A Fuzzy Programming Technique in Agriculture Using Triangular Membership Function

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Abstract: ***This study examines decision-making problems in which the best possible land allocation for a variety of crops, including rice, groundnut, and black gram, is determined by a fuzzy linear programming (FLP) approach using triangular membership functions. Consideration has been given to the solution of converting FLP into clear multi-objective linear programming problems. In order to compare the outcomes, the mean and median of triangular fuzzy numbers are also taken into account.***

Keywords: Fuzzy linear programming, Triangular membership function, Maximizing income, Fuzzy set, Crop combination.

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1. Introduction

In practical applications, a linear programming model uses parameters provided by experts with unknown values. The manufacturer describes the uncertainties that the decision maker must cope with, and these assigned figures are not precise. To address these uncertainties, sensitivity analysis can be employed to evaluate how changes in the parameters affect the overall solution. This approach allows decision makers to understand the robustness of their strategies and make more informed choices under varying conditions. The study focuses on fuzzy linear programming (FLP) models where the parameters are only partially known with a certain level of accuracy. Smallholder farmers make up the majority of the agricultural industry, and one of their biggest challenges is figuring out how to use their property as efficiently as possible to maximize their profits. Despite having superior plantation skills, farmers still struggle with making decisions about what and when to plant. Cost fluctuations and product costs are major contributors to these issues. There are numerous causes for these variations. Some farmers cultivate plants according to custom or the prices of the market at the time. In order to maximize farmers' profits given their resources, this research examines a way for solving the fuzzy linear programming (FLP) problem with triangular membership. Three crops are taken into consideration in this study: rice, groundnut, and black gram.

2. Literature Survey

Alsheikh and Ahmad (2002) describe how linear programming has been employed in literature as a strategy to

achieve optimal results. The linear programming technique was introduced by Radhakrishnan (1962) and Krishna (1963) to determine the best farming planning. Felix and Judith (2010) employ a linear programming model to allocate farm resources. In order to optimize the overall returns at the end of the planning horizon, Mohamad and Said (2010) employed the mathematical programming approach. According to Keith Butterworth (1985), linear programming (LP) would be worth reevaluating as a maximizing strategy in agricultural planning given the state of the economy. Annetts and Audeley (2002) created an LP model that allows for profit maximization by taking into account a variety of farming situations. Using conventional approaches, farmers have made judgments based on rivalry with neighbors and experience, as noted by Hazzel and Norton (1986). To identify the ideal enterprise combination, Igwe et al. (2011) and Lone et al. (2014) employed the LP approach. According to Higgins et al. (2004), preserving production efficiency in agricultural planning is essential to company profitability since it can boost an operation's returns with minimal additional expenses. Zadeh (1965) was the first to formulate fuzziness mathematically. To address the many kinds of FLP challenges, several writers employ alternative approaches. Almost every area of decision-making problems has seen the development of fuzzy approaches, as demonstrated by Tamiz (1996), Zimmermann (1991), Lone et al. (2016), and Ross (1995). Senthilkumar and Rajendran (2010) use fuzzy variables in the parametric form to solve the FLPP. Numerous attempts were made by Orlovsky (1980) to investigate whether fuzzy set theory could be a helpful tool for sufficient mathematical analysis of real-world problems.

3. Mathematical Model

3.1 Definition of Membership function

1) Let U be the universal set $\widetilde{A}$ is referred to as a fuzzy set if it is a set of ordered pairs

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x) \mid R \in U\}$$

where $\mu_{\widetilde{A}}(x)$ is the membership function.

2) Membership: The purpose of the membership function is to indicate whether or not an element x is a part of the set, say A. For a set A, we define membership function of μ_A such as

$$\mu_A = \begin{cases} 1 \ \text{if and only if } x \in A \\ 0 \ \text{if and only if } x \notin A \end{cases}$$

Every element in a fuzzy set is mapped to the [0, 1] membership function $\mu_A : x \to [0,1]$. The real number between 0 and 1 (including 0 and 1) is denoted by [0,1].

3) If $\mu_A(x) = 1$, then a fuzzy set $\widetilde{A}$ is normal.

4) A fuzzy set $\widetilde{A}$ is convex if and only if $x_1, x_2 \in U$

$$\mu_{\widetilde{A}}\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}, \lambda \in [0,1]$$

5) A fuzzy set is referred to as a fuzzy number if it is convex, normalized, and has a piecewise continuous membership function specified in R. A real number interval with a fuzzy limit is represented as a fuzzy number (fuzzy set).

