Mixed Picture Fuzzy Graphs for Consumer Influence Modeling in Recommendation Systems

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Abstract: ***This study develops the theoretical framework of Mixed Picture Fuzzy Graphs by integrating picture fuzzy sets with mixed graph structures to model uncertainty and directional influence. Fundamental structural properties including degree relations and order size bounds are established. An algorithmic recommendation model is formulated and demonstrated through a numerical product ranking example. The findings indicate that the proposed framework improves representation of uncertain preference interactions and supports influence aware recommendation analysis. The results suggest broader applicability of the model in decision support systems involving imprecise relational data.***

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1. Introduction

Graph theory is widely used to model relationships in complex systems; however, classical graphs are inadequate for handling uncertainty in real-world data. The concept of fuzzy sets introduced by Zadeh [21] laid the foundation for uncertainty modeling, which was extended to graph structures through fuzzy graphs by Rosenfeld [18]. Further developments include intuitionistic fuzzy sets by Atanassov [2], incorporating membership and non-membership degrees, and picture fuzzy sets by Cuong [5], which additionally include a neutral membership degree for more flexible representation. Significant research has been carried out on fuzzy and intuitionistic fuzzy graphs, including their structures, operations, and generalizations [1,13,14,15,3]. Fundamental contributions to fuzzy graphs and hypergraphs are discussed in [11,16], while complementary properties and related concepts are studied in [19]. The development of picture fuzzy sets has led to various extensions such as operators and computational models [6,7,16,17]. These frameworks have been successfully applied in multiple domains including decision-making, social networks, communication systems, and road network analysis [9,8,20,4]. Advanced aggregation and decision-making techniques based on picture fuzzy environments have also been proposed in [10,17]. Despite these advancements, most existing models consider only a single type of edge. However, real-world systems often involve both directed and undirected relationships simultaneously. To address this limitation, Mixed Picture Fuzzy Graphs (MPFGs) were introduced by Myithili Kothandapani and Nandhini Chandrasekar [12], integrating both edge types within a picture fuzzy framework. Although initial definitions have been established, their structural properties and applications remain underexplored. Therefore, this study focuses on establishing fundamental properties of Mixed Picture Fuzzy Graphs (MPFGs) and demonstrates their applicability in online shopping systems for product influence analysis and recommendation. The proposed model provides an effective framework for handling uncertainty in complex decision-making environments.

2. Basic Definitions

Definition 2.1 [21]: A **fuzzy set** A in a universe $X$is defined as $A = \{ (x, \mu_A(x)) \mid x \in X \}$where $\mu_A: X \to [0,1]$represents the degree of membership of element $x$in A.

Definition 2.2 [11]: A **fuzzy graph** is defined as a pair $G = (V, E)$, where $V$is a fuzzy set of vertices and $E$is a fuzzy relation on V, such that $\mu_E(u,v) \leq \min\{\mu_V(u), \mu_V(v)\}, \forall u, v \in V$

Definition 2.3 [2]: An **intuitionistic fuzzy set** Ain $X$is defined as $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$, where $\mu_A(x)$and $\nu_A(x)$represent membership and non-membership degrees respectively, satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$

Definition 2.4[5]: A picture fuzzy set A is defined as $A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X\}$
where, $\mu_A(x)$: positive membership , $\eta_A(x)$: neutral membership and $\nu_A(x)$: negative membership such that $0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1$.

Definition 2.4[12]: A picture fuzzy graph is a pair $G = (A, B)$, where $A$is a picture fuzzy set on vertices and $B$is a picture fuzzy relation on edges satisfying:

$$\mu_B(u,v) \leq \min\{\mu_A(u), \mu_A(v)\}$$
$$\eta_B(u,v) \leq \min\{\eta_A(u), \eta_A(v)\}$$
$$\nu_B(u,v) \geq \max\{\nu_A(u), \nu_A(v)\}$$

Definition 2.5 [6]: A **Mixed Picture Fuzzy Graph** is defined as $G = (A, B, \tilde{B})$, where:

- A is a picture fuzzy set of vertices,

- B is a set of undirected picture fuzzy edges,
- $\tilde{B}$ is a set of directed picture fuzzy edges.

