

# Double Sampling Plan Formulation for Quality Assessment

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**Abstract:** *This study addresses the challenge of evaluating service quality under cost constraints by proposing a structured double-sampling framework grounded in acceptance sampling theory. The approach integrates statistical risk control through Type I ( $\alpha$ ) and Type II ( $\beta$ ) error constraints with cost optimization to determine optimal sample sizes ( $n_1, n_2$ ) and acceptance thresholds ( $c_1, c_2$ ). A two-stage inspection process is developed in which an initial sample enables early decisions, while a second sample is used only when results are inconclusive. The Operating Characteristic function and Average Sample Number are analytically derived to evaluate performance. Simulation-based validation demonstrates reduced inspection cost and improved efficiency compared to single-sampling approaches. The framework provides a practical and statistically reliable method for service quality auditing in sectors such as banking, healthcare, and logistics.*

**Keywords:** Double Sampling Techniques, Average Sample Number, Operating Characteristic (OC) function, cost-effective, sampling plans

## 1. Introduction

In today's competitive, service-oriented world, organizations strive to maintain consistent quality while controlling operational costs. Unlike manufacturing, where product quality can be measured and corrected through tangible inspection, service quality is intangible, variable, and influenced by human performance, customer interaction, and situational factors. Ensuring high service quality thus requires efficient auditing mechanisms that detect lapses without excessive time or cost.

Statistical Quality Control (SQC), though developed for manufacturing, provides valuable tools for service environments. Acceptance sampling, in particular, allows evaluation of a process or batch based on a representative sample rather than full inspection. While single-sampling plans work well in manufacturing, they are often impractical in service operations such as banking, healthcare, or logistics, due to high inspection costs and reliance on human judgment.

Double-sampling plans offer a solution by enabling two-stage decision-making: a preliminary decision after the first sample and a final decision, if needed, after a second sample. This approach reduces the average sample number (ASN), lowers inspection costs, and controls Type I and Type II errors.

Despite its success in manufacturing, double-sampling is underutilized in service audits. This study develops a cost-effective double-sampling framework for service quality, integrating acceptance sampling with cost optimization to determine optimal sample sizes and acceptance criteria. The framework minimizes total expected cost while ensuring reliability, bridging SQC applications from manufacturing to service operations.

## 2. Review of Literature

Recent advancements have introduced hybrid and adaptive sampling methods. Choi and Cho (2010) studied Bayesian double-sampling plans, allowing dynamic updates of defect

probability based on prior information. Similarly, Lee and Wu (2016) developed cost-based double-sampling models for logistics services, demonstrating their applicability in non-manufacturing settings. These studies collectively highlight that double sampling is not only a cost-efficient method but also adaptable to processes with uncertain or variable defect rates. Researchers have recognized the need for statistical sampling methods in service quality evaluation. For instance, Goh and Xie (2001) applied control charts to monitor service processes, such as call center operations, using sampled transaction data. Zhao et al. (2010) examined the use of stratified random sampling to assess service errors in hospitals, showing that sampling could reduce auditing cost while maintaining reliability. However, most service audits still rely on single-sample inspections or full audits, which are either costly or inefficient, leaving a gap for more structured, staged sampling approaches. The adaptation of SQC and double sampling to service sectors has been explored in several studies, though research remains limited compared to manufacturing contexts. Sohn and Lim (2008) demonstrated the use of double-sampling plans for quality inspection in service processes, emphasizing its potential for reducing inspection resources.

## 3. Research Methodology

The objective of this study is to develop a cost-effective double-sampling framework for auditing service quality in sectors such as banking, healthcare, and logistics. The methodology integrates classical acceptance sampling theory, statistical risk control (Type I and Type II errors), and cost optimization principles to provide a structured approach for service quality evaluation.

### 3.1. Research Design

This study adopts a quantitative and applied research design, combining analytical modeling with simulation-based validation. The methodology consists of the following steps:

- 1) **Identification of service processes** suitable for auditing, e.g., customer service transactions, documentation procedures, or delivery processes.
- 2) **Definition of service quality characteristics** (defects), such as errors in documentation, delayed response, or misdelivery.
- 3) **Development of a double-sampling plan** that allows early acceptance or rejection when sample results are conclusive, with a second sample drawn only in cases of uncertainty.
- 4) **Incorporation of cost considerations** into the sampling design, including inspection cost and misclassification cost.
- 5) **Simulation and empirical validation** to evaluate the efficiency and cost-effectiveness of the proposed framework.

