

An Introduction to Gravitational Waves

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Abstract: *This review presents an introduction to gravitational waves, outlining their theoretical basis in general relativity, major astrophysical sources, principal detection methods, and scientific implications. It discusses ground-based and space-based observatories, including interferometric and pulsar timing approaches, and highlights the role of gravitational-wave astronomy in modern astrophysics and cosmology. The review aims to provide a concise overview of foundational concepts and recent developments in the field.*

Keywords: Gravitational Waves; General Relativity; LIGO; Black Hole Mergers; Pulsar Timing Arrays; Multimessenger Astronomy

1. Introduction

Albert Einstein introduced the general theory of relativity in 1915, offering a fundamentally new perspective on gravity. Unlike the classical view of Isaac Newton, where gravity is treated as a force acting between masses, Einstein described it as a geometric property of spacetime [1]. In this framework, the presence of mass and energy distorts the structure of spacetime, and this curvature determines how objects move. As a result, bodies such as stars and planets bend the surrounding spacetime, guiding the motion of other objects along curved trajectories within this distorted geometry.

A major implication of General Relativity is the prediction of gravitational waves. The theory states that when massive bodies undergo acceleration, they generate disturbances in spacetime that propagate outward as waves. These gravitational waves move at the speed of light and transport energy away from their source. A helpful analogy is the behavior of water in a pond: when a stone is dropped into still water, circular ripples spread across the surface. In a similar way, accelerating massive systems such as black holes or neutron stars produce ripples in spacetime that travel through the universe as gravitational waves [24].

Although General Relativity provides a compelling theoretical framework that predicts gravitational waves, observing them directly has been extraordinarily difficult. These waves are much weaker than the effects associated with other fundamental interactions, such as electromagnetism or strong and weak nuclear forces. Their faintness arises from the inherently weak nature of gravity, especially when signals travel across immense cosmic distances. As a result, the distortions they produce in spacetime are exceedingly small, making their direct detection exceptionally challenging.

It was not until the late twentieth and early twenty-first centuries that advances in technology, together with large-scale scientific collaboration, made the direct observation of gravitational waves possible. The Laser Interferometer Gravitational-Wave Observatory (LIGO), which consists of two large interferometers located in the United States— one in Hanford, Washington, and the other in Livingston, Louisiana—was specifically designed for this purpose. On September 14, 2015, LIGO achieved a landmark result by recording gravitational waves produced by the merger of two black holes, marking the first direct detection of such signals

[4,5].

The first observation of gravitational waves marked the fulfillment of a long-standing scientific goal and provided direct confirmation of a key prediction made by Albert Einstein in his theory of General Relativity over a century ago. This achievement not only verified a fundamental aspect of our understanding of gravity but also opened an entirely new way of exploring the universe and established the foundation of modern gravitational-wave astronomy [23,24]. Because gravity influences all forms of matter and energy, the study of gravitational waves, an area now known as Gravitational Wave Astronomy, offers a powerful tool for observing cosmic phenomena. It enables scientists to investigate events that were previously inaccessible, providing a deeper understanding of the structure and evolution of the universe including recent advances in precision cosmology using gravitational waves [2,24].

Gravitational wave astronomy offers a unique way of observing the universe, allowing scientists to study previously undetectable phenomena using traditional electromagnetic methods. By directly observing the ripples in spacetime caused by events like black hole mergers and neutron star collisions, gravitational wave detectors provide a new perspective on the universe's most energetic and violent events and even the mysterious processes unfolding in the universe's earliest moments. Gravitational waves provide a unique window into the fabric of spacetime itself, offering insights into the fundamental workings of gravity and the structure of the universe.

This section briefly introduced the theory of General Relativity and gravitational waves, outlining key concepts and their mathematical description. It then reviews major sources of gravitational waves, detection methods, and their scientific implications, before concluding with a summary of the main points.