Each edge is associated with a triple (μ, η, ν) representing positive, neutral, and negative membership values satisfying picture fuzzy conditions.

Example.

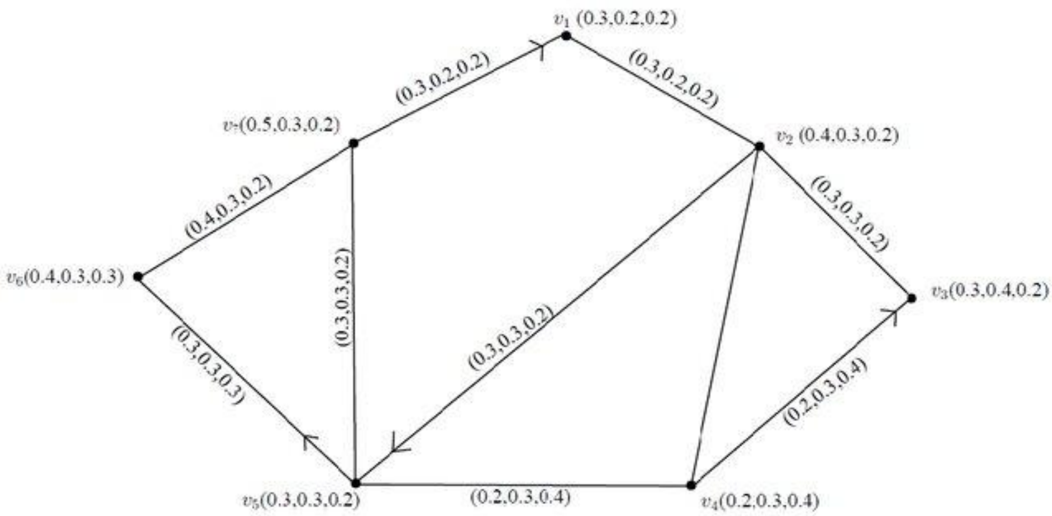

Figure 2.1: MPFG

3. Main Results

Theorem 3.1 (Handshaking Lemma)
Let $G = (V, E, \tilde{E})$be a Mixed Picture Fuzzy Graph (MPFG). Then:

$$\sum_{v \in V} \deg(v) = 2\sum_{e \in E} \mu(e) + \sum_{e \in \tilde{E}} \mu(e)$$
$$\sum_{v \in V} \deg(v) = 2\sum_{e \in E} \eta(e) + \sum_{e \in \tilde{E}} \eta(e)$$
$$\sum_{v \in V} \deg(v) = 2\sum_{e \in E} \nu(e) + \sum_{e \in \tilde{E}} \nu(e)$$

Proof: In classical graph theory, the handshaking lemma states:

$$\sum_{v \in V} \deg(v) = 2 \mid E \mid$$

In MPFG, let Vbe the set of vertices, $E$the set of undirected edges, and $\tilde{E}$the set of directed edges.
For an undirected edge $e = \{u, v\} \in E$:
It is incident on both vertices $u$and v.

Hence, its membership value contributes twice to the total degree.
Therefore, total contribution:

$$2\sum_{e \in E} \mu(e)$$

For a directed edge $e = (u, v) \in \tilde{E}$:

It contributes only once (either incoming or outgoing).

Therefore, total contribution:

$$\sum_{e \in \tilde{E}} \mu(e)$$

Combining both:

$$\sum_{v \in V} \deg(v) = 2\sum_{e \in E} \mu(e) + \sum_{e \in \tilde{E}} \mu(e)$$

Similarly, the results hold for $\eta$and ν.

Hence proved.

Theorem 3.2 (Isolated Vertex Condition)
Let $G = (V, E, \tilde{E})$be an MPFG. A vertex $v \in V$is an isolated vertex if and only if:

$$d_\mu(v) = d_\eta(v) = d_\nu(v) = 0$$

Proof: **Necessary Condition:**

Assume $v$is an isolated vertex.