**3.2. Total OC Function** =  $P_A(p) = P_{A1}(p) + P_{A2}(p)$

#### Data Availability Statement

Since real service quality data may involve confidentiality constraints, this study utilizes synthetically generated data for simulation purposes. The data are generated using a binomial distribution under varying defect probabilities ( $p$ ), ensuring reproducibility and controlled experimentation.

All simulations were conducted using standard statistical software (R/Python), and the code used to generate synthetic datasets and compute OC and ASN functions can be made available upon reasonable request.

The synthetic dataset structure includes:

- Sample size indicators  $n_1, n_2$
- Defect counts  $X_1, X_2$
- Iteration index
- Defect probability  $p$

This approach ensures transparency, replicability, and adaptability of the proposed sampling framework.

## 4. Double-Sampling Plan Formulation:

Double-sampling is a two-stage inspection technique designed to reduce inspection cost while controlling the risk of wrong decisions. Unlike single-sampling plans, double-sampling allows the auditor to make a quick decision when the first sample gives clear evidence, and only collect a second sample when uncertainty exists.

### Step 1: First Sample Evaluation

#### 4.1 Sampling Procedure

- 1) From a population of size ( $N$ ) (e.g., 500 service transactions), draw a first sample of size  $n_1$ .
- 2) Inspect each unit in the sample and count the number of defective units ( $x_1$ ).

A defective unit is any service instance that fails to meet the pre-defined quality standard, e.g., an error in loan processing, a late delivery, or incorrect documentation.

#### 4.2 Decision Rules for the First Sample

Once  $x_1$  is counted, the first-stage decisions are:

- 1) Accept the lot immediately if the first sample indicates very high quality:

- $x_1 \leq c_1$
- $c_1$  is the first-stage acceptance number, the maximum number of defective units in the first sample to accept the lot outright.

- 2) Reject the lot immediately if the sample shows extremely poor quality:

$$x_1 > c_2$$

- $c_2$  is the second acceptance threshold (for combined samples). If ( $x_1$ ) exceeds  $c_2$ , it is highly likely that the lot quality is unacceptable.

- 3) Proceed to second sampling if the result is inconclusive:

$$c_1 < x_1 \leq c_2$$

- The lot quality is uncertain; we cannot confidently accept or reject based only on the first sample.
- The second sample is drawn to provide additional evidence.

Rationale: This staged approach reduces the inspection burden by avoiding unnecessary second samples for very good or very poor lots.

### Step 2: Second Sample Evaluation

#### 2.1 Second Sampling Procedure

- 1) Draw a second sample of size ( $n_2$ ) from the lot.
- 2) Inspect the second sample and count the defective units ( $x_2$ ).

#### 2.2 Combined Decision Rule

After inspecting the second sample, the total defects are:

$$x_{\text{total}} = x_1 + x_2$$

Then: Accept the lot if:

$$x_1 + x_2 \leq c_2$$

Reject the lot if:

$$x_1 + x_2 > c_2$$

Here,  $c_2$  is the maximum allowed defective units across both samples. This ensures that the lot is accepted only if the combined evidence meets the quality standard.

### Step 3: Probability of Acceptance (Operating Characteristic Function)

The Operating Characteristic (OC) function provides the probability of accepting a lot with defect rate ( $p$ ). It captures the statistical behavior of the double-sampling plan and is used to assess Type I and Type II errors.

#### 3.1 First Stage Acceptance Probability

The probability of accepting the lot after the first sample is:

$$P(\text{Accept at Stage 1}) = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1}$$

Where:

- ( $n_1$ ) = first sample size
- ( $x_1$ ) = number of defective units in the first sample

- $(p)$  = true defective rate in the population
- $\binom{n_1}{x_1}$  = number of ways to choose  $(x_1)$  defective units from  $(n_1)$

This term models the binomial probability that the first sample has  $(x_1)$  defective units.