2. Theory of General Relativity and Gravitational Waves

2.1 Basics of Einstein's theory of general relativity

In classical physics, space and time are treated as separate entities. However, in general relativity, Einstein proposed that they are intimately connected to form a four-dimensional

continuum called spacetime. This unified framework allows us to describe events using their spatial and temporal coordinates (time coordinates). According to general relativity, the presence of mass and energy causes spacetime to curve. Imagine placing a heavy object on a rubber sheet. The mass of the object causes the sheet to curve around it. Similarly, massive objects curve spacetime in their vicinity. This curvature influences the motion of other objects, causing them to follow curved paths in spacetime.

Albert Einstein's general theory of Relativity introduced a fundamentally new way of interpreting gravity. Instead of viewing it as a force acting between objects over a distance, the theory describes gravity as a consequence of the curvature of spacetime produced by mass and energy. The equations of general relativity establish a relationship between this curvature and the distribution of matter and energy, offering a geometric framework for understanding gravitational phenomena.

The equations of general relativity, known as Einstein's field equations, provide a mathematical framework for describing the relationship between the distribution of matter and energy and the curvature of spacetime. This unified description of gravity as geometry has profound implications for our understanding of the universe, from the behaviour of celestial bodies to the nature of black holes and the universe's expansion.

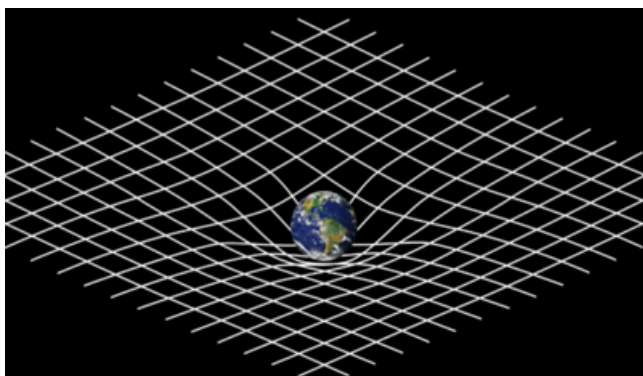


Figure 1: In general relativity, gravity is viewed as a movement of particle in the space- time [7]

This curvature affects the trajectories of other objects nearby. For example, smaller objects like planets follow curved paths around the star due to the curvature of spacetime. This is what we observe as the gravitational force. The curvature of spacetime determines how objects move within it, leading to phenomena such as the bending of light and the formation of orbits for celestial bodies. Understanding spacetime curvature helps us explain various gravitational phenomena, from the motion of planets around stars to the bending of light around massive objects like black holes. It's a fundamental concept in modern physics, essential for understanding gravity's nature and objects' behaviour in the universe.

2.2 Mathematical formulation of gravitational wave solutions

Gravitational waves arise in General Relativity as perturbations in the curvature of spacetime that travel outward from accelerating masses. From a mathematical perspective,

these waves emerge as specific solutions of Einstein's field equations. These equations consist of ten coupled, nonlinear partial differential equations that relate the geometry of spacetime to the distribution of matter and energy. They are commonly expressed in the following form:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

In this expression $G_{\mu\nu}$ denotes the Einstein tensor, which characterizes the curvature of spacetime. The term $T_{\mu\nu}$ represents the stress-energy tensor, accounting for the distribution of mass, energy, momentum, and pressure. The symbol Λ corresponds to the cosmological constant, often introduced to describe the energy associated with empty space. The constant G known as the gravitational constant, determines the strength of the gravitational interaction within the framework of General Relativity.

Solving Einstein's field equations for gravitational waves is challenging, often requiring advanced mathematical techniques and numerical simulations. However, these solutions provide crucial insights into the properties of gravitational waves and their interactions with matter and spacetime.