Then:
- No undirected edges are incident on v
- No directed edges are entering or leaving v

Thus,

$$\mu_E(v, u) = \eta_E(v, u) = \nu_E(v, u) = 0$$
$$\mu_{\tilde{E}}(v, u) = \eta_{\tilde{E}}(v, u) = \nu_{\tilde{E}}(v, u) = 0$$
$$\mu_{\tilde{E}}(u, v) = \eta_{\tilde{E}}(u, v) = \nu_{\tilde{E}}(u, v) = 0$$

Summing over all vertices: $d_\mu(v) = d_\eta(v) = d_\nu(v) = 0$

Sufficient Condition:
Assume: $d_\mu(v) = d_\eta(v) = d_\nu(v) = 0$

Then no edge contributes to the degree of v, implying:
- No incident edges exist

Hence, $v$is isolated.

This completes the proof.

Theorem 3.3 (Order–Size Bound in MPFG)
Let $G = (A, B, \tilde{B})$be an MPFG with $n$vertices. Then:

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$$S_\mu(G) \le \frac{n-1}{2} O_\mu(G)$$
$$S_\eta(G) \le \frac{n-1}{2} O_\eta(G)$$
$$S_\nu(G) \ge \frac{n-1}{2} O_\nu(G) \text{(for complete graphs)}$$

Proof: From MPFG definition:

$$\mu_B(u,v) \le \min\{\mu_A(u), \mu_A(v)\}$$

Using inequality:

$$\min(a,b) \le \frac{a+b}{2}$$

Thus,

$$\mu_{edge}(u,v) \le \frac{\mu_A(u) + \mu_A(v)}{2}$$

Summing over all edges:

$$S_\mu = \sum \mu_{edge}(u,v)$$
$$S_\mu \le \sum \frac{\mu_A(u) + \mu_A(v)}{2}$$

Each vertex appears at most $n-1$times, hence:

$$\sum(\mu_A(u) + \mu_A(v)) \le (n-1) \sum_{v \in V} \mu_A(v)$$

Therefore, $2S_\mu \le (n-1)O_\mu$

$$S_\mu \le \frac{n-1}{2} O_\mu$$

Similarly,

$$S_\eta \le \frac{n-1}{2} O_\eta$$

From: $\nu_B(u,v) \ge \max\{\nu_A(u), \nu_A(v)\}$

Using: $\max(a,b) \ge \frac{a+b}{2}$

We get: $S_\nu \ge \frac{n-1}{2} O_\nu$ (for complete graphs)

Thus, the order–size relationship is established for MPFG.
Hence proved.

Theorem 3.4 (Degree Sum Inequality)
Let $G = (V, E, \tilde{E})$be an MPFG. Then:

$$\sum_{v \in V} d_\mu(v) \ge \sum_{v \in V} d_\eta(v)$$
$$\sum_{v \in V} d_\eta(v) \ge \sum_{v \in V} d_\nu(v)$$

Proof: From the definition of picture fuzzy sets, for every edge:

$$\mu(e) \ge \eta(e) \ge \nu(e)$$

Summing over all edges:
For undirected edges: $2\sum \mu(e) \ge 2\sum \eta(e) \ge 2\sum \nu(e)$
For directed edges: $\sum \mu(e) \ge \sum \eta(e) \ge \sum \nu(e)$
Adding both contributions: $\sum d_\mu(v) \ge \sum d_\eta(v) \ge \sum d_\nu(v)$
Hence proved.

Theorem 3.5 (Maximum Degree Bound)
Let $G = (V, E, \tilde{E})$be an MPFG with $n$vertices. Then for any vertex $v \in V$:

$$d_\mu(v) \le (n-1)$$
$$d_\eta(v) \le (n-1)$$
$$d_\nu(v) \le (n-1)$$

Proof: A vertex can be connected to at most $n-1$other vertices.
Since, $0 \le \mu(e), \eta(e), \nu(e) \le 1$

Maximum occurs when:
- Every possible edge exists
- Membership values are equal to 1

Thus: $d_\mu(v) = \sum \mu(e) \le n-1$

Similarly: $d_\eta(v) \le n-1, d_\nu(v) \le n-1$

Hence proved.