6) A fuzzy number $\mu_{\widetilde{A}}$ is called L/R fuzzy number if its membership function satisfies the following properties:
is continuous function from R to the closed interval [0,1].
$\mu_{\widetilde{A}}$ is strictly decreasing for all $x \in [a_3, a_4]$.
$\mu_{\widetilde{A}}$ is strictly increasing for all $x \in [a_1, a_2]$.
$\mu_{\widetilde{A}}(x) = 1 \ m \ or \ all \ x \in [a_2, a_3]$
noted by $\widetilde{A} = [a_1, a_2, a_3, a_4]$, where (a_1, a_2, a_3, a_4) are real numbers. The parametric form of a fuzzy number is an ordered pair of functions $(a^L(\alpha), a^R(\alpha))$, where α lies between 0 and 1, and $a^L(\alpha) \leq a^R(\alpha)$. $a^L(\alpha)$ and $a^R(\alpha)$ are continuous non-decreasing bounded left function and continuous non-increasing bounded right function over closed interval [0,1] respectively. Interval numbers, trapezoidal fuzzy numbers, and triangular fuzzy numbers are the most often utilized fuzzy mathematical programming problems in practice; the decision maker can specify them with ease.

A fuzzy number $\widetilde{A}$ is said to be a trapezoidal fuzzy number denoted by $(a_1, a_2, a_3, a_4 : \omega)$ if its membership is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x-a_1)}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \dfrac{(a_4 - x)}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases} \quad (1)$$

A fuzzy number $\widetilde{A}$ is said to be a triangular fuzzy number denoted by (a_1, a_2, a_3) if its membership is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x-a_1)}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & x = a_2 \\ \dfrac{(a_3 - x)}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & otherwise \end{cases} \quad (2)$$

3.2 Some Properties

1) A trapezoidal fuzzy number $\widetilde{A} = [a_1, a_2, a_3, a_4]$ is said to be zero trapezoidal fuzzy number if $a_1 = a_2 = a_3 = a_4 = 0$.
2) A trapezoidal fuzzy number $\widetilde{A} = [a_1, a_2, a_3, a_4]$ is said to be non-negative trapezoidal fuzzy number if $a_1 - a_3 \geq 0$.
3) Two trapezoidal fuzzy number $\widetilde{A} = [a_1, a_2, a_3, a_4]$ and $\widetilde{B} = [b_1, b_2, b_3, b_4]$ is said to be equal if $a_1 = b_1, a_2 = b_2, a_3 = b_3 \ and \ a_4 = b_4$.
4) Addition and subtraction between fuzzy numbers become a trapezoidal fuzzy number.
5) Multiplication, division, and inverse need not be a trapezoidal fuzzy number.
6) Minimum and Maximum of fuzzy number is not always in the form of a trapezoidal fuzzy number.
7) The results from addition or subtraction between triangular fuzzy numbers result also triangular fuzzy numbers.
8) The results from multiplication or division are not triangular fuzzy numbers.
9) The minimum or maximum operation does not give a triangular fuzzy number (TFN). But we often assume that the operational results of multiplication or division to be TFNs as approximation values.

3.3 Fuzzy Linear Programming Problem (FLPP)

Consider the following linear programming model

$$\begin{aligned} & Max \ Z = CX \\ & subject \ to \\ & AX \leq B \\ & X \geq 0 \end{aligned} \quad (3)$$

Where $C = 1 \times n$ vector of components. $B = n \times 1$ a vector of all crisp values. X is a decision variable vector and $A = m \times n$ matrix of coefficients. The fuzzy coefficients are determined in such a way that the fuzzy output has the minimum fuzzy at a target degree of belief $h \ or \ \alpha$. The parameter h can be chosen by the decision maker and represents the desired degree of belief. The value of h is between 0 and 1. If the degree of confidence (or degree of belief) is set to zero, then the assumed model is extremely compatible with the data. If the degree of confidence is set to higher h =1, then the assumed model is extremely incompatible with the data and the upper and lower fuzzy bounds are widened in order to embed all the observations at the h-level set.

