Kulli-Basava Indices and Polynomials for Spherical Fuzzy Graph

N. K. Raut

Ex. Head, Department of Physics, Sunderrao Solanke Mahavidyalaya Majalgaon, Dist. Beed (M.S.) India
Email: *rautnk87[at]gmail.com*

Abstract: ***In a spherical fuzzy environment uncertainty is captured through the interaction of three independent degrees with every vertex and every edge having three degree independent membership values as membership degree, neutral degree and non-membership degree. In this paper first, second Kulli-Basava indices, modified first, second Kulli-Basava indices, F_1-Kulli-Basava polynomial, square Kulli-Basava polynomial and first Zagreb polynomial and corresponding index for spherical fuzzy graph are investigated.***

Keywords: Degree, Kulli-Basava indices, membership, neutral, non-membership, polynomials, spherical fuzzy graph

1. Introduction

A fuzzy graph $\widetilde{G}$= (σ, μ) is a pair of functions σ: V → [0, 1] and μ: V × V → [0, 1] such that for all u, v ∈ V, μ(u, v) ≤ σ(u) ∧ σ(v) and μ is a symmetric fuzzy relation on σ. Here σ(v) and μ(u, v) represent the membership values of vertex v and edge (u, v) respectively [1-3]. The concept of spherical fuzzy graph (SFG) is applicable in decision making, which give an outstanding research direction for the uncertain environmental optimization problems [4]. Spherical fuzzy set is an extension of picture fuzzy set and Pythagorean fuzzy set [5]. To address the uncertainty parameters and concepts in the world, graphs such as fuzzy, neutroscopic, Turiyam neutrosophic graphs, and plithogenic graphs are studied in [6]. A Turiyam neutrosophic graph is a generalization of the fuzzy graph. In a spherical fuzzy environment uncertainty is captured through the interaction of three independent degrees. Spherical fuzzy graphs are an extension of intuitionistic fuzzy graphs and Pythagorean fuzzy graphs, designed to handle human uncertainty through three degrees of membership: membership (α), neutral (β) and non-membership (γ). The hesitancy represents the margin of uncertainty or the refusal degree when assigning membership and non-membership values. The intuitionistic fuzzy graph hesitancy α+ β+ γ=1 is linear and in SFG $\alpha^2 + \beta^2 + \gamma^2 = 1$ is quadratic. The effective degree, neighbourhood degree, closed neighbourhood degree and their minimum and maximum neighbourhood degrees are defined for spherical fuzzy graph in [7]. The concept of energy of spherical fuzzy graph can be studied by computing the energy using adjacency matrix. The principal disparity with the IFS, Pythagorean fuzzy set (PyFS), picture fuzzy set (PFS) and spherical fuzzy set (SFS) is their constraint conditions. A spherical fuzzy graph's energy is determined by adding up all of the absolute values of its adjacency matrix's eigen values. Additionally, the energy of spherical fuzzy graphs is bounded from below and above, providing insight into the possible range of energy values for these graphs [8].Spherical fuzzy graphs are easier to utilize than image fuzzy graphs for variety of real–world circumstances [9]. The spherical fuzzy set, created by Gundogdu and Kahraman is an extension of the picture fuzzy set since it increases the space of membership degree (α), neutral membership degree (β) and non-membership degree (γ) in the [0, 1] with a condition [10-17],
$0 \le \alpha^2 + \beta^2 + \gamma^2 \le 1$.

The constraints forms a unit sphere, hence the name spherical fuzzy.

The complement of a spherical fuzzy graph G = (V, E) is a graph $\overline{G}$ =(V, $\overline{E}$) where the vertex membership functions remain the same,but the edge membership values α, β and γ are defined by taking the minimum of vertex values, minus the original edge values. The complement typically satisfies

$$\alpha_{\overline{E}}(uv) = \min(\alpha_p(u), \alpha_p(v)) - \alpha_E(uv).$$

A spherical fuzzy graph (SFG) is of the form G = (V, E),
(1) V = {v = $v_1, v_2, v_3, \cdot\ \cdot\ \cdot, v_n$} such that α_V : V → [0, 1] and β_V : V → [0, 1],
γ_V : V → [0, 1] denotes the degree of membership, neutral degree and non-membership of the element v_i ∈ V respectively and

$$0 \le \alpha_V^2(v_i) + \beta_V^2(v_i) + \gamma_V^2(v_i) \le 1 \ \forall v_i \in V.$$

