass sub-2-kg are particularly vulnerable to external disturbances, actuator limitations, and energy constraints, which impose stricter requirements on controller performance and robustness.

Among commonly used approaches, Proportional–Integral–Derivative (PID) controllers remain dominant due to their simplicity and ease of implementation, but their performance can degrade significantly under disturbances, noise, and model uncertainties [5], [12], [16]. As an alternative, the Linear Quadratic Regulator (LQR) has been investigated for its ability to produce smooth, optimal state-feedback control; however, its effectiveness depends strongly on the accuracy of the linearized model and the availability of reliable state information [4], [6], [24].

Although both PID and LQR have been extensively studied, many existing works evaluate them separately or under inconsistent assumptions, limiting the comparability of results and leading to inconclusive insights regarding their relative strengths [17]-[19]. Furthermore, only a limited number of studies explicitly focus on lightweight UAV platforms, where robustness, noise attenuation, and energy efficiency are critical considerations. To address this gap, this study offers a unified and directly comparable evaluation of PID and LQR under a single quadrotor model and a full range of noise conditions.

This paper presents a systematic and fair comparison of PID and LQR for quadrotor attitude control, specifically targeting UAVs with a mass sub-2-kg. The analysis is conducted using a unified simulation framework, a consistent linearized model, and quantitative performance metrics including transient response, steady-state accuracy, control energy, and disturbance-rejection capability [20]-[23].

The remainder of this paper is organized as follows: Section II introduces the UAV linearization method, Section III presents the controller design procedures, Section IV provides simulation results and performance analysis, and Section V concludes the study.

2. Linearization of the UAV

2.1 UAV characteristics and utilized parameters

A quadrotor in an "X" configuration with an approximate mass of 2 kg is considered. The parameters used are intended for simulation purposes and can be replaced with experimentally measured values to improve model accuracy.

Table 1: UAV utilized parameters

Parameter	*Symbol*	*Value*	*Unit*
Roll, pitch, yaw damping coefficient	a_p, a_q, a_r	2.5, 2.8, 1.7	s^{-1}
Roll, Pitch, Yaw control effectiveness	b_p, b_q, b_r	84.75, 84.75, 86.21	rad/s²
PID proportional gain	K_p	[3, 3, 3]	–
PID integral gain	K_i	[0.2, 0.2, 0.2]	–
PID derivative gain	K_d	[0.5, 0.5, 0.5]	–
LQR state weighting	Q	diag (120, 120, 120, 35, 35, 35)	–
LQR control weighting	R	diag(10,10,10)	–
Simulation time	T	10	s
Attitude sensor noise	–	10^{-4}, 10^{-4}, 2×10^{-4}	rad
Angular rate noise	–	10^{-3}	rad/s
Actuator noise	–	0.02	–
External disturbance	–	0.05	–

2.2 Linear State-Space Model

The quadrotor is a six-degree-of-freedom underactuated system [1], [2], [9], whose motion comprises both translational and rotational dynamics. Let (1) denote the position expressed in the inertial frame, and (2) the angular velocity expressed in the body frame.

$$r = \begin{bmatrix} x & y & z \end{bmatrix}^T \quad (1)$$

x,y,z: Position in the inertial frame.

$$\omega = \begin{bmatrix} p & q & r \end{bmatrix}^T \quad (2)$$

p,q,r: Angular velocity about the body axes.

Linearization is performed around a steady hover equilibrium point [1], [2]. Define the attitude state vector as:

$$x = \begin{bmatrix} \varphi & \theta & \psi & p & q & r \end{bmatrix}^T \quad (3)$$

ϕ,θ,ψ: Roll, pitch, and yaw Euler angles.

$$u = \begin{bmatrix} \tau_\varphi & \tau_\theta & \tau_\psi \end{bmatrix} \quad (4)$$

$\tau_\phi,\tau_\theta,\tau_\psi$: Control moments for roll, pitch, and yaw.