2.3 Characteristics of gravitational waves

Gravitational waves possess several key characteristics. They travel at the speed of light, meaning they propagate through spacetime with the maximum possible velocity. Gravitational waves propagate as transverse waves, meaning the oscillations occur perpendicular to the direction of propagation. This is in contrast to longitudinal waves, where the oscillations are parallel to the direction of propagation. As a gravitational wave passes through spacetime, it alternately stretches and compresses space along orthogonal directions. This stretching and compression cause the changes in distance between freely falling test masses, which gravitational wave detectors measure. Gravitational waves carry energy away from their sources, causing the objects emitting them to lose energy and move closer together. Gravitational waves have characteristic frequencies and amplitudes that depend on the properties of their sources. The frequency of a gravitational wave is related to the orbital period of the source. At the same time, the amplitude is determined by the strength of the gravitational field and the distance from the source. A clear understanding of these features is crucial for both theoretical analysis and the experimental detection of gravitational waves. It helps deepen our knowledge of the fundamental nature of spacetime and improves our ability to study dynamic astrophysical processes such as merging black holes and neutron stars.

3. Sources of Gravitational waves

Gravitational waves are generated by various astrophysical phenomena involving the acceleration or motion of massive objects. Some of the primary sources of gravitational waves include:

3.1 Binary Systems

Binary systems are fascinating sources of gravitational waves due to their dynamic interactions [6,8]. They consist of two massive objects in orbit around their common centre of mass. These objects can be black holes, neutron stars, or white dwarfs. As they orbit each other, they form a gravitationally bound system, with their mutual gravitational attraction keeping them in orbit. According to general relativity, accelerating masses generate gravitational waves. In a binary system, the orbital motion of the two objects causes them to accelerate relative to each other. This acceleration leads to the emission of gravitational radiation. Gravitational waves are produced by the changing quadrupole moments of the binary system. As the objects orbit each other, their relative positions and velocities change, causing their quadrupole moments (which describe the distribution of mass and energy within the system) to vary over time. These variations in the quadrupole moments result in the emission of gravitational waves. This radiation carries away energy and angular momentum, causing the binary system to spiral inward and eventually merge. The inspiral and merger phases of binary systems are particularly strong sources of gravitational waves. Gravitational wave observatories like LIGO (Laser Interferometer Gravitational-Wave Observatory) [9] and Virgo [10] are specifically designed to detect the tiny distortions in spacetime caused by passing gravitational waves. By measuring the changes in the distances between freely falling test masses, these detectors can detect the gravitational waves emitted during binary systems. Binary systems are key sources of gravitational waves that provide invaluable insights into the dynamics and properties of compact objects and the nature of gravity itself [4].

3.2 Supernovae

Supernovae, the explosive deaths of massive stars, can also generate gravitational waves as the collapsing core undergoes rapid changes in density and asymmetrical motions. When a massive star undergoes core collapse and explodes as a supernova, the violent processes involved can produce disturbances in the fabric of spacetime, leading to the emission of gravitational waves. These waves carry information about the dynamics of the collapse and explosion, offering us a unique window into the cataclysmic events that mark the end of massive stars' lives. If we can detect these gravitational waves, it would confirm our understanding of supernova physics and provide valuable insights into the behaviour of matter under extreme conditions and the formation of compact objects like neutron stars and black holes. The gravitational waves produced by individual supernovae are generally weaker than those from binary mergers; they contribute to the universe's overall background of gravitational wave signals [11].

3.3 Cosmic Inflation

Cosmic inflation is a theory that suggests the universe underwent an extremely rapid and exponential expansion in the first fraction of a second after the Big Bang. According to quantum field theory, even empty space is not truly empty but is filled with energy fluctuations and fields. During inflation, these quantum fluctuations were stretched to cosmological scales, becoming the seeds for the large-scale structure observed in the universe today, including galaxies and galaxy

clusters. Quantum fluctuations during inflation would have included fluctuations in the gravitational field itself. These fluctuations, amplified by the rapid expansion of space during inflation, would have given rise to gravitational waves. As space expanded, it caused ripples in the fabric of spacetime, generating gravitational waves.