(2) E ⊆ V ×V where α_E : V ×V → [0, 1], β_E : V ×V → [0, 1] and γ_E : V ×V → [0, 1] are such that

$$\alpha_E(v_i, v_j) \le \min\{\alpha_V(v_i), \alpha_V(v_j)\},$$

$$\beta_E(v_i, v_j) \le \min\{\beta_V(v_i), \beta_V(v_j)\},$$

$$\gamma_E(v_i, v_j) \le \max\{\gamma_V(v_i), \gamma_V(v_j)\}, \ \forall\ v_i, v_j \in V$$

and $0 \le \alpha_E^2(v_i, v_J) + \beta_E^2(v_i, v_j) + \gamma_E^2(v_i, v_j) \le 1$

$(v_i, v_j) \le 1, \forall (v_i, v_j) \in V \times V$,

where α_E : V × V → [0, 1], β_E : V × V → [0, 1], and γ_E : V × V → [0, 1], represent the membership, neutral and non-membership grades of E respectively.

Membership degree of a vertex is calculated by summing the membership values of all edges incident to that vertex $d_\alpha(v)=\sum_{uv\in E}\alpha(uv)$,
and similarly for neutral degree $d_\beta(v)=\sum_{uv\in E}\beta(uv)$,
and non-membership degree $d_\gamma(v)=\sum_{uv\in E}\gamma(uv)$.

The neutral degree or β-degree in a spherical fuzzy graph measures the strength of neutral associates for each vertex summing β values of incident edges, is the most distinctive feature that separates spherical fuzzy graphs from standard intuitionistic fuzzy graphs. The degrees in spherical fuzzy graphs are computed separately for α, β, γ, reflecting the three dimensional uncertainty structure. The degree of a vertex v in an spherical fuzzy graph is denoted by $d(v) = (d_\alpha(v), d_\beta(v), d_\gamma(v))$.

The basic vertex degree sums edge values incident to a vertex, while advanced degrees like neighbourhood or effective degrees refine this for specific graph properties. Order of the graph is the sum of the membership triplets of all vertices in the graph. Size of the graph is the sum of the membership triplets of all edges in the graph. The refusal degree (R) is the actual leftover space, calculated as
$R = \sqrt{1-(\alpha^2+\beta^2+\gamma^2)}$.

The degree of an edge e= uv in G is defined by
$d_G(e) = d_G(u) + d_G(v) - 2$.

Let $S_e(v)$ denote the sum of degrees of edges incident to a vertex v [18-19]. $S_e(v)$ is defined as
$S_e(v) = \sum_{e\in N_e(v)} d_G(e)$,

where $N_e(v)$ is edge neighbourhood of a vertex v∈ V(G) as a set $N_e(v)$ consisting of all edges e which are incident with v and the edge neighbourhood degree sum of a vertex v∈ V(G).The score function is used to covert spherical fuzzy value into a single real number, which helps in comparison, ranking and decision making [20-21]. The score function is often used to interpret topological indices like Zagreb or Kulli-Basava indices in a fuzzy environment. Some topological indices and polynomials are defined for the spherical fuzzy graph as follows.

Kulli-Basava indices are defined as

$$KB_1(G) = \sum_{uv\in E(G)}[S_e(u) + S_e(v)] \quad (1)$$

$$KB_2(G) = \sum_{uv\in E(G)}[S_e(u) \times S_e(v)] \quad (2)$$

where $S_e(v)$ denote the sum of degrees of edges incident to a vertex v.

Modified first and second Kulli-Basava indices are defined as

$$^{m}KB_1(G) = \sum_{uv\in E(G)} \frac{1}{[S_e(u)+S_e(v)]} \quad (3)$$

$$^{m}KB_2(G) = \sum_{uv\in E(G)} \frac{1}{[S_e(u)\times S_e(v)]} \quad (4)$$

F_1-Kulli-Basava polynomial is defined as

$$F_1\text{-}KB(G,x) = \sum_{uv\in(G)} x^{S_e(u)^2+S_e(v)^2} \quad (5)$$

Square Kulli-Basava polynomial is

$$QKB(G,x) = \sum_{uv\in(G)} x^{[S_e(u)-S_e(v)]^2} \quad (6)$$

First Zagreb polynomial in spherical fuzzy graph extends classical graph invariants, incorporates three-dimensional uncertainty and provides a powerful analytical and comparative tool which is defined as
$M_1^{SFG}(G,x) = \sum_{uv\in E} x^{\alpha(uv)(d_\alpha(u)+d_\alpha(v))}$, and similarly for β, γ. (7)

The score function S(A) for a spherical fuzzy number A = (α,β,γ) is given by

$$\text{Score function } S(A) = \alpha^2 - \gamma^2. \quad (8)$$

The symbols and notations follow standard conventions as established [22-24]. In this paper first, second Kulli-Basava indices, modified first, second Kulli-Basava indices, F_1-Kulli-Basava polynomial, square Kulli-Basava polynomial and first Zagreb polynomial and corresponding index for spherical fuzzy graph are investigated.

