Graphical Representation of Inverse Matrix using Mathematica Software

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Abstract: ***Matrix inversion is a method that plays a very vital role in linear algebra with vast applications such as differential equations, computer graphics and engineering, cryptography, statistics, and probability. The uniqueness of the matrix inversion helps in providing consistency in inversion-based calculations. However, numerical methods usually lack conceptual clarity. This paper explores a graphical method using mathematica software that helps to illustrate matrices and their uniqueness of matrix inverse through different color-based visualizations to optimize conceptual clarity and Interpreting data. The method is efficient for teaching, learning and Investigative in field of research in linear algebra.***

Keywords: Matrix Inverse, Linear Algebra, Mathematica, Visualization, Matrix Plot, inverse of matrix, color matrix

1. Introduction

Matrix inverse theory, including the classical inverse and its many generalizations such as the Moore-Penrose pseudoinverse, has been a central theme in linear algebra and applied mathematics for over a century, with foundational work by Moore (1920) and Penrose (1955) establishing the basis for generalized inverses used in singular and non-square systems. Research continues to expand both theoretical understanding and practical utility: for instance, convolution-based methods for Moore-Penrose inverses optimize analytical computation of singular matrices in applied mathematics and computing contexts, avoiding high numerical cost in large problems ; neural network models have been proposed to compute time-varying complex matrix inverses in real time ; and iterative and secant methods have been developed to approximate generalized inverses efficiently . Applications span solving over- and under-determined linear systems via generalized inverses and singular value decompositions, improving computational efficiency of inverse algorithms, and extending to cryptology through specialized inverse constructions such as those based on Fibonacci matrices. Beyond pure mathematics, the Moore-Penrose inverse plays a significant role in statistical methods, such as least squares estimation and multivariate testing frameworks, and continues to be leveraged in physics research for solving differential and structural problems. The literature also includes extensive surveys of generalized matrix inverses, illustrating their theoretical properties and broad applicability in least squares, signal processing, and optimization problems. Collectively, these studies attest to the enduring importance of matrix inverse concepts in both abstract algebraic theory and diverse applied domains, motivating further research into more robust, efficient, and domain-specific inversion techniques.

Despite its significance, matrix inversion is frequently thought of as abstract and challenging to comprehend. Mathematica's visualization tools make it possible to graphically display matrices, which facilitates the interpretation of abstract numerical data. In order to improve comprehension and learning, this work investigates the graphical depiction of matrices and their inverses.

2. Mathematical Background

A. Inverse of a Matrix
For a square matrix A, the inverse A^{-1} exists if and only if:
$det(A)\neq 0$
The inverse satisfies the condition: $AA^{-1} = I$
where I is the identity matrix.

Software Use
The implementation uses Mathematica due to its powerful symbolic computation and visualization features.

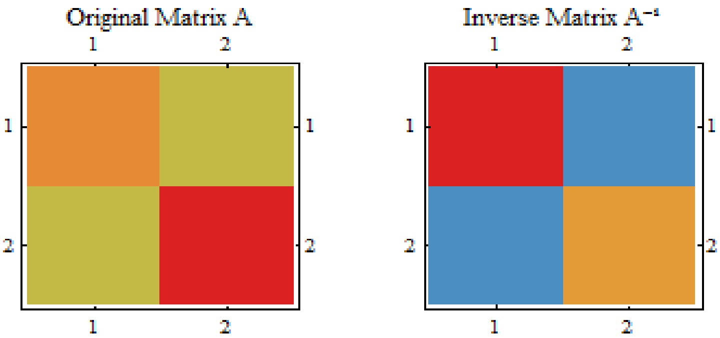
Problems
[i] Matrix inverse 2 by 2

```
A = {{2, 1}, {1, 3}};
Ainv = Inverse[A]
GraphicsRow[{MatrixPlot[A, PlotLabel → "Original Matrix A", ColorFunction → "Rainbow"],

 MatrixPlot[Ainv, PlotLabel → "Inverse Matrix A^-1", ColorFunction → "Rainbow"]}]
```

Out[2]= $\left\{\left\{\frac{3}{5}, -\frac{1}{5}\right\}, \left\{-\frac{1}{5}, \frac{2}{5}\right\}\right\}$

Out[3]=

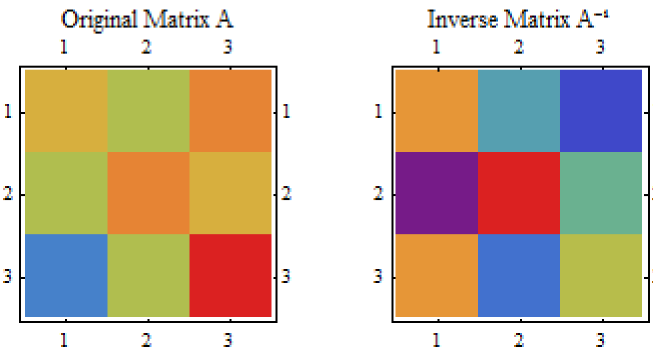


[ii] Matrix inverse 3 by 3

In[7]:=
```
A = {{2, 1, 3}, {1, 3, 2}, {-4, 1, 5}};
Ainv = Inverse[A]
GraphicsRow[{MatrixPlot[A, PlotLabel → "Original Matrix A", ColorFunction → "Rainbow"],
```