For finding optimal solution of the FLPP, we define some definitions as

1) A fuzzy vector X is a basic solution of the FLPP if it satisfies set of constraints $(AX \le B)$.
2) A fuzzy vector X is a feasible solution of the FLPP if it satisfies set of constraints $(AX \le B)$ non-negativity condition $X \ge 0$.
3) A feasible solution of the fuzzy vector X that optimizes the objective function $Z = CX$ is the optimal solution to the FLPP.
4) The value of the objective function provided by the optimal solution is called an optimal value.
5) Let $\sum_{j}^{n} a_{ij} x_j \le b_i$, where $b_i \ge 0$ be the i^th^ fuzzy constraint of a fuzzy linear programming problem than a fuzzy variable S_i such that $S_i \ge 0$ and $\sum_{j}^{n} a_{ij} x_j - S_i = b_i$ is called fuzzy slack variable.
6) Let $\sum_{j}^{n} a_{ij} x_j \ge b_i$, where $b_i \ge 0$ be the i^th^ fuzzy constraint of a fuzzy linear programming problem than a fuzzy variable S_i such that $S_i \ge 0$ and $\sum_{j}^{n} a_{ij} x_j + S_i = b_i$ is called fuzzy surplus variable.
7) Consider the system $AX = B$ with $X \ge 0$ where A is $m \times n$ matrix of m. Let the columns of A corresponding to fuzzy variables $X_{\lambda 1}, X_{\lambda 2}, \ldots\ldots, X_{\lambda m}$ are linearly independent and then $X_{\lambda 1}, X_{\lambda 2}, \ldots\ldots, X_{\lambda m}$ are called fuzzy basic variables and the remaining $(m-n)$ variables are called non-basic variables.

Now, we have already discussed that fuzzy numbers can be written in parametric form as an ordered pair of functions. Here for a triangular fuzzy number $B = (b_1, b_2, b_3)$ we have

$$\frac{b_1^{\alpha} - b_1}{b_2 - b_1} = \alpha, \frac{b_2 - b_3^{\alpha}}{b_3 - b_2} = \alpha$$

Writing in parametric form

$$B = (\alpha(b_2 - b_1) + b_1, b_3 - \alpha(b_3 - b_2)), 0 \le \alpha \le 1.$$

Similarly, we can write the trapezoidal fuzzy number $B = (b_1, b_2, b_3)$ in parametric form as

$$B = (\alpha(b_2 - b_1) + b_1, b_4 - \alpha(b_4 - b_3)), 0 \le \alpha \le 1.$$

For different values of α the FLPP can be divided into several auxiliary FLPPs and each of them becomes crisp linear programming problem; i.e. at each value of α we obtain a crisp linear programming problem. The problem (3) can be written in parametric form as

$$Maximize\ Z = c_1(x_1{}^l, x_1{}^r) + \ldots\ldots + c_n(x_n{}^l, x_n{}^r)$$
$$subject\ to$$
$$a_{h1}(x_1{}^l, x_1{}^r) + \ldots\ldots + c_{hn}(x_n{}^l, x_n{}^r) \le (b_h{}^l, b_h{}^r)$$
$$x_j{}^l, x_j{}^r \ge 0. For\ all\ h = 1,2,\ldots\ldots m\ and\ j = 1,2,\ldots\ldots\ldots, n$$

3.4 Formulation of the Problem

The information regarding cropping pattern was collected through an interview in the District Thanjavur of state Tamil Nadu and it was found that the major crops grown are paddy, groundnut and black gram. In this investigation the considered household has around 6 acres of land that is used for growing paddy, groundnut and black gram. The household: (i) expect to get a maximum gross income, (ii) interested in cropping combination that helps them to maximize their net profit. In this study, a plan of the crop plantation was considered and is given in Table 1.

Table 1: Plan of the crop plantation

Resources/ Activities	Black gram (acre)	Groundnut (acre)	Paddy (acre)	RHS (constraints)
Crop land*	2	2	2	$\le$ around 6
No. of Laborers	130	85	340	$\le$ around 540
Capital available*	70000	50000	180000	$\le$ around 280000
Gross Income*	57200	59000	40000	

*Crop land is in acres and, Capital available and Gross income is in Rs.

Solution Procedure

Let x_1, x_2 *and* x_3 are decision variables.

x_1 = area required for black gram crop.

x_2 = area required for groundnut crop.

x_3 = area required for Paddy crop.