2. Materials and Methods

A spherical graph with V_1,V_2,V_3 and V_4 vertices and e_{12},e_{23},e_{34},e_{41} and e_{13} edges is shown in figure (1).The vertex memberships and edge memberships are represented in tables 1-2 with constraints: $\alpha^2 + \beta^2 + \gamma^2 \leq 1$.The degree of each vertex $d(v)=(d_\alpha(v), d_\beta(v), d_\gamma(v))$ computed from definition of degree of vertex and is given table (3) which in turn is used to compute Kulli-Basava indices and corresponding polynomials.

3. Results and Discussion

The degree of an edge e= uv in G is defined by $d_G(e) = d_G(u) + d_G(v) - 2$.Let $S_e(v)$ denote the sum of degrees of edges incident to a vertex v. $S_e(v)$ is defined as $S_e(v) = \sum_{e\in N_e(v)} d_G(e)$,where $N_e(v)$ is edge neighbourhood of a vertex v∈ V(G) as a set $N_e(v)$ consisting of all edges e which are incident with v and the edge neighbourhood degree sum of a vertex v∈ V(G). $S_e(v)$ is not the degree of v itself, it is the sum of degrees of all vertices adjacent to v.

Theorem 1. First Kulli-Basava index for spherical fuzzy graph is (59.2,53.2,38.4).

Proof. Using figure (1), tables 1-3 and equation (1)**,** we get, $KB_1(G)$.

The edge neighbourhood for each edge e belong to the edges incident to v. In spherical fuzzy graph this often calculated component-wise. Difference between d(v) and $S_e(v)$ is that: d(v) is weights of edges incident to vertex v and $S_e(v)$ is weights of edges adjacent to the edges incident to v.

If neighbours of V_1 are V_2, V_3, V_4 then
$S_e(V_1)$ =d(V_2)+d(V_3)+d(V_4)=(0.9,0.8,0.5)+(1.3,1.3,1.0)+(1.1,0.6,0.5)
=(0.9+1.3+1.1,0.8+1.3+0.6,0.5+1.0+0.5)=(3.3,2.7,2.2).

If neighbours of V_2 are V_1, V_3 then
$S_e(V_2)$ = d(V_1)+d(V_3)=(1.3,1.3,0.8)+(1.3,1.3,1.0)=(2.6,2.6,1.8).

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If neighbours of V_3 are V_2, V_4, V_1 then $S_e(V_3)$ = $d(V_2)+d(V_4)+d(V_3)=(0.9,0.8,0.5)+(1.1,0.6,0.5)+(1.3,1.3,0.8)=(3.3,2.7,1.8)$.

If neighbours of V_4 are V_1, V_3 then $S_e(V_4)$ = $d(V_1)+d(V_3)=(1.3,1.3,0.8)+(1.3,1.3,1.0)=(2.6,2.6,1.8)$.

$KB_1(G) = \sum_{uv\in E(G)}[S_e(u)+S_e(v)]=$
$\{[S_e(V_1)+S_e(V_2)] + [S_e(V_1)+S_e(V_3)] + [S_e(V_1)+S_e(V_4)]\}$
$+\{[S_e(V_1)+S_e(V_2)] + [S_e(V_2)+S_e(V_3)]\}$
$+\{[S_e(V_2)+S_e(V_3)] + [S_e(V_1)+S_e(V_3)] + [S_e(V_3)+S_e(V_4)]\}$
$+\{[S_e(V_1)+S_e(V_4)] + [S_e(V_3)+S_e(V_4)]\}$
$=\{[(3.3,2.7,2.2)+(2.6,2.6,1.8)]+[(3.3,2.7,2.2)+(3.3,2.7,1.8)]+[(3.3,2.7,2.2)+(2.6,2.6,1.8)]\}$
$+\{[(3.3,2.7,2.2)+(2.6,2.6,1.8)]+[(2.6,2.6,1.8)+(3.3,2.7,1.8)]\}$
$+\{[(2.6,2.6,1.8)+(3.3,2.7,1.8)]+[(3.3,2.7,2.2)+(3.3,2.7,1.8)]+[(3.3,2.7,1.8)+(2.6,2.6,1.8)]\}$
$+\{[(3.3,2.7,1.8)+(2.6,2.6,1.8)]+[(3.3,2.7,2.2)+(2.6,2.6,1.8)]\}$
$=(18,16,12)+(11,10.6,7.6)+(18.4,16,11.2)+(11.8,10.6,7.6)$
$=(59.2,53.2,38.4)$.

Theorem 2: Second Kulli-Basava index for spherical fuzzy graph is (90.42,70.92,35.92).