The linearized system is written as [4]:

$$\dot{x} = Ax + Bu \quad (5)$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -a_p & 0 & 0 \\ 0 & 0 & 0 & 0 & -a_q & 0 \\ 0 & 0 & 0 & 0 & 0 & -a_r \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ b_p & 0 & 0 \\ 0 & b_q & 0 \\ 0 & 0 & b_r \end{bmatrix}$$

The attitude subsystem can be separated into three independent channels.

The linearization is carried out around a steady hover equilibrium under the standard small-angle assumptions (ϕ, $\theta \ll 1$ rad) and negligible coupling between translational and rotational dynamics. Aerodynamic cross-coupling terms and higher-order nonlinearities are omitted, consistent with commonly adopted small-scale quadrotor models. This yields a decoupled attitude subsystem suitable for state-space controller design.

3. Controller design

3.1 PID controller

For each axis, a classical PID control law is applied [5], [12], [16]:

$$\tau_i = K_{p,i} e_i + K_{d,i} \dot{e}_i + K_{i,i} \int e_i dt, i \in \{\varphi, \theta, \psi\} \quad (6)$$

$$e_i = i_r - i_e \quad (7)$$

i_r, i_e: Reference and measured value of angle $i \in \{\phi,\theta,\psi\}$.
$K_{p,i}$, $K_{i,i}$, $K_{d,i}$ - proportional, derivative, and integral gains.

The PID gains were tuned manually through iterative trial-and-error procedures to achieve acceptable transient behavior in the linearized model. The tuning process prioritized fast rise time while avoiding excessive overshoot and maintaining stable closed-loop behavior. Final values listed in Table 1 represent the parameter set that produced the best compromise between responsiveness and stability.

3.2 LQR controller

The control law is defined as in (20), where the gain matrix K is obtained by minimizing a quadratic cost function [4], [6], [15]:

$$u = -Kx + K_r r \quad (8)$$

where K is the state feedback gain obtained by minimizing the quadratic cost function [5]:

$$J = \int_0^\infty (x^T Q x + u^T R u) \quad (9)$$

Q: State-weighting matrix, R: Control-weighting matrix.

The optimal gain matrix is given by:

$$K = R^{-1} B^T P \quad (10)$$

The matrix P is obtained from the solution of the Algebraic Riccati Equation (ARE) [6]:

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \quad (11)$$

Since the standard LQR formulation does not inherently guarantee reference tracking, a feedforward gain K_r is introduced, defined as:

$$K_r = -(C(A - BK)^{-1} B)^{-1} \quad (12)$$

C: Output matrix mapping states to measurable outputs.

The feedforward gain ensures zero steady-state error for constant reference inputs.

The weighting matrices Q and R were selected through an iterative search process. Higher weights were assigned to attitude angles to emphasize tracking accuracy, while lower weights on angular-rate states allowed smoother transients. The control weighting matrix R was adjusted to regulate actuator activity and prevent excessive control effort. The final Q and R values in Table 1 correspond to the configuration that achieved the best balance between tracking precision, robustness, and control energy in simulation. A qualitative sensitivity check around the selected Q and R values confirmed that the controller performance deteriorates for significantly smaller angle weights or excessively large control penalties, supporting the suitability of the final parameter choice.

4. Simulation Results

4.1 Simulation setup

All simulations are conducted in MATLAB/Simulink using the quadrotor model with identical parameters listed in Table 1, under both noise-free and noisy conditions. Performance is evaluated using time-domain and energy-based metrics, including rise time, settling time, overshoot, steady-state error, IAE, ITAE, and control effort.

Sensor noise, actuator noise, and external disturbances were modeled using band-limited white-noise sources in Simulink, following common practice in recent quadrotor control studies such as Chen et al. [25]. Each noise channel used different amplitudes consistent with the values in Table

1. Sensor and angular-rate noise were injected at the measurement interface, actuator noise was added directly to the control input, and external disturbances were applied as additive white-noise torques on the attitude dynamics. All noise signals were generated using identical random-seed settings to ensure reproducibility.

