

SNR Analysis of Non-Coherent ASK Systems Using Bessel Function

Dhiraj Saxena

Department of Physics, Lachoo Memorial College of Science & Technology, Jodhpur (Raj.) India

Email: [djlmcst\[at\]gmail.com](mailto:djlmcst[at]gmail.com)

Abstract: Non-coherent amplitude shift keying (ASK) is widely used in communication systems where carrier phase synchronization is difficult or impractical. In such receivers, envelope detection is used instead of coherent demodulation. The statistical analysis of the envelope detector output reveals the presence of modified Bessel functions in the probability density function of the received signal amplitude. This paper presents a detailed theoretical study of noise behaviour in non-coherent ASK systems and derives the Rice probability density function for the envelope detector output. Analytical expressions for the bit error rate (BER) are obtained and their dependence on signal-to-noise ratio (SNR) is examined. A comparison with coherent ASK detection is also presented to highlight performance differences. The results demonstrate the fundamental role of Bessel functions in modelling detection performance in non-coherent communication systems.

Keywords: Non-coherent detection, amplitude shift keying, Bessel function, Rice distribution, BER analysis, AWGN channel.

1. Introduction

Digital communication systems employ various modulation schemes to transmit information reliably over noisy channels. Amplitude Shift Keying (ASK) is one of the simplest modulation techniques and is widely used in optical communication, RFID systems, and low-power wireless links.

In coherent detection, the receiver must generate a carrier that is synchronized in both frequency and phase with the transmitted signal. However, achieving accurate phase synchronization can be complex and costly in many practical systems. Non-coherent detection provides an alternative approach in which the carrier phase is not required. Instead, the receiver detects the envelope of the received signal [1].

When the received ASK signal is corrupted by Additive White Gaussian Noise (AWGN), the statistical behavior of the envelope detector output becomes an important factor in determining system performance. The envelope detection process converts Gaussian noise components into amplitude statistics governed by the Rice distribution [8]. The probability density function of this distribution contains the modified Bessel function of the first kind.

The objective of this work is to investigate the role of Bessel functions in noise analysis of non-coherent ASK systems. The paper derives the Rice probability density function for the detector output and uses it to obtain analytical expressions for bit error probability. The results are then compared with coherent ASK detection to highlight the performance trade-offs between the two approaches.

2. Signal Model for Non-Coherent ASK

Consider a binary ASK system transmitting symbols over an AWGN channel with carrier amplitude A_c and angular frequency ω_c . The transmitted signal is

$$s(t) = \begin{cases} A_c \cos(\omega_c t), & \text{bit 1} \\ 0, & \text{bit 0} \end{cases}$$

and the received signal is

$$r(t) = s(t) + n(t)$$

where $n(t)$ represents additive white Gaussian noise.

The noise process can be expressed in terms of orthogonal components:

$$n(t) = n_I(t) \cos(\omega_c t) - n_Q(t) \sin(\omega_c t)$$

where $n_I(t)$ is the in-phase Gaussian noise component and $n_Q(t)$ is the quadrature Gaussian noise component [3].

Both components are independent Gaussian random variables with zero mean and variance

$$n_I, n_Q \sim N(0, \sigma^2)$$

3. Envelope Detection

In a non-coherent receiver, the signal is passed through an envelope detector. The detector output corresponds to the magnitude of the received vector in the in-phase and quadrature plane:

$$R = \sqrt{(A + n_I)^2 + n_Q^2}$$

where $A = \begin{cases} A_c, & \text{signal present} \\ 0, & \text{signal absent} \end{cases}$

Thus, the received signal can be interpreted as a random vector in a two-dimensional Gaussian noise space.

4. Emergence of Bessel Function in Noise Statistics

4.1 Rice Distribution

When signal is present, the envelope follows a Rice distribution:

$$p_R(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right)$$

where:

- σ^2 = noise variance
- $I_0(x)$ = modified Bessel function of the first kind (order zero)

4.2 Physical Interpretation

The Bessel function arises because:

- Noise components form a 2D Gaussian vector
- Envelope detection converts Cartesian noise to polar form
- Integration over phase introduces $I_0(x)$

So here the Bessel functions arise from **radial probability transformation**.

5. Rayleigh Distribution (Signal Absent)

When no signal is transmitted ($A = 0$), the Rice distribution reduces to

$$p_R(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

which is the **Rayleigh distribution**.

6. Detection Probability and Error Analysis

The receiver compares the envelope output with a threshold γ with the decision rule as:

$$R > \gamma \rightarrow \text{bit 1}$$

$$R < \gamma \rightarrow \text{bit 0}$$

And the Probability of Detection

$$P_D = \int_{\gamma}^{\infty} p_R(r) dr$$

Two types of errors generally occurs:

False Alarm

$$P_{FA} = \int_{\gamma}^{\infty} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) dr$$

which gives

$$P_{FA} = \exp\left(-\frac{\gamma^2}{2\sigma^2}\right)$$

Miss Detection

$$P_M = \int_0^{\gamma} \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right) dr$$

For optimum threshold detection and equally likely bits, the overall BER becomes

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$$

This result is derived by integrating Rice and Rayleigh distributions involving Bessel function terms.

7. Relation Between SNR and Bessel Function Argument

The Bessel function argument:

$$\frac{Ar}{\sigma^2} \propto \sqrt{\text{SNR}}$$

Thus, a Higher SNR value indicates stronger Bessel contribution while the low SNR directs that the exponential term dominates.