Theorem 3.6 (Complement of MPFG)
Let $G = (A, B, \tilde{B})$be an MPFG. The complement graph $\bar{G}$is defined as:

$$\bar{G} = (A, \bar{B}, \bar{\tilde{B}})$$

such that for all $u, v \in V$:

For undirected edges:

$$\mu_{\bar{B}}(u,v) = \min\{\mu_A(u), \mu_A(v)\} - \mu_B(u,v)$$
$$\eta_{\bar{B}}(u,v) = \min\{\eta_A(u), \eta_A(v)\} - \eta_B(u,v)$$
$$\nu_{\bar{B}}(u,v) = \max\{\nu_A(u), \nu_A(v)\} - \nu_B(u,v)$$

For directed edges:

$$\mu_{\bar{\tilde{B}}}(u,v) = \min\{\mu_A(u), \mu_A(v)\} - \mu_{\tilde{B}}(u,v)$$
$$\eta_{\bar{\tilde{B}}}(u,v) = \min\{\eta_A(u), \eta_A(v)\} - \eta_{\tilde{B}}(u,v)$$
$$\nu_{\bar{\tilde{B}}}(u,v) = \max\{\nu_A(u), \nu_A(v)\} - \nu_{\tilde{B}}(u,v)$$

Proof: From picture fuzzy graph properties:

$$\mu_B(u,v) \le \min\{\mu_A(u), \mu_A(v)\}$$

Thus: $\mu_{\bar{B}}(u,v) \ge 0$
Similarly: $\eta_{\bar{B}}(u,v) \ge 0$ and $\nu_{\bar{B}}(u,v) \ge 0$

Also, $\mu + \eta + \nu \le 1$ is preserved under subtraction.
Hence, the complement satisfies all conditions of MPFG.
Thus, $\bar{G}$is also a Mixed Picture Fuzzy Graph.
Hence proved.

4. Smart Recommendation Modeling Using Mixed Picture Fuzzy Graphs in Online Shopping Systems

Online shopping platforms have become a central component of modern commerce, making effective recommendation systems essential for predicting customer preferences and suggesting relevant products. However, traditional methods such as collaborative filtering and content-based approaches rely on precise data and fail to capture the uncertainty inherent in human decision-making. In practice, customer opinions often involve varying degrees of preference, hesitation, and rejection, which cannot be adequately represented using classical or basic fuzzy graph models.

To address this limitation, advanced frameworks like picture fuzzy sets allow the incorporation of positive, neutral, and negative membership values, enabling a more realistic representation of user behavior. Additionally, product relationships in e-commerce are often directional as well as mutual, requiring a model that can simultaneously handle influence and similarity. The Mixed Picture Fuzzy Graph (MPFG) model effectively integrates these aspects by combining picture fuzzy information with mixed graph structures. This approach captures multi-dimensional uncertainty and complex product interactions, leading to more accurate and reliable recommendation outcomes.

4.1 Theoretical Model

Let $G = (A, B, \tilde{B})$ be a Mixed Picture Fuzzy Graph, where:

- A: Set of products (vertices)
- B: Undirected edges (similarity between products)
- $\tilde{B}$: Directed edges (recommendation/influence)

Each vertex v is assigned the triple $v = (\mu_v, \eta_v, \nu_v)$
Each edge $e$is assigned: $e = (\mu_e, \eta_e, \nu_e)$

Here,

- μ: Customer liking or purchase tendency
- η: Uncertainty or hesitation
- ν: Dislike or rejection

Score Function
To rank products: $S(v) = \mu_v - \nu_v + \frac{\eta_v}{2}$
A higher score indicates a better recommendation priority.

4.2 Algorithm: MPFG-Based Product Recommendation

Step 1: Define the Graph
Let $G = (V, E, \tilde{E})$ be a Mixed Picture Fuzzy Graph, where:

- $V = \{v_1, v_2, \dots, v_n\}$is the set of products
- $E$is the set of undirected edges
- $\tilde{E}$is the set of directed edges

Each vertex $v_i \in V$is associated with $v_i = (\mu_i, \eta_i, \nu_i)$ such that $0 \le \mu_i + \eta_i + \nu_i \le 1$

Step 2: Define Edge Memberships
For each edge e, assign $e = (\mu_e, \eta_e, \nu_e)$
For directed edge $e = (v_i \to v_j)$