4.2 Performance without noise

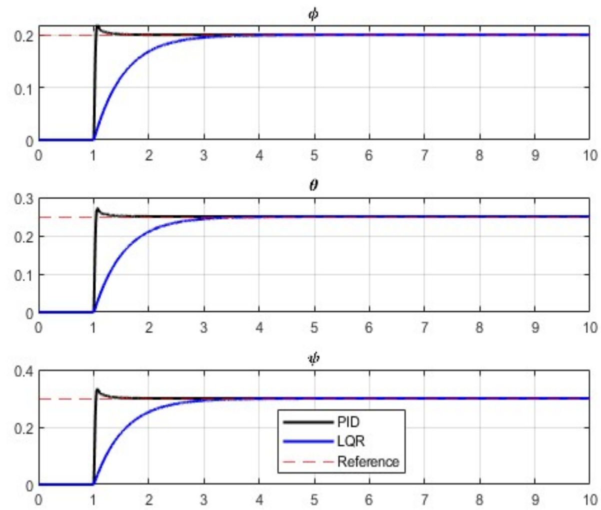

Figure 1: Attitude tracking

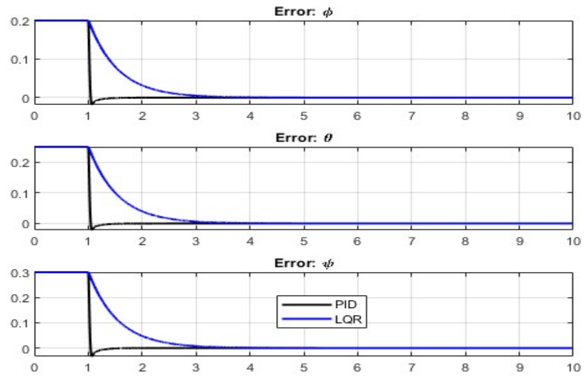

Figure 2: Tracking error comparison

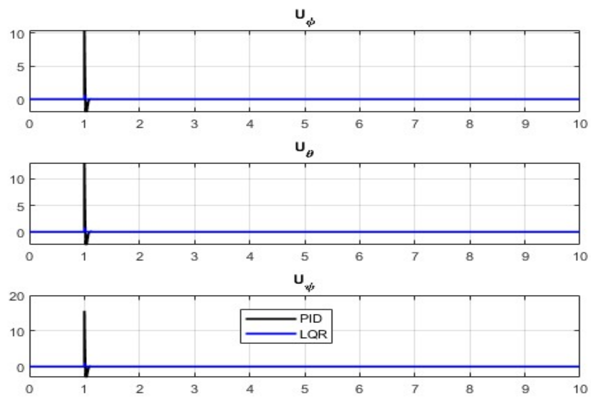

Figure 3: Angular rate responses

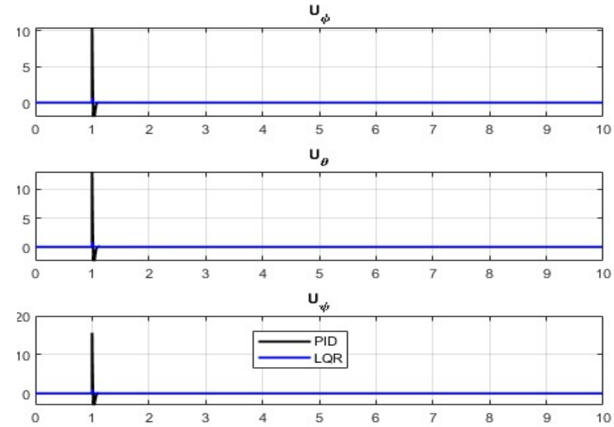

Figure 4: Control input

PID achieves acceptable tracking performance but exhibits several limitations, including overshoot, longer settling time, initial oscillations, and higher control effort, all of which indicate reduced stability and lower energy efficiency. LQR provides smoother and more balanced behavior, characterized by minimal or zero overshoot, stable convergence, smoother angular rates, and substantially lower control effort. Overall, LQR consistently demonstrates superior tracking quality, robustness to noise, and energy efficiency, PID remains more sensitive to disturbances and requires greater control activity to maintain performance.