Proof. Using figure (1), tables 1-3 and the equation (2), we have neighbours of V_1 are V_2, V_3, V_4 then $S_e(V_1)=d(V_2)+d(V_3)+d(V_4)=(0.9,0.8,0.5)+(1.3,1.3,1.0)+(1.1,0.6,0.5)$
$=(0.9+1.3+1.1,0.8+1.3+0.6,0.5+1.0+0.5)=(3.3,2.7,2.2)$.

If neighbours of V_2 are V_1, V_3 then $S_e(V_2)$ = $d(V_1)+d(V_3)=(1.3,1.3,0.8)+(1.3,1.3,1.0)=(2.6,2.6,1.8)$.

If neighbours of V_3 are V_2, V_4, V_1 then $S_e(V_3)$ = $d(V_2)+d(V_4)+d(V_3)=(0.9,0.8,0.5)+(1.1,0.6,0.5)+(1.3,1.3,0.8)=(3.3,2.7,1.8)$.

If neighbours of V_4 are V_1, V_3 then $S_e(V_4)$ = $d(V_1)+d(V_3)=(1.3,1.3,0.8)+(1.3,1.3,1.0)=(2.6,2.6,1.8)$.

$KB_2(G) = \sum_{uv\in E(G)}[S_e(u)\times S_e(v)] = \{[S_e(V_1)\times S_e(V_2)] + [S_e(V_1)\times S_e(V_3)] + [S_e(V_1)\times S_e(V_4)]\}$
$+\{[S_e(V_1)\times S_e(V_2)] + [S_e(V_2)\times S_e(V_3)]\}$
$+\{[S_e(V_2)\times S_e(V_3)] + [S_e(V_1)\times S_e(V_3)] + [S_e(V_3)\times S_e(V_4)]\}$
$+\{[S_e(V_1)\times S_e(V_4)] + [S_e(V_3)\times S_e(V_4)]\}$
$=\{[(3.3,2.7,2.2)\times(2.6,2.6,1.8)]+[(3.3,2.7,2.2)\times(3.3,2.7,1.8)]+[(3.3,2.7,2.2)\times(2.6,2.6,1.8)]\}$
$+\{[(3.3,2.7,2.2)\times(2.6,2.6,1.8)]+[(2.6,2.6,1.8)\times(3.3,2.7,1.8)]\}$
$+\{[(2.6,2.6,1.8)\times(3.3,2.7,1.8)]+[(3.3,2.7,2.2)\times(3.3,2.7,1.8)]+[(3.3,2.7,1.8)\times(2.6,2.6,1.8)]\}$
$+\{[(3.3,2.7,1.8)\times(2.6,2.6,1.8)]+[(3.3,2.7,2.2)\times(2.6,2.6,1.8)]\}$
$=[(8.58,7.02,3.2)+(10.89,7.29,3.96)+(8.58,7.02,3.96)]$
$+[(8.58,7.2,3.96)+(8.58,7.02,3.24)]$
$+[(8.58,7.02,3.2)+(10.89,7.29,3.96)+(8.58,7.02,3.24)]$
$+[(8.58,7.02,3.24)+(8.58,7.02,3.96)]$
$=(28.05,21.33,11.12)+(17.16,14.22,7.2)+(28.05,21.33,10.4)+(17.16,14.04,7.2)$
$=(90.42,70.92,35.92)$.

Theorem 3. Modified first Kulli-Basava index for spherical fuzzy graph is $\frac{1}{(59.2,53.2,38.4)}$.

Proof. Using figure (1), tables 1-3 and the equation (3)**,** we have
neighbours of V_1 are V_2,V_3,V_4 then $S_e(V_1)=d(V_2)+d(V_3)+d(V_4)=(3.7,2.7,2.2)$,
neighbours of V_2 are V_1,V_3 then $S_e(V_2)$ = $d(V_1)+d(V_3)=(2.6,2.6,1.8)$,
neighbours of V_3 are V_2,V_4, V_1 then $S_e(V_3)$= $d(V_2)+d(V_4)+d(V_3)=(3.3,2.7,1.8)$,
neighbours of V_4 are V_1,V_3 then $S_e(V_4)$ = $d(V_1)+d(V_3)=(2.6,2.6,1.8)$.Then

$$^{m}KB_1(G) = \sum_{uv\in E(G)}\frac{1}{[S_e(u)+S_e(v)]} = \left\{\frac{1}{[S_e(V_1)+S_e(V_2)]} + \frac{1}{[S_e(V_1)+S_e(V_3)]} + \frac{1}{[S_e(V_1)+S_e(V_4)]}\right\}$$
$$+\left\{\frac{1}{[S_e(V_1)+S_e(V_2)]} + \frac{1}{[S_e(V_2)+S_e(V_3)]}\right\}$$
$$+\left\{\frac{1}{[S_e(V_2)+S_e(V_3)]} + \frac{1}{[S_e(V_1)+S_e(V_3)]} + \frac{1}{[S_e(V_3)+S_e(V_4)]}\right\}$$
$$+\left\{\frac{1}{[S_e(V_1)+S_e(V_4)]} + \frac{1}{[S_e(V_3)+S_e(V_4)]}\right\}$$