Gravitational waves generated during inflation are called primordial gravitational waves. They differ from the gravitational waves produced by astrophysical phenomena like merging black holes or neutron stars. Primordial gravitational waves carry information about the very early universe and the physics of inflation itself. The cosmic microwave background (CMB) radiation is the oldest light in the universe, emitted when the universe cooled enough for photons to travel freely. Primordial gravitational waves would have imprinted a distinct pattern, known as B-mode polarization, on the CMB. Detecting this polarization pattern would provide direct evidence for the existence of primordial gravitational waves [12].

3.4 Cosmic Strings and Phase Transitions

Cosmic strings are hypothetical one-dimensional topological defects that may have formed during phase transitions in the early universe. These phase transitions occur as the universe cools and undergoes symmetry-breaking changes in its fundamental forces. During certain phase transitions, regions of space can become trapped in a different vacuum state, causing defects like cosmic strings to form. These strings are highly energetic and thin, extending across vast cosmic distances. Cosmic strings possess significant energy density and curvature of spacetime along their length. As cosmic strings move through the universe or interact with other cosmic structures, they can produce gravitational waves. Detecting gravitational waves from cosmic strings presents significant challenges due to their expected low amplitudes and characteristic signatures. Various experiments and observatories, including ground-based detectors like LIGO and space-based missions like LISA (Laser Interferometer Space Antenna), are designed to search for these signals.

These examples represent only a small subset of the many energetic astrophysical processes that can produce gravitational waves. Observations of these signals provide powerful insights into extreme gravitational environments, the behaviour of compact objects such as black holes and neutron stars, and the evolution of the universe, while also opening new avenues for research in astronomy and cosmology.

4. Methods of detection

Gravitational waves are incredibly subtle phenomena that require incredibly sensitive equipment to detect them. Several methods have been developed to detect these elusive signals.

4.1 Ground-based Interferometry

Interferometers, such as those used by the Laser Interferometer Gravitational-Wave Observatory (LIGO), consist of long arms set perpendicular to each other. A laser beam is split and sent down these arms, reflecting off mirrors and returning to a central detector. When a gravitational wave

passes through the detector, it causes the fabric of spacetime to stretch and compresses ever so slightly, altering the arms' lengths. This alteration leads to a difference in the time it takes for the laser beams to travel down the arms and return, which is detected as an interference pattern when the beams recombine. By analysing these interference patterns, scientists can precisely measure the properties of the gravitational waves, such as their frequency and amplitude. LIGO made history by detecting gravitational waves for the first time in 2015, confirming a prediction made by Einstein's general theory of relativity.

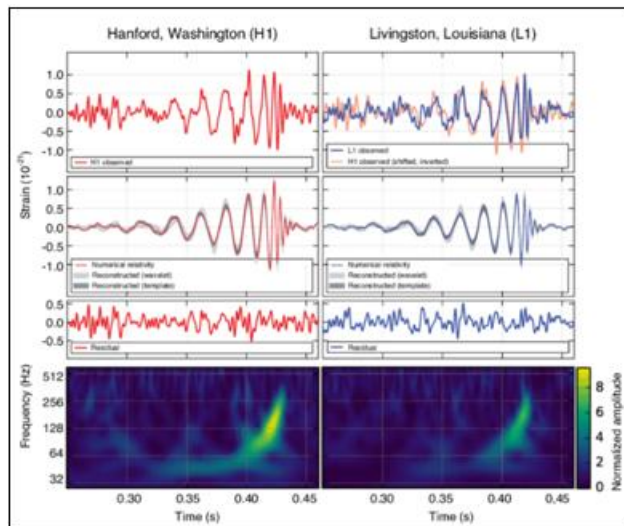


Figure 2: The gravitational wave event GW 150914 observed by the LIGO Hanford (H1, left column panels) and Livingston (L1, right column panels) detectors. [4]

Virgo is another ground-based interferometric gravitational wave detector in Italy, near Pisa. Like LIGO, Virgo consists of two perpendicular arms that are several kilometres long, with mirrors at their ends. It operates on the same principle of laser interferometry to detect gravitational waves. Virgo collaborates closely with LIGO, providing additional data and helping to triangulate the sources of detected gravitational waves. Having multiple detectors allows scientists to improve the localization of gravitational wave sources in the sky [21].