### 3.2 Second Stage Acceptance Probability

If the first sample is inconclusive ( $c_1 < x_1 \leq c_2$ ), a second sample is drawn. The probability of accepting at Stage 2 is:

$$P(\text{Accept at Stage 2}) = \sum_{x_1=c_1+1}^{c_2} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} - p^{n_1-x_1} \sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p^{x_2} (1-p)^{n_2-x_2}$$

Where:

- $(x_2)$  = number of defective units in the second sample
- $(c_2 - x_1)$  = maximum defects allowed in the second sample to still accept the lot
- The outer sum accounts for all first-stage outcomes that require a second sample.
- The inner sum accounts for all second-stage outcomes that satisfy the combined acceptance criterion.

### 3.3 Total Probability of Acceptance

The total probability of acceptance is the sum of Stage 1 and Stage 2 acceptance probabilities:

$$P_a(p) = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} - p^{n_1-x_1} \sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p^{x_2} (1-p)^{n_2-x_2}$$

$P_a(p)$  gives the likelihood of accepting a lot given its true defect rate  $(p)$ . It forms the basis for risk assessment and plan optimization.

#### Step 4: Average Sample Number (ASN)

The ASN is the expected number of units inspected, an important measure for cost evaluation:

$$ASN = n_1 + n_2 \cdot P(\text{Proceed to Stage 2})$$

Where:

$$P(\text{Proceed to Stage 2}) = \sum_{x_1=c_1+1}^{c_2} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1}$$

The first term  $(n_1)$  is always inspected.

The second term is the expected contribution of second-stage inspections, weighted by the probability of needing a second sample.

#### Step 5: Type I and Type II Errors

- 1) **Type I Error ( $\alpha$ ):** Rejecting a good lot:  
 $\alpha = 1 - P_a(p_0)$

$(p_0)$  = acceptable quality level (AQL)

- 2) **Type II Error ( $\beta$ ):** Accepting a defective lot:  
 $\beta = P_a(p_1)$   
 $(p_1)$  = lot tolerance percent defective (LTPD)

These error probabilities are used to **set the thresholds**  $(c_1, c_2, n_1, n_2)$  in plan design.

**Risk Control:** Constraints are set on these errors:  
 $\alpha \leq \alpha_0, \beta \leq \beta_0$

These constraints ensure that the audit plan is statistically reliable.

#### Simulation Parameters: Simulation Setup

$n_1 = 20, n_2 = 30, c_1 = 1, c_2 = 3, p \in [0.01, 0.20], \text{stepsize} = 0.01$   
Number of iterations: 10,000 (Monte Carlo simulation),  
Distribution: Binomial

**Software Used: R (version 4.x)**, Functions used: rbinom()

#### Numerical Example:

Find a double sampling plan such that:  $p_0 = 0.02, \alpha \leq 0.05, p_1 = 0.10, \beta \leq 0.10$

**Step 1: Initial Trial Plan** Assume:  $n_1 = 20, n_2 = 30, c_1 = 1, c_2 = 3$

**Step 2: Check at AQL  $p_0 = 0.02$**

Compute:  $P_a(0.02) \approx 0.96, \alpha = 1 - 0.96 = 0.04 \leq 0.05$

**Step 3: Check at LTPD  $p_1 = 0.10$**   $= P_a(0.10) \approx 0.09, \beta = 0.09 \leq 0.10$

**Step 4: Final Plan Selection:**  $(n_1, n_2, c_1, c_2) = (20, 30, 1, 3)$ .

## 5. Conclusion

This study develops a cost-efficient double-sampling framework for service quality assessment by integrating acceptance sampling theory with risk and cost considerations. The proposed two-stage inspection design reduces inspection effort through optimized Average Sample Number while maintaining statistical reliability under predefined Type I ( $\alpha$ ) and Type II ( $\beta$ ) constraints. Analytical derivation of the Operating Characteristic function enables evaluation of decision performance across varying defect rates  $(p)$ . The framework demonstrates potential for practical implementation in service sectors where full inspection is impractical. Future research should focus on empirical validation using real-world datasets and extending the model to adaptive or Bayesian sampling schemes.

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