Step 3: Compute Score Function
For each vertex v_i, compute:

$$S(v_i) = \mu_i - \nu_i + \frac{\eta_i}{2}$$

Step 4: Compute Influence Strength
For each directed edge $(v_j \to v_i)$, define:

$$I(v_j, v_i) = \mu_{ji} - \nu_{ji}$$

Step 5: Aggregate Influence
For each vertex v_i, calculate total incoming influence:

$$I_{total}(v_i) = \sum_{(v_j \to v_i) \in \tilde{E}} (\mu_{ji} - \nu_{ji})$$

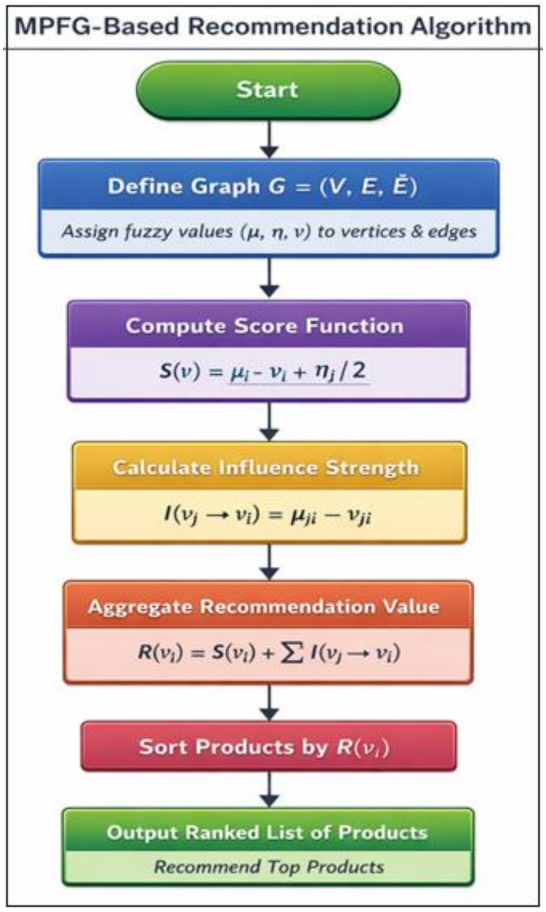
Step 6: Compute Recommendation Value
For each vertex v_i, compute:

$$R(v_i) = S(v_i) + I_{total}(v_i)$$

Step 7: Ranking
Arrange all vertices in descending order based on:

$$R(v_1) \ge R(v_2) \ge \cdots \ge R(v_n)$$

Step 8: Output
The vertex with the highest value of $R(v_i)$is the most recommended product.



1.4 Real-life example:

Consider an online shopping system with the following set of products:

$$V = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}\}$$

The screenshot of the first data entry has been attached for reference. This data is extracted from the Flipkart product interface by analyzing customer interaction details such as ratings, reviews, and recommendation indicators. The positive membership value $\mu$is determined from the overall product rating (for instance, a 4.4★ rating corresponds to a higher μvalue, such as 0.7). The neutral membership $\eta$is assigned based on moderate or uncertain feedback, reflecting user hesitation, while the negative membership $\nu$is derived from lower ratings and critical reviews, indicating dissatisfaction. Similarly, the remaining data entries are computed using the same methodology.

Step 1: Assign Picture Fuzzy Values to Vertices

Each product is assigned a triple (μ, η, ν):

Table 1: *Picture Fuzzy Values* of Products

S. No.	Product	μ (Positive)	η (Neutral)	*V* (Negative)
1	P_1	0.70	0.20	0.10
2	P_2	0.60	0.20	0.10
3	P_3	0.50	0.30	0.30
4	P_4	0.80	0.10	0.10
5	P_5	0.65	0.20	0.15
6	P_6	0.55	0.25	0.20
7	P_7	0.75	0.15	0.20
8	P_8	0.60	0.20	0.10
9	P_9	0.70	0.10	0.20
10	P_{10}	0.70	0.10	0.20
11	P_{11}	0.50	0.30	0.20

Step 2: Directed Edges (Influence Relationships)