$$=\left\{\frac{1}{[(3.3,2.7,2.2)+(2.6,2.6,1.8)]} + \frac{1}{[(3.3,2.7,2.2)+(2.6,2.6,1.8)]} + \frac{1}{[(3.3,2.7,2.2)+(2.6,2.6,1.8)]}\right\}$$
$$+\left\{\frac{1}{[(3.3,2.7,2.2)+(2.6,2.6,1.8)]} + \frac{1}{[(3.3,2.7,2.2)+(2.6,2.6,1.8)]} + \frac{1}{[(2.6,2.6,1.8)+(3.3,2.7,1.8)]}\right\}$$
$$+\left\{\frac{1}{[(2.6,2.6,1.8)+(3.3,2.7,1.8)]} + \frac{1}{[(3.3,2.7,2.2)+(3.3,2.7,1.8)]} + \frac{1}{[(3.3,2.7,1.8)+(2.6,2.6,1.8)]}\right\}$$
$$+\left\{\frac{1}{[(3.3,2.7,1.8)+(2.6,2.6,1.8)]} + \frac{1}{[(3.3,2.7,2.2)+(2.6,2.6,1.8)]}\right\}$$
$$=\frac{1}{(59.2,53.2,38.4)}.$$

Theorem 4. Modified second Kulli-Basava index for spherical fuzzy graph is $\frac{1}{(90.42,70.92,35.92)}$.

Proof. Using figure (1), tables 1-3 and the equation (4)**,** we have
neighbours of V_1 are V_2,V_3,V_4 then $S_e(V_1)=d(V_2)+d(V_3)+d(V_4)=(3.7,2.7,2.2)$,
neighbours of V_2 are V_1,V_3 then $S_e(V_2)$ = $d(V_1)+d(V_3)=(2.6,2.6,1.8)$,
neighbours of V_3 are V_2,V_4, V_1 then $S_e(V_3)$= $d(V_2)+d(V_4)+d(V_3)=(3.3,2.7,1.8)$,
neighbours of V_4 are V_1,V_3 then $S_e(V_4)$ = $d(V_1)+d(V_3)=(2.6,2.6,1.8)$.

Modified second Kulli-Basava index

$$^{m}KB_2(G) = \sum_{uv\in E(G)}\frac{1}{[S_e(u)\times S_e(v)]} = \left\{\frac{1}{[S_e(V_1)\times S_e(V_2)]} + \frac{1}{[S_e(V_1)\times S_e(V_3)]} + \frac{1}{[S_e(V_1)\times S_e(V_4)]}\right\}$$
$$+\left\{\frac{1}{[S_e(V_1)\times S_e(V_2)]} + \frac{1}{[S_e(V_2)\times S_e(V_3)]}\right\}$$

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$+\{\frac{1}{[S_e(V_2)\times S_e(V_3)]}+\frac{1}{[S_e(V_1)\times S_e(V_3)]}+\frac{1}{[S_e(V_3)\times S_e(V_4)]}\}+\{\frac{1}{[S_e(V_1)\times S_e(V_4)]}+\frac{1}{[S_e(V_3)\times S_e(V_4)]}\}$

$=\{\frac{1}{[(3.3,2.7,2.2)\times(2.6,2.6,1.8)]}+\frac{1}{[(3.3,2.7,2.2)\times(3.3,2.7,1.8)]}+\frac{1}{[(3.3,2.7,2.2)\times(2.6,2.6,1.8)]}\}$

$+\{\frac{1}{[(3.3,2.7,2.2)\times(2.6,2.6,1.8)]}+\frac{1}{[(2.6,2.6,1.8)\times(3.3,2.7,1.8)]}\}$

$+\{\frac{1}{[(2.6,2.6,1.8)\times(3.3,2.7,1.8)]}+\frac{1}{[(3.3,2.7,2.2)\times(3.3,2.7,1.8)]}+\frac{1}{[(3.3,2.7,1.8)\times(2.6,2.6,1.8)]}\}$

$+\{\frac{1}{[(3.3,2.7,1.8)\times(2.6,2.6,1.8)]}+\frac{1}{[(3.3,2.7,2.2)\times(2.6,2.6,1.8)]}\}$

$=\frac{1}{(90.42,70.92,35.92)}$.