PID offers the fastest rise time but suffers from significant overshoot (8–11%), nonzero steady-state error, and the highest control effort, making it quick but less stable and less efficient. Conversely, LQR delivers the most stable and balanced performance, achieving zero overshoot, zero steady-state error, smooth transients, and the lowest control effort, albeit with the slowest rise time. In summary, PID provides rapid responsiveness at the expense of stability and energy consumption, while LQR offers the most reliable and efficient attitude-control behavior.

4.3 Performance with noise

Case of sensor noise:

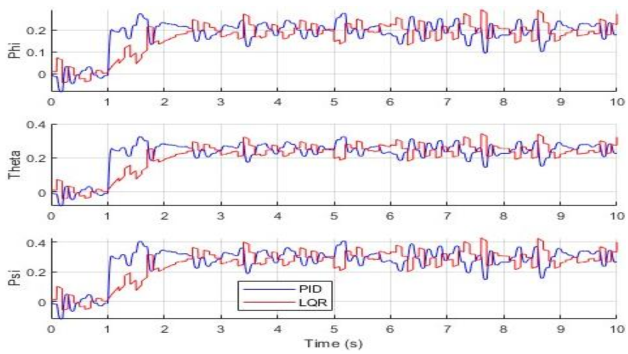

Figure 5: Attitude tracking

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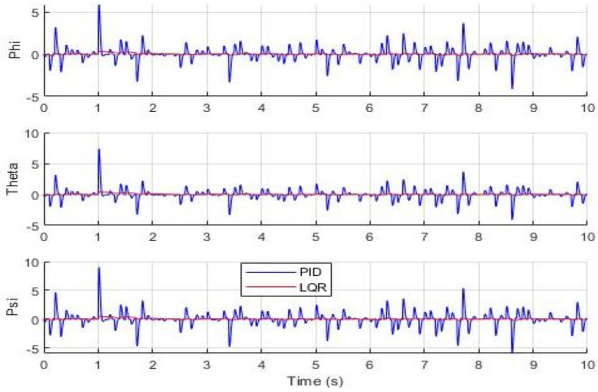

Figure 6: Angular rate responses

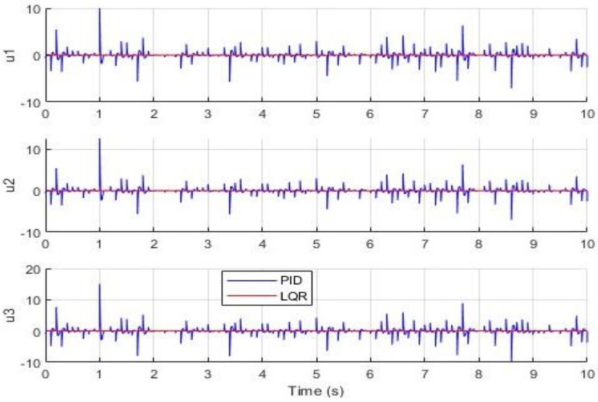

Figure 7: Tracking error comparison

Under sensor noise, PID responds quickly but suffers from noticeable oscillations and strong noise amplification, whereas LQR maintains the smoothest and most stable response with significantly lower control effort. Overall, LQR is more robust while PID is more sensitive to noise.

Case of actuator noise:

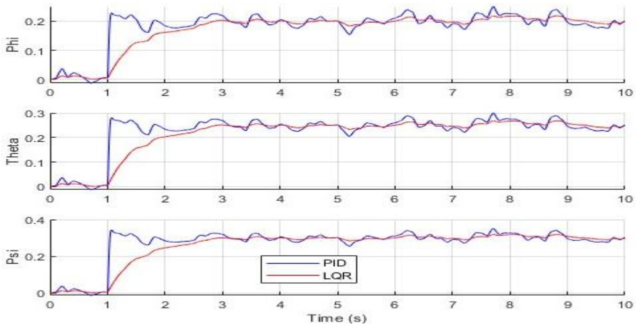

Figure 8: Attitude tracking

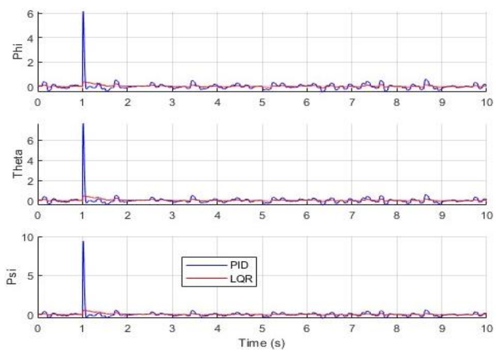

Figure 9: Angular rate responses

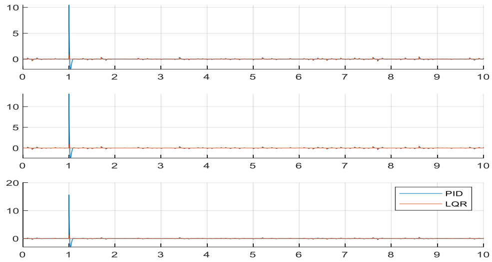

Figure 10: Tracking error comparison

Under actuator noise, the PID controller reacts quickly but induces significant oscillations and amplifies disturbances, leading to larger tracking errors and reduced stability. In contrast, the LQR controller maintains a smoother and more stable response, effectively attenuating the impact of noise and producing smaller tracking errors, resulting in more consistent overall control performance.

Case of disturbance noise:

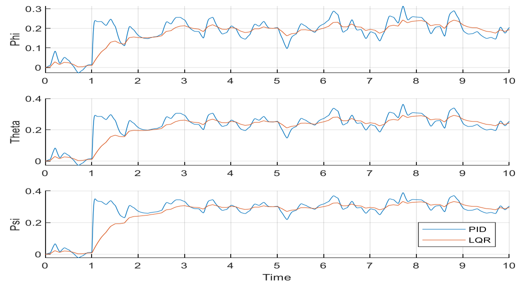

Figure 11: Attitude tracking

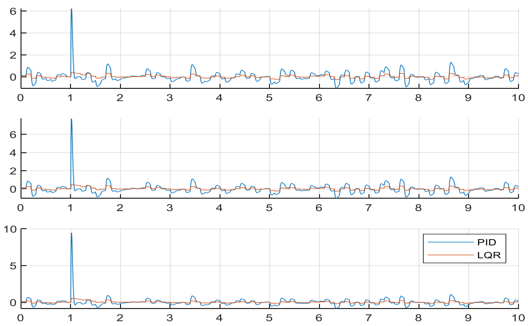

Figure 12: Angular rate responses

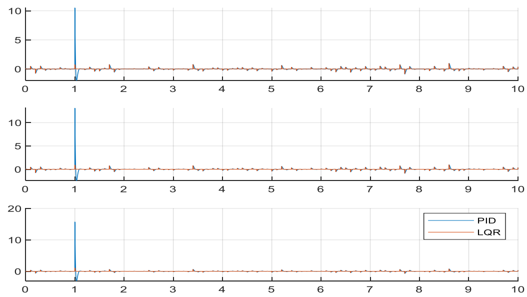

Figure 13: Tracking error comparison

Under actuator noise, PID exhibits overshoot, oscillations, and large variations in control input, whereas LQR remains more stable and suppresses noise more effectively, although with a slightly slower response. Overall, LQR performs better and is more robust than PID.

Case of full noise:

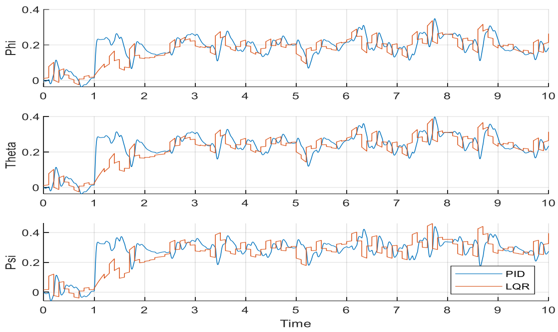

Figure 14: Attitude tracking

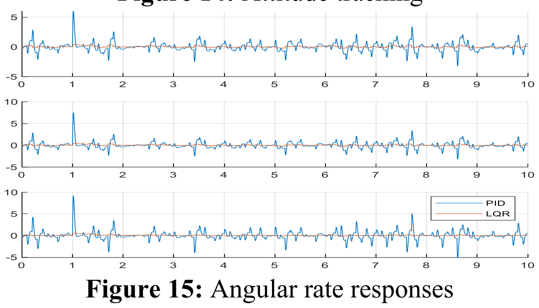

Figure 15: Angular rate responses

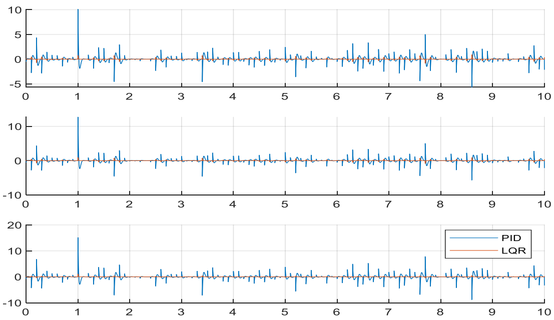

Figure 16: Tracking error comparison

With combined sensor, the PID controller responds quickly but exhibits large oscillations and pronounced variations in control input, resulting in degraded tracking performance. In contrast, the LQR controller maintains a much smoother response with smaller tracking errors, albeit with slightly slower dynamics. Overall, LQR demonstrates higher robustness and superior performance compared to PID under combined noise conditions.

Dynamic performance comparison:

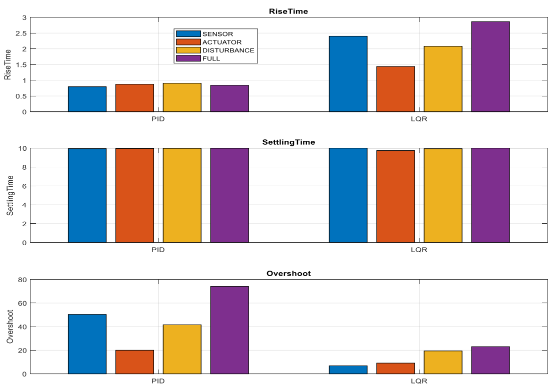

Figure 17: Dynamic performance of PID vs LQR

Between the two controllers, PID achieves the fastest rise time but exhibits the poorest stability margin, making it highly sensitive to noise and model uncertainties. LQR, in contrast, shows a slower rise time but provides a smoother and more stable response due to its optimal state-feedback formulation.

Regarding settling behavior, PID may converge more quickly at the initial stage but often experiences prolonged oscillations under disturbances. LQR achieves a more consistent and stable convergence, though with slower overall dynamics.

In terms of overshoot, PID generally produces noticeably larger overshoot as a consequence of its proportional–integral structure, whereas LQR maintains smaller overshoot by optimally distributing control effort across system states.
Overall, PID is suitable for scenarios requiring rapid initial response and operating in low-noise, low-uncertainty environments. LQR delivers smoother, more stable, and more robust performance, making it preferable when robustness and stability are prioritized over response speed.

Table 2: Performance degradation under sensor noise

	IAE ↑ (%)	*ITAE ↑ (%)*	*Overshoot ↑ (abs %)*	*u_rms ↑ (%)*	*Energy ↑ (%)*
PID	106.11	1079.53	40.63	132.90	442.58
LQR	57.77	421.39	6.82	135.00	452.24

Table 3: Performance degradation under actuator noise

	IAE ↑ (%)	*ITAE ↑ (%)*	*Overshoot ↑ (abs %)*	*u_rms ↑ (%)*	*Energy ↑ (%)*
PID	46.62	525.09	10.31	2.16	4.15
LQR	10.14	93.68	9.10	100.95	294.03

Table 4: Performance degradation under disturbance

	IAE ↑ (%)	*ITAE ↑ (%)*	*Overshoot ↑ (abs %)*	*u_rms ↑ (%)*	*Energy ↑ (%)*
PID	96.92	1089.68	31.91	8.76	17.00
LQR	22.38	198.94	19.41	272.23	1262.81