Out[8]= $\left\{\left\{\frac{1}{4}, -\frac{1}{26}, -\frac{7}{52}\right\}, \left\{-\frac{1}{4}, \frac{11}{26}, -\frac{1}{52}\right\}, \left\{\frac{1}{4}, -\frac{3}{26}, \frac{5}{52}\right\}\right\}$

Out[9]=

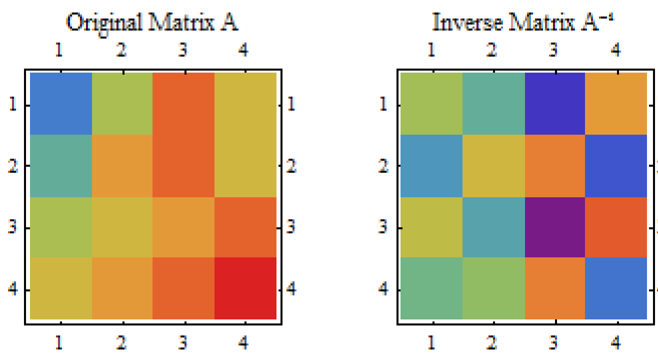


[iii] Matrix inverse 4 by 4

In[4]:=
```
A = {{-3, 1, 4, 2}, {-1, 3, 4, 2}, {1, 2, 3, 4}, {2, 3, 4, 5}};
Ainv = Inverse[A]
```

Out[5]= $\left\{\left\{\frac{1}{4}, -\frac{5}{12}, -2, \frac{5}{3}\right\}, \left\{-\frac{3}{4}, \frac{11}{12}, 2, -\frac{5}{3}\right\}, \left\{\frac{3}{4}, -\frac{7}{12}, -3, \frac{7}{3}\right\}, \left\{-\frac{1}{4}, \frac{1}{12}, 2, -\frac{4}{3}\right\}\right\}$

Out[6]=

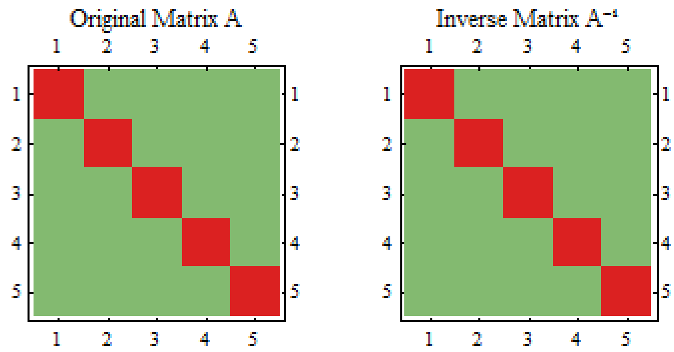


[iv] Matrix inverse 5 by 5

In[10]:=
```
A = {{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}};
Ainv = Inverse[A]
```

Out[11]=
```
{{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}
```

Out[12]=



3. Methodology

The proposed approach follows these steps:

1) Define a square matrix
2) Compute its inverse using built-in functions
3) Visualize the matrix and its inverse using graphical plots
4) Analyze visual patterns and relationships

4. Implementation

a) Matrix Definition

A = {{2, 1}, {1, 3}};

b) Inverse Calculation

Ainv = Inverse[A];

c) Graphical Visualization

```
GraphicsRow[
{
    MatrixPlot[A, PlotLabel -> "Original Matrix A", ColorFunction -> "Rainbow"],
    MatrixPlot[Ainv, PlotLabel -> "Inverse Matrix A⁻¹", ColorFunction -> "Rainbow"]
}
]
```

5. Results and Discussion

The graphical plots represent matrix elements through varying color intensities. The visualization reveals:

- Clear differences between original and inverse matrices
- Distribution of positive and negative values
- Structural relationships between matrix elements

These insights are difficult to observe through numerical values alone, highlighting the benefit of graphical representation.

6. Advantages of the Proposed Approach

- Enhances conceptual understanding
- Reduces abstraction in learning linear algebra
- Useful for teaching and presentations
- Encourages interactive exploration

7. Limitations

- Applicable only to non-singular matrices
- Visualization may become complex for large matrices
- Requires access to Mathematica software

8. Applications

The approach is suitable for:

- Mathematics and computer science education
- Engineering research
- Data visualization
- Algorithm analysis

9. Conclusion

This paper demonstrates that graphical visualization of matrix inversion using Mathematica significantly improves understanding and interpretation. The method serves as an effective complement to traditional numerical techniques and provides valuable support for education and research in linear algebra.

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