Theorem 5. F_1-Kulli-Basava polynomial for spherical fuzzy graph is $[x^{(5.9,5.3,4)^2}+x^{(6.6,5.4,4)^2}+x^{(5.9,5.3,4)^2}]+[x^{(5.9,5.3,4)^2}+x^{[(5.9,5.3,3.6)^2}]+[x^{[(5.9,5.3,4)^2}+x^{(6.6,5.4,4)^2}+x^{(5.9,5.3,3.6)^2}]+[x^{(5.9,5.3,4)^2}+x^{(5.9,5.3,3.6)^2}]$.

Proof. Using figure (1), tables 1-3 and the equation (5), we have
neighbours of V_1 are V_2,V_3,V_4 then $S_e(V_1)=d(V_2)+d(V_3)+d(V_4)=(3.7,2.7,2.2)$,
neighbours of V_2 are V_1,V_3 then $S_e(V_2)=d(V_1)+d(V_3)=(2.3,2.0,1.4)$,
neighbours of V_3 are V_2,V_4, V_1 then $S_e(V_3)= d(V_2)+d(V_4)+d(V_3)=(3.3,2.7,1.8)$,
neighbours of V_4 are V_1,V_3 then $S_e(V_4)=d(V_1)+d(V_3)=(2.6,2.6,1.8)$.

Then F_1-Kulli-Basava polynomial of spherical fuzzy graph
$F_1KB(G,x)=\sum_{uv\in(G)}x^{S_e(u)^2+S_e(v)^2}$

$=\{x^{[S_e(V_1)^2+S_e(V_2)^2]}+x^{[S_e(V_1)^2+S_e(V_3)^2]}+x^{[S_e(V_1)^2+S_e(V_4)^2]}\}$

$+\{x^{[S_e(V_1)^2+S_e(V_2)^2]}+x^{[S_e(V_2)^2+S_e(V_3)^2]}\}$

$+\{x^{[S_e(V_2)^2+S_e(V_3)^2]}+x^{[S_e(V_1)^2+S_e(V_3)^2]}+x^{[S_e(V_3)^2+S_e(V_4)^2]}\}$

$+\{x^{[S_e(V_1)^2+S_e(V_4)^2]}+x^{[S_e(V_3)^2+S_e(V_4)^2]}\}$

$=\{x^{[(3.3,2.7,2.2)^2+(2.6,2.6,1.8)^2]}+x^{[(3.3,2.7,2.2)^2+(3.3,2.7,1.8)^2]}+x^{(3.3,2.7,2.2)^2+(2.6,2.6,1.8)^2]}\}$

$+\{x^{[(3.3,2.7,2.2)^2+(2.6,2.6,1.8)^2]}+x^{[(2.6,2.6,1.8)^2+(3.3,2.7,1.8)^2]}\}$

$+\{x^{[(2.6,2.6,1.8)^2+(3.3,2.7,1.8)^2]}+x^{[(3.3,2.7,2.2)^2+(3.3,2.7,1.8)^2]}+x^{[(3.3,2.7,1.8)^2+(2.6,2.6,1.8)^2]}\}$

$+\{x^{[(3.3,2.7,2.2)^2+(2.6,2.6,1.8)^2]}+x^{[(3.3,2.7,1.8)^2+(2.6,2.6,1.8)^2]}\}$.

$=[x^{(5.9,5.3,4)^2}+x^{(6.6,5.4,4)^2}+x^{(5.9,5.3,4)^2}]+[x^{(5.9,5.3,4)^2}+x^{[(5.9,5.3,3.6)^2]}]$

$+[x^{[(5.9,5.3,4)^2}+x^{(6.6,5.4,4)^2}+x^{(5.9,5.3,3.6)^2}]+[x^{(5.9,5.3,4)^2}+x^{(5.9,5.3,3.6)^2}]$.

Theorem 6. Square Kulli-Basava polynomial for sherical fuzzy graph is $[x^{(0.7,0.1,0.4)^2}+x^{(0,0,0.4)^2}+x^{(0.7,0.1,0.4)^2}]+[x^{(-0.7,-0.1,-0.4)^2}+x^{(-0.7,-0.1,0.0)^2}]+[x^{(-0.7,0.1,0.0)^2}+x^{(0,0,-0.4)^2}+x^{(0.7,0.1,0.0)^2}]+[x^{(-0.7,-0.1,-0.4)^2}+x^{(-0.7,-0.1,0)^2}]$.

Proof. From figure (1) and tables (1-3) we get, square Kulli-Basava polynomial as,
$S_e(V_1)=d(V_2)+d(V_3)+d(V_4)=(3.7,2.7,2.2)$,
$S_e(V_2)= d(V_1)+d(V_3)=(2.3,2.0,1.4)$,
$S_e(V_3)= d(V_2)+d(V_4)+ d(V_3)=(3.3,2.7,1.8)$,
$S_e(V_4)= d(V_1)+d(V_3)=(2.6,2.6,1.8)$.