Some additional ground-based gravitational-wave detectors complement the efforts of LIGO and Virgo in the global network of gravitational-wave observatories:

KAGRA: Located in Japan, KAGRA is a cryogenic interferometric gravitational wave detector that aims to achieve greater sensitivity by operating at cryogenic temperatures.

GEO600: Located in Germany, GEO600 is a smaller scale gravitational wave detector that serves as a testbed for advanced technologies and techniques.

INDIGO: The Indian Initiative in Gravitational wave Observations (INDIGO) aims to develop gravitational wave detectors in India, contributing to the global effort in gravitational wave astronomy.

4.2 Pulsar timing arrays

Pulsar Timing Arrays (PTAs) is a novel method of detecting low-frequency gravitational waves by observing the timing of pulses emitted by pulsars, which are rapidly rotating neutron stars. Pulsars emit beams of radiation that sweep across space like cosmic lighthouses as they rotate. These beams are observed as periodic radiation pulses as the beam crosses the Earth's line of sight. Pulsars are incredibly stable cosmic clocks, emitting pulses with precise regularity. Gravitational waves passing through space cause minute distortions in the fabric of spacetime. When a gravitational wave passes between a pulsar and the Earth, it causes a slight stretching or compression of space, which alters the arrival times of the pulses observed on Earth. By comparing the expected arrival times of pulses from multiple pulsars with their actual arrival times, astronomers can detect deviations caused by gravitational waves. These deviations manifest as tiny fluctuations in the arrival times, known as timing residuals. PTAs are sensitive to gravitational waves with frequencies lower than those detectable by ground-based detectors. Recent pulsar timing array collaborations (North American Nanohertz Observatory for Gravitational Waves (NANOGrav), Parkes Pulsar Timing Array (PPTA) and European Pulsar Timing Array (EPTA)) have reported compelling evidence (2023) for a nanohertz stochastic gravitational-wave background [24–26].

4.3 Space-based detectors

Space-based detectors are a class of gravitational-wave observatories that operate in orbit around the Earth, offering unique advantages over ground-based detectors. LISA (Laser Interferometer Space Antenna) is a planned space-based gravitational-wave observatory led by the European Space Agency (ESA), with contributions from NASA. It consists of three spacecraft arranged in a triangular formation separated by millions of kilometres. Each spacecraft contains freely falling test masses that serve as inertial reference points. Laser interferometry measures the distances between the test masses with extreme precision, allowing LISA to detect gravitational waves by monitoring changes in the relative distances caused by passing gravitational waves. LISA is designed to be sensitive to lower frequency gravitational waves (millihertz to 1 Hz), detecting sources such as binary supermassive black hole mergers, extreme mass ratio inspirals, and compact white dwarf binaries [22].

By placing detectors in orbit, space-based observatories can achieve long observation baselines, allowing for precise measurements of gravitational waves from distant sources. Space-based detectors operate in a near-perfect vacuum environment, minimizing noise sources that can interfere with gravitational wave measurements. Space-based detectors complement ground-based detectors by observing different frequency ranges and types of sources, providing a more comprehensive view of the gravitational wave universe.

The Laser Interferometer Space Antenna (LISA) is planned for launch in the 2030s with updated mission sensitivity and design forecasts described in recent studies [26]. Once deployed, it is expected to significantly advance our understanding of the universe by observing gravitational

waves from a wide variety of astrophysical sources. In addition, other proposed space-based missions, such as the Deci-hertz Interferometer Gravitational Wave Observatory (DECIGO), are designed to probe gravitational waves at even lower frequencies, particularly in the sub-hertz range, complementing the capabilities of LISA.