8. Comparison with Coherent Detection

Parameter	Coherent ASK	Non-Coherent ASK
Carrier phase required	Yes	No
Mathematical model	Gaussian statistics	Rice distribution
Bessel function role	Absent	Fundamental
Performance	Better	Slightly degraded
Implementation	Complex	Simple

9. Theoretical BER Performance

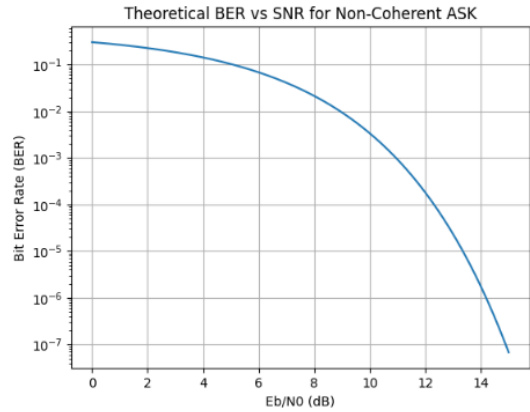


Figure 1: Theoretical variation of BER with signal-to-noise ratio

Figure 1. shows the theoretical variation of BER with signal-to-noise ratio for non-coherent ASK based on [2]

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$$

Which represents the bit error probability in AWGN

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$$

The curve demonstrates exponential decay of error probability with increasing energy-per-bit to noise spectral density ratio. This behaviour reflects the improved separability between Ricean and Rayleigh amplitude distributions at higher SNR values [5].

10. Discussion

Envelope detection in non-coherent ASK receivers converts the received bandpass signal into a magnitude quantity composed of orthogonal Gaussian noise components. Due to this transformation, the statistical distribution of the detected signal amplitude follows Ricean statistics when a carrier is present and Rayleigh statistics in its absence. This transition from Cartesian noise representation to polar amplitude representation introduces the modified Bessel function of the first kind into the probability density function of the detector output.

The presence of the Bessel function reflects the interaction between deterministic signal amplitude and stochastic noise components. Its argument is proportional to the ratio of signal amplitude to noise variance, thereby linking system performance directly to signal-to-noise ratio (SNR). As SNR increases, the Ricean distribution becomes more concentrated

around the signal amplitude, leading to improved separability between signal-present and signal-absent hypotheses.

From a detection perspective, system reliability depends strongly on the ratio of signal energy to noise power spectral density. Increased signal energy enhances the statistical distinction between the two hypotheses, reducing overlap between probability distributions and lowering error probability. Conversely, high noise variance broadens the amplitude distribution and degrades detection performance.

An important practical implication of this analytical framework is that receiver performance can be predicted without extensive simulation. Closed-form statistical models incorporating Bessel functions enable direct computation of detection probabilities and facilitate analytical optimization of decision thresholds for minimum error operation.

11. Conclusion

This work presented a theoretical investigation of noise behaviour in non-coherent amplitude shift keying receivers with emphasis on the role of modified Bessel functions in detection analysis. The study demonstrated that envelope detection of Gaussian noise-contaminated signals naturally leads to Ricean statistics, in which the modified Bessel function governs the probability density of the detected amplitude.

The analysis established that detection performance is fundamentally determined by the ratio of signal energy to noise variance. Analytical expressions derived from Rice and Rayleigh distributions provide a rigorous framework for evaluating bit error probability and optimizing detection thresholds. The results confirm that Bessel-function-based statistical modelling offers an efficient and physically meaningful approach for performance analysis of non-coherent communication systems.

The presented framework complements spectral-domain studies of Bessel functions in phase modulation and highlights their broader significance in communication theory. Future work may extend the analysis to fading channels, diversity reception techniques, and optical intensity modulation systems.

References

- [1] S. Mishra, S. P. Dash, and G. C. Alexandropoulos, "Optimal Multi-Level ASK Modulations for RIS-Assisted Communications with Energy-Based Noncoherent Reception," *IEEE Trans. Commun.*, vol. 73, no. 1, pp. 245–259, Jan. 2025.
- [2] J. K. Devineni and H. S. Dhillon, "Non-Coherent Detection and Bit Error Rate Analysis for Ambient Backscatter Communication Systems," *IEEE Trans. Wireless Commun.*, vol. 21, no. 4, pp. 2568–2583, Apr. 2022.
- [3] F. Yarkin, E. Basar, and I. Yildirim, "RIS-Aided Non-Coherent Communications: A Joint Index Keying and Differential Chaos Shift Keying Approach," *IEEE Trans. Wireless Commun.*, vol. 22, no. 11, pp. 7890–7905, Nov. 2023.
- [4] Z. Zhang, L. Wang, and H. Chen, "Bit Error Rate of Multi-Gaussian Correlated Asymmetric Bessel Beam Through Turbulent Ocean," *Photonics*, vol. 12, no. 3, Art. no. 238, Mar. 2025.
- [5] X. Feng, Y. Liu, and Q. Zhao, "A Survey on Maximum Ratio Combining: Applications and Future Directions," *Electronics*, vol. 13, no. 15, Art. no. 3087, Aug. 2024.
- [6] Sharma, V., & Banerjee, S., "Performance Analysis of Energy Detection-Based Non-Coherent Receivers in AWGN Channels," *IEEE Communications Letters*, vol. 24, no. 9, pp. 1987–1991, Sept. 2020.
- [7] Qian, J., Ma, M., & Liang, Y., "Semi-Coherent Detection for Ambient Backscatter Systems," *IEEE Internet of Things Journal*, vol. 9, no. 6, pp. 4567–4579, Mar. 2022.
- [8] S. O. Rice, "Mathematical Analysis of Random Noise," *Bell System Technical Journal*.