Table 5: Performance degradation under combined noise

	IAE ↑ (%)	ITAE ↑ (%)	Overshoot ↑ (abs %)	u_rms ↑ (%)	Energy ↑ (%)
PID	120.16	1339.76	64.38	105.12	324.65
LQR	64.32	503.08	23.04	233.35	1028.23

Under sensor noise, the PID controller experiences the most severe degradation. Both IAE and ITAE increase sharply-ITAE grows by more than 1000% and IAE more than doubles-indicating strong sensitivity to measurement noise. This degradation is mainly caused by the derivative term, which amplifies high-frequency noise and induces large oscillations in the control input. The LQR controller maintains substantially better robustness, showing only a small increase in overshoot (6.82%) and a moderate rise in tracking error. Despite a higher control effort, its overall performance remains significantly more stable than PID.

Under actuator noise, both controllers show performance deterioration, but LQR still outperforms PID. For LQR, IAE increases remain below 11%, demonstrating good robustness against noise injected into the actuator channel. PID exhibits larger error growth and higher overshoot due to its limited ability to regulate noisy control commands. Although actuator noise affects both controllers, LQR preserves a better trade-off between accuracy and control effort, while PID displays noticeably poorer tracking behavior.

Under external disturbances, PID undergoes substantial degradation, with ITAE increasing by more than 1000%, reflecting weak disturbance rejection capability. Overshoot and settling time also worsen markedly. LQR provides improved stability compared to PID and limits increases in tracking error more effectively, although disturbances still reduce its performance. Under disturbance inputs, LQR exhibits stronger robustness, PID performs the weakest.

Under combined noise, both controllers deteriorate significantly; however, the performance gap becomes more pronounced. PID shows the most severe degradation, with overshoot rising by 64% and ITAE increasing by more than 1300%, indicating extreme sensitivity to uncertain environments. LQR maintains smoother behavior and better overall stability, although its control energy increases due to stronger corrective actions. In highly uncertain conditions, LQR remains clearly more robust, while PID performs the worst across all evaluated metrics.

Table 6: Controller robustness ranking

Noise type	*Best*	*Worst*
Sensor noise	LQR	PID
Actuator noise	LQR	PID
External disturbance	LQR	PID
Combined noise	LQR	PID

The ranking indicates that no controller appears universally optimal within the scope of our experiments, as the selection depends on the dominant noise characteristics observed in the tested scenarios.

5. Conclusion

This study compared PID and LQR controllers for quadrotor attitude control using a unified linearized model and consistent performance metrics evaluated under both noise-free and noisy conditions. Under nominal operation, PID achieved the fastest rise time but exhibited overshoot, oscillatory transients, and higher control effort. In contrast, LQR produced smoother and more stable responses, characterized by zero overshoot and lower steady-state error. Across all noise scenarios, the quantitative results consistently show that PID undergoes more severe degradation. Under sensor noise, PID experienced the largest deterioration, with tracking error increasing sharply (IAE +106.11%, ITAE +1079.53%). LQR maintained substantially better robustness, with considerably lower error growth (IAE +57.77%, ITAE +421.39%). Under actuator noise, LQR again showed superior resilience, exhibiting only a +10.14% increase in IAE compared with +46.62% for PID. External disturbances and combined noise further amplified the performance gap: PID reached up to +1339.76% ITAE and +64.38% overshoot, whereas LQR-although demanding higher control energy- preserved comparatively stable tracking behavior.

Overall, the results indicate that LQR provides a more balanced trade-off between accuracy, robustness, and control energy, making it more suitable for lightweight, sub-2-kg UAVs operating in uncertain environments. PID remains advantageous only in scenarios where rapid transient response is prioritized and noise levels are low. These findings highlight that controller selection for small UAV platforms should explicitly account for noise characteristics and robustness requirements rather than relying solely on nominal performance metrics.

While the findings are conclusive within the simulation framework, practical deployment requires further consideration of sensor latency, actuator dynamics, real-time computation limits, and parameter variations in small UAV platforms. Future work will incorporate hardware-in-the-loop (HIL) experiments and real flight testing to validate the comparative performance of PID and LQR under real-world noise and disturbances, and to assess their robustness in practical implementation.

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Author Profile

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