5. Future Concepts

5.1 Einstein Telescope

The Einstein Telescope (ET) is a proposed third-generation gravitational-wave detector that aims to enhance our capabilities for observing and studying gravitational waves significantly. The Einstein Telescope is designed to have significantly improved sensitivity compared to current gravitational-wave detectors like LIGO and Virgo. It aims to detect a broader range of frequencies with higher precision, enabling the observation of a wider variety of astrophysical sources. The Einstein Telescope will be built underground to minimize the effects of terrestrial noise and environmental disturbances. This underground design reduces seismic noise and thermal fluctuations, allowing for more precise measurements of gravitational waves [22].

Unlike the L-shaped configurations of current detectors, the Einstein Telescope is proposed to have a triangular configuration of three arms arranged in an equilateral triangle. This design offers several advantages, including increased sensitivity to gravitational waves from all directions and improved localization of sources. The Einstein Telescope will utilize advanced mirror and suspension technologies to minimize noise and maximize sensitivity. These mirrors will be suspended in a vacuum to isolate them from external disturbances and equipped with advanced coatings to reduce thermal noise. The Einstein Telescope aims to detect gravitational waves across a wide frequency range, from low-frequency waves generated by sources like binary supermassive black hole mergers to high-frequency waves from compact binary mergers and cosmic string events. Einstein Telescope represents an exciting prospect for the future of gravitational-wave astronomy, offering the potential to revolutionize our understanding of the universe by directly observing gravitational waves across a wide range of frequencies and sources as outlined in recent design and science case studies [26].

5.2 Cosmic Explorer

Cosmic Explorer is another proposed third-generation ground-based detector, aiming to push the sensitivity of gravitational-wave detection even further than current detectors like LIGO and Virgo. Cosmic Explorer aims to push the boundaries of gravitational-wave detection by improving sensitivity and extending the frequency range accessible to ground-based detectors. It seeks to observe a broader spectrum of gravitational waves, including signals from more distant and lower-mass sources. Cosmic Explorer is expected to achieve greater sensitivity than existing detectors by employing advanced technologies and larger detector configurations. This increased sensitivity will enable the detection of weaker gravitational-wave signals from a wider range of astrophysical sources.

Each of these methods offers unique advantages and challenges in detecting gravitational waves, and combining data from multiple detectors using different techniques allows scientists to build a more comprehensive understanding of these elusive phenomena.

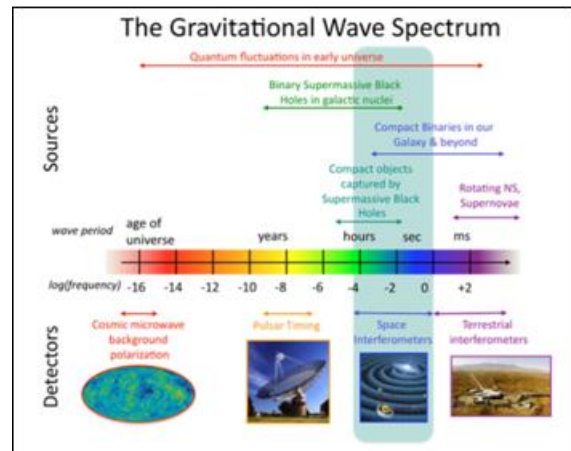


Figure 3: The gravitational wave spectrum with sources and detectors [7]

6. Implication and application

The discovery of gravitational waves has profound implications across various fields of science and technology. Here are some of the key implications and applications:

6.1 Confirmation of General Relativity

Gravitational waves directly confirm Einstein's theory of General Relativity, which predicted their existence over a century ago. By observing gravitational waves from various sources and comparing them with the predictions of General Relativity, scientists can test the theory under extreme conditions, such as strong gravitational fields and high velocities. Any discrepancies between the observed signals and theoretical predictions could indicate the need for modifications to our current understanding of gravity, potentially leading to the development of new theories.