$QKB(G,x)=\sum_{uv\in(G)}x^{[S_e(u)-S_e(v)]^2}=\{x^{[S_e(V_1)-(V_2)]^2}+x^{[S_e(V_1)-(V_3)]^2]}+x^{[S_e(V_1)-(V_4)]^2}\}+\{x^{[S_e(V_1)-(V_2)]^2}+x^{[S_e(V_2)-(V_3)]^2}\}$

$+\{x^{[S_e(V_2)-(V_3)]^2}+x^{[S_e(V_1)-(V_3)]^2}+x^{[S_e(V_3)-(V_4)]^2}\}$

$+\{x^{[S_e(V_1)-(V_4)]^2}+x^{[S_e(V_3)-(V_4)]^2}\}$

$=\{x^{[(3.3,2.7,2.2)-(2.6,2.6,1.8)]^2}+x^{[(3.3,2.7,2.2)-(3.3,2.7,1.8)]^2}+x^{[(3.3,2.7,2.2)-(2.6,2.6,1.8)]^2}\}$

$+\{x^{[(2.6,2.6,1.8)-(3.3,2.7,2.2)]^2}+x^{[(2.6,2.6,1.8)-(3.3,2.7,1.8)]^2}\}$

$+\{x^{[(3.3,2.7,1.8)-(2.6,2.6,1.8)]^2}+x^{[(3.3,2.7,1.8)-(3.3,2.7,2.2)]^2}+x^{[(3.3,2.7,1.8)-(2.6,2.6,1.8)]^2}\}$

$+\{x^{[(2.6,2.6,1.8)-(3.3,2.7,2.2)]^2}+x^{[(2.6,2.6,1.8)-(3.3,2.7,1.8)]^2}\}$

$=[x^{(0.7,0.1,0.4)^2}+x^{(0,0,0.4)^2}+x^{(0.7,0.1,0.4)^2}]$

$+[x^{(-0.7,-0.1,-0.4)^2}+x^{(-0.7,-0.1,0.0)^2}]$

$+[x^{(-0.7,0.1,0.0)^2}+x^{(0,0,-0.4)^2}+x^{(0.7,0.1,0.0)^2}]$

$+[x^{(-0.7,-0.1,-0.4)^2}+x^{(-0.7,-0.1,0)^2}]$.

Theorem 7. First Zagreb polynomial for spherical fuzzy graph is
$x^{1.1,0.63,0.26}+x^{0.88,1.05,0.45}+x^{1.2,0.38,0.45}+x^{1.2,0.76,0.26}+x^{0.78,1.56,0.72}$ and first Zagreb index is $(5.16,4.38,2.14)$.

Proof. Using figure (1) and table 3, we get degrees of V_1, V_2, V_3 and V_4 vertices.
$M_1^{SFG}(G,x)=\sum_{uv\in E}x^{\alpha(uv)(d_\alpha(u)+d_\alpha(v))}$, and similarly for β, γ.

$M_1^{SFG}(G,x)$ s$=[x^{\alpha(uv)(d(V_1)+d(V_2))}+x^{\beta(uv)(d(V_1)+d(V_2))}+x^{\gamma(uv)(d(V_1)+d(V_2))}]+$
$[x^{\alpha(uv)(d(V_2)+d(V_3))}+x^{\beta(uv)(d(V_2)+d(V_3))}+x^{\gamma(uv)(d(V_2)+d(V_3))}]+$
$[x^{\alpha(uv)(d(V_3)+d(V_4))}+x^{\beta(uv)(d(V_3)+d(V_4))}+x^{\gamma(uv)(d(V_3)+d(V_4))}]+$
$[x^{\alpha(uv)(d(V_4)+d(V_1))}+x^{\beta(uv)(d(V_4)+d(V_1))}+x^{\gamma(uv)(d(V_4)+d(V_1))}]+$
$[x^{\alpha(uv)(d(V_1)+d(V_3))}+x^{\beta(uv)(d(V_1)+d(V_3))}+x^{\gamma(uv)(d(V_1)+d(V_3))}]$

$=[x^{0.5((1.3)+(0.9))}+x^{0.3((1.3)+(0.8))}+x^{0.2((0.8)+(0.5))}]+$
$[x^{0.4((0.9)+(1.3))}+x^{0.5((0.8)+(1.3))}+x^{0.3((0.5)+(0.1))}]+$
$[x^{0.6((1.3)+(1.1))}+x^{0.2((1.3)+(0.6))}+x^{0.3((1.0)+(0.5))}]+$
$[x^{0.5((1.1)+(1.3))}+x^{).4((0.6)+(1.3))}+x^{0.2((0.5)+(0.8))}]+$
$[x^{0.3((1.3)+(1.3))}+x^{0.6((1.3)+(1.3))}+x^{0.4((1.0)+(0.8))}]$

$=x^{1.1,0.63,0.26}+x^{0.88,1.05,0.45}+x^{1.2,0.38,0.45}+x^{1.2,0.76,0.26}+x^{0.78,1.56,0.72}$.