Let the following directed edges exist:

$$P_1 \to P_2 = (0.6,0.2,0.2)$$
$$P_2 \to P_3 = (0.5,0.3,0.2)$$
$$P_3 \to P_4 = (0.6,0.2,0.2)$$
$$P_4 \to P_5 = (0.7,0.1,0.2)$$
$$P_5 \to P_6 = (0.6,0.2,0.2)$$
$$P_6 \to P_7 = (0.5,0.3,0.2)$$
$$P_7 \to P_8 = (0.7,0.1,0.2)$$
$$P_8 \to P_9 = (0.6,0.2,0.2)$$
$$P_9 \to P_{10} = (0.5,0.3,0.2)$$
$$P_{10} \to P_1 = (0.6,0.2,0.2)$$

Step 3: Compute Score Function

Using:

$$S(v_i) = \mu_i - \nu_i + \frac{\eta_i}{2}$$
$$S(P_1) = 0.7 - 0.1 + 0.1 = 0.7$$
$$S(P_2) = 0.6 - 0.1 + 0.15 = 0.65$$
$$S(P_3) = 0.5 - 0.3 + 0.1 = 0.3$$
$$S(P_4) = 0.8 - 0.1 + 0.05 = 0.75$$
$$S(P_5) = 0.65 - 0.15 + 0.1 = 0.6$$
$$S(P_6) = 0.55 - 0.2 + 0.125 = 0.475$$
$$S(P_7) = 0.75 - 0.1 + 0.075 = 0.725$$
$$S(P_8) = 0.6 - 0.2 + 0.1 = 0.5$$
$$S(P_9) = 0.7 - 0.2 + 0.05 = 0.55$$
$$S(P_{10}) = 0.5 - 0.2 + 0.15 = 0.45$$

Step 4: Compute Influence Values

Using:

$$I(u,v) = \mu - \nu$$

Each directed edge gives:

$$0.6 - 0.2 = 0.4,\ 0.5 - 0.2 = 0.3,\ 0.6 - 0.2 = 0.4,\ 0.7 - 0.2 = 0.5,$$
$$0.6 - 0.2 = 0.4,\ 0.5 - 0.2 = 0.3,\ 0.7 - 0.2 = 0.5,\ 0.6 - 0.2 = 0.4,$$
$$0.5 - 0.2 = 0.3,\ 0.6 - 0.2 = 0.4$$

Step 5: Aggregate Recommendation Value

$$R(v_i) = S(v_i) + \sum I(u, v_i)$$
$$R(P_1) = 0.7 + 0.4 = 1.1$$
$$R(P_2) = 0.65 + 0.4 = 1.05$$
$$R(P_3) = 0.3 + 0.3 = 0.6$$
$$R(P_4) = 0.75 + 0.4 = 1.15$$
$$R(P_5) = 0.6 + 0.5 = 1.1$$
$$R(P_6) = 0.475 + 0.4 = 0.875$$
$$R(P_7) = 0.725 + 0.3 = 1.025$$
$$R(P_8) = 0.5 + 0.5 = 1.0$$
$$R(P_9) = 0.55 + 0.4 = 0.95$$
$$R(P_{10}) = 0.45 + 0.3 = 0.75$$

Final Ranking

$$P_4 > P_1 = P_5 > P_2 > P_7 > P_8 > P_9 > P_6 > P_{10} > P_3$$

4.5 Result and Discussion

The proposed MPFG-based recommendation model is applied to a dataset of ten products, demonstrating its ability to handle uncertainty and influence simultaneously. The dataset reflects realistic e-commerce behavior, where picture

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fuzzy values capture positive, neutral, and negative customer preferences, and relationships are modeled based on common recommendation patterns.

From the results, P_4 achieves the highest recommendation score due to strong positive membership and minimal negative influence, further supported by incoming relationships. Products P_1 and P_5 also rank highly due to balanced preferences and moderate influence. In contrast, P_3 and P_{10} rank lower due to higher negative membership and weaker connections.

Overall, the model effectively utilizes directional influence and neutral membership to produce realistic and reliable product rankings.

5. Conclusion

This study established theoretical properties of Mixed Picture Fuzzy Graphs and demonstrated their applicability to influence aware recommendation modeling. The framework enables representation of positive, neutral, and negative preference interactions within mixed relational structures. Although the numerical illustration supports conceptual feasibility, empirical validation on real datasets remains necessary. Future research should focus on computational optimization, scalability analysis, and comparative evaluation with existing fuzzy recommendation approaches to confirm practical effectiveness.

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