6.2 Astrophysics and Cosmology

Gravitational wave detection offers a new window into the universe, allowing scientists to explore space regions inaccessible to traditional telescopes. By studying the properties of gravitational wave sources, such as their masses, spins, and distances, astronomers can gain insights into the formation and evolution of galaxies, black holes, neutron stars, and other cosmic structures. Gravitational waves also provide information about the universe's expansion rate, the distribution of dark matter and dark energy, and the nature of cosmic inflation.

6.3 Black Hole Physics

Observations of gravitational waves from merging black holes have offered remarkable new insights into their physical properties. From these signals, scientists can determine key parameters such as masses and spins with high accuracy, analyse the behaviour of binary systems, and test predictions

of General Relativity- including the characteristic “ringdown” phase that follows a merger. In addition, gravitational wave data from these events help place limits on alternative theories of gravity and improve our understanding of how black holes form and evolve in the universe.

6.4 Neutron Star Physics

The detection of gravitational waves from neutron star mergers, such as the landmark observation of GW170817, has revolutionized our understanding of these dense stellar remnants. Gravitational wave signals from neutron star mergers provide insights into the equation of state of nuclear matter, the behaviour of ultra-dense neutron star cores, and the production of heavy elements in the universe through r-process nucleosynthesis. Neutron star mergers are also believed to be sources of short gamma-ray bursts, making multimessenger observations crucial for studying these energetic phenomena.

6.5 Cosmic inflation

Gravitational waves generated during cosmic inflation leave distinct signatures on the polarization of the cosmic microwave background radiation. By analysing the polarization patterns in the cosmic microwave background, scientists can test inflation models, probe the energy scale of inflationary physics, and constrain the properties of hypothetical inflationary fields. Detecting primordial gravitational waves would provide direct evidence for the inflationary paradigm and offer insights into the earliest moments of the universe's history.

6.6 Fundamental physics

Observations of gravitational waves provide a powerful way to examine the predictions of General Relativity and to investigate possible extensions or alternatives, including modified gravity models and scenarios involving extra spatial dimensions. By analysing the properties of detected signals and looking for any departures from the expectations of General Relativity, researchers can place limits on competing theories and potentially uncover evidence of new physics beyond the Standard Model.

6.7 Technological Development

The development of gravitational wave detectors, such as LIGO and Virgo, has driven advances in precision measurement techniques, laser technology, vibration isolation, and data analysis algorithms. These technological innovations have applications beyond astrophysics, including in fields such as quantum metrology, precision engineering, medical imaging, and geophysical monitoring. Future improvements in detector sensitivity and observational capabilities will continue to rely on cutting-edge technologies and interdisciplinary collaborations.

6.8 Multimessenger Astronomy

Gravitational wave observations are often combined with data from other astronomical observatories, such as telescopes that detect electromagnetic radiation (e.g., optical, infrared, X-ray,

radio) and neutrino detectors. Multimessenger observations provide complementary information about astrophysical phenomena, allowing scientists to study cosmic events from multiple perspectives and test theoretical models. By integrating data from different sources, researchers can gain a more comprehensive understanding of the underlying physics, dynamics, and environments associated with gravitational wave sources.

These implications and applications highlight the transformative impact of gravitational wave astronomy on our understanding of the universe and the advancement of scientific knowledge.

7. Conclusion

Gravitational-wave astronomy has transformed modern physics by providing a direct probe of compact objects, strong-field gravity, and cosmological phenomena. Advances in observational facilities have confirmed key predictions of general relativity while opening new opportunities in multimessenger astronomy. Future detectors are expected to expand sensitivity and frequency coverage, further deepening our understanding of the universe.

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