The LP model for first year is given by:

FLP with a triangular membership function can be written as

$$Max\ Z = 57200x_1 + 59000x_2 + 40000x_3$$
$$subject\ to$$
$$2x_1 + 2x_2 + 2x_3 \le (5,6,7)$$
$$130x_1 + 85x_2 + 340x_3 \le (530,540,550)$$
$$70000x_1 + 50000x_2 + 180000x_3 \le (270000,280000,290000)$$
$$x_1, x_2\ and\ x_3 \ge 0$$

Table 2: Result of FLPP

x_1	0	0	0
x_2	2.5	3	3.5
x_3	0	0	0
Maximize Z	147500	177000	206500

Converting this FLPP into a crisp multi-objective linear programming problem

$$Maximize\ Z_1 = 57200x_1{}^{l} + 59000x_2{}^{l} + 40000x_3{}^{l}$$

$$Maximize\ Z_2 = 57200x_1{}^{r} + 59000x_2{}^{r} + 40000x_3{}^{r}$$

$$subject\ to$$

$$0 \le \alpha \le 1$$

$$2x_1{}^{l} + 2x_2{}^{l} + 2x_3{}^{l} \le \alpha + 5$$

$$2x_1{}^{r} + 2x_2{}^{r} + 2x_3{}^{r} \le 7 - \alpha$$

$$130x_1{}^{l} + 85x_2{}^{l} + 340x_3{}^{l} \le 10\alpha + 530$$

$$130x_1{}^{r} + 85x_2{}^{r} + 340x_3{}^{r} \le 550 - 10\alpha$$

$$70000x_1{}^{l} + 50000x_2{}^{l} + 180000x_3{}^{l} \le 1000\alpha + 270000$$

$$70000x_1{}^{r} + 50000x_2{}^{r} + 180000x_3{}^{r} \le 290000 - 10000\alpha$$

Table 3: Results of FLPP for different values of λ

TFNs	λ	0	0.5	1	Median of TFN s	*
(5, 6, 7) (530, 540, 550) (270000, 280000, 290000)	x_1^l	0	0	0	6 540 280000	x_1= 0 x_2= 3 x_3= 0 Max Z= 177000
	x_2^l	2.5	2.75	3		
	x_3^l	0	0	0		
	x_1^r	0	0	0		
	x_2^r	3.5		3		
	x_3^r	0	0	0		
	Max Z_1	147500	162250	177000		
	Max Z_2	206500	191750	177000		

*represents the solution of the original problem when median of TFNs are considered

The conversion of FLP into a multi-objective linear programming problem with a triangle membership function is demonstrated above. LINGO software is used to obtain the results for the various values of α, as shown in Table 2. The best value in each problem is equal to the median of TFNs when the decision maker sets α = 1. If we take the problem, the best answer is one of these (147500, 162250, 177000) and the farmer achieves greatest yield by growing groundnut crops on 2 acres of land.

4. Result and Discussion

The ideal solution of the problems can be simply determined from Table 2. If we study the outcomes of problem it is clear that optimal solution (147500, 162250, 177000) of the original problem is one of them. If we take the median of TFN's, the farmer earns the most profit by employing median of the TFN's. It has been proven that the farmer gets the same profit when α=1 and when median of the TFN's is included. At the conclusion of every year, our mission is to provide the greatest technique of usage of land so that the farmer can reach his goal. In the Problem, the maximum number of laborers is eaten by paddy and from the outcome Table 2; the farmer gets his maximum profit by planting groundnut crops. In a similar vein, growing groundnut crops can yield the highest profit for the second year. The number of laborers is currently declining for the third year, but farmers are making more money by using the best planting method for paddy and groundnuts. Paddy crops can be uprooted and planted after ten or twelve years, as was previously described. However, the remaining crops can be grown every year. The results table indicates that the paddy crops can yield increasing profits for the farmer. Thus, it indicates that farmer profits are trending upward. The ideal decision to the farmer is to utilize the additional area for paddy crops rather than wasting land for other crops. Additionally, the result table makes it clear which crop combination yields the highest return. It is evident from the aforementioned study that growing paddy crops in agricultural lands can also boost the state's economy.

5. Conclusion

The explanation above leads to the conclusion that when uncertainty occurs in real-life scenarios, how can the farmer maximize profit while utilizing limited resources? Thus, we deduce that the best course of action for the farmer is to use more land for paddy crops in order to maximize his profit.

6. Future Scope

This study can be further improved by incorporating real-time data such as weather, soil conditions, and market prices to enhance decision-making. The model can also be extended using advanced techniques like machine learning to improve prediction accuracy. Additionally, applying the model to different crops and regions, along with developing user-friendly tools, can make it more practical and beneficial for framers.

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