The first Zagreb index is $M_1^{SFG}(G)=M_1^{SFG}(G,x)|_{x=1}$
$=(1.1+0.88+1.2+1.2+0.78)+(0.63+1.05+0.38+0.76+1.56)+(0.26+0.45+0.45+0.26+0.72)=(5.16,4.38,2.14)$.

Theorem 8. Score function S(A) for spherical fuzzy graph with vertices V_1, V_2, V_3 and V_4 is 0.4,0.2,0.16 and 0.6 respectively.

Proof. Using formula (8) and table (3), we get score function for vertices: V_1, V_2, V_3, V_4 as:
Score function $S(A)=\alpha^2-\gamma^2$,
V_1: $S(A)=\alpha^2-\gamma^2=0.49-0.09=0.4$,
V_2: $S(A)=\alpha^2-\gamma^2=0.36-0.16=0.2$,
V_3: $S(A)=\alpha^2-\gamma^2=0.25-0.09=0.16$,

V_4: S(A)= $\alpha^2 - \gamma^2$ =0.64-0.04=0.6.

Higher truth membership (α) increases score and higher falsity (γ) decrease score.

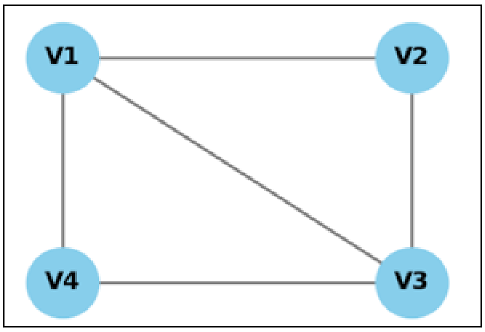

Figure 1: Spherical fuzzy graph.

Table 1: Vertex memberships

Vertex	α_v	β_v	γ_v	$\alpha^2 + \beta^2 + \gamma^2 \leq 1$
V_1	0.7	0.4	0.3	0.49+0.16+0.09=0.74≤1
V_2	0.6	0.5	0.4	0.36+0.25+0.16=0.77≤1
V_3	0.5	0.6	0.3	0.25+0.36+0.09=0.7≤1
V_4	0.8	0.3	0.2	0.64+0.09+0.04=0.77≤1

Table 2: Edge memberships

Edge	α_{uv}	β_{uv}	γ_{uv}	$\alpha^2 + \beta^2 + \gamma^2 \leq 1$
e_{12}	0.5	0.3	0.2	0.25+0.09+0.04=0.38≤1
e_{23}	0.4	0.5	0.3	0.16+0.25+0.09=0.5≤1
e_{34}	0.6	0.2	0.3	0.36+0.04+0.09=0.49≤1
e_{41}	0.5	0.4	0.2	0.25+0.16+0.04=0.45≤1
e_{13}	0.3	0.6	0.4	0.09+0.36+0.16=0.61≤1

Table 3: Degree of each vertex

Vertex	$d_\alpha(v)$	$d_\beta(v)$	$d_\gamma(v)$	d(v)
V_1	0.5+0.3+0.5=1.3	0.3+0.6+0.4=1.3	0.2+0.4+0.2=0.8	(1.3,1.3,0.8)
V_2	0.5+0.4=0.9	0.3+0.5=0.8	0.2+0.3=0.5	(0.9,0.8.0.5)
V_3	0.6+0.3+0.4=1.3	0.2+0.6+0.5=1.3	0.3+0.4+0.3=1.0	(1.3,1.3,1.0)
V_4	0.5+0.6=1.1	0.4+0.2=0.6	0.2+0.3=0.5	(1.1,0.6,0.5)

4. Conclusion

Fuzzy graph theory is very vast field in Mathematics. Kulli-Basava indices and corresponding polynomials are investigated for spherical fuzzy graph. These indices and polynomials encode full uncertainty in molecular structure. Neutrality introduces an extra degree of freedom enabling independent adjustment of uncertainty, better modeling of multidimensional vagueness. In a spherical fuzzy environment uncertainty is captured through the interaction of three independent degrees. The spherical fuzzy graph allows more flexibility because $\alpha^2 + \beta^2 + \gamma^2 \leq 1$, which gives a 3D spherical space.

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