

Constraints on Neutron Star Maximum Mass and Implications for the Compact Object “Mass-Gap” from Packing Radius Analysis

Ramen Kumar Parui

ARC, Room No-F101, Block-F, Mall Enclave, 13, K. B. Sarani, Kolkata- 700080, India

Corresponding Author Email: [rkparuidr\[at\]yahoo.com](mailto:rkparuidr[at]yahoo.com)

ORCID ID: 0000 – 0001 – 6838 – 3341

Abstract: “The compact object mass gap between neutron stars and stellar black holes remains a major unresolved problem in relativistic astrophysics. Motivated by the GW190814 event, this study examines neutron star compactness using a packing-radius framework combined with hoop conjecture considerations. Analytical estimates suggest an upper limit of approximately $2.65 M_{\odot}$ for observable neutron stars and propose a transitional compact configuration termed as “black neutron star” in the range $2.9 M_{\odot}$ to $3.2 M_{\odot}$. The results imply that certain compact objects within the mass gap may represent non-singular ultra-compact states rather than classical black holes. These findings provide testable implications for gravitational wave observations and constraints on the neutron star equation of state.”
The author encourages the GW community to observed the “Black Neutron Star” during their observations

Keywords: Neutron star structure, compact object mass-gap, gravitational waves, equation of state, hoop conjecture, ultra-compact stars.

1. Introduction

The “mass-gap” [2, 3] i.e. a gap that lies between maximum mass of Neutron Star and lowest mass of Black-hole, creates a puzzle to the astronomers for long decades. When more massive stars die through collapse under their own gravity they leave behind Black-holes. When the less massive stars end their lives through the explosion of supernovae they leave behind compact remnant, called Neutron Star. Our present understanding of compact objects indicates that the heaviest known Neutron Star mass is not more than $2.5M_{\odot}$ and the lightest known Black-hole mass is about $5M_{\odot}$ [4]. Analysis of some of the stellar evolution models predicted that the Black-holes with masses in two ranges cannot be formed through the gravitational collapse of a star. These two mass-gaps are roughly represented as $(2- 5) M_{\odot}$ (known as lower mass-gap) and $(50 - 150) M_{\odot}$ (called the upper mass-gap) [4,5]. These mass-gaps raised the question: does anything lie in this so called mass-gap? The answer of this question remains unsolved. [6,7].

The “mass gap” creates a puzzle regarding the identification of the heaviest known neutron star mass and the lightest black holes. Detection of GW190814 [8] through gravitational waves by LIGO-VIRGO detector offers a challenge to the scientists in the form of puzzle indicating whether the companion compact object with mass $(2.5 - 2.67) M_{\odot}$ be a (i) heaviest neutron star [8,9], or (ii) black hole [10], or (iii) primordial black hole [11,12], or strange quark star [13], or (iv) Triaxial star [14], or quark star [15], or rotating neutron star [16], or non-spinning neutron star [17], or isolated neutron star [18], or neutron star composed of hadron matter [19], or any still unknown, undetected compact object [20]. Analyzing the consistency of electro-magnetic (EM) counterpart in the

case of joint gravitational wave (GW) detection and electro-magnetic (EM) counterpart Parui [14] suggested that the companion of GW190814 binary system with mass $(1.5 - 2.67) M_{\odot}$ was a triaxial star.

After the detection of GW190814 the main problem was to identify the low mass counterpart i.e. what is the nature or type of the compact object in this binary system. If this compact object is neutron star then it will be the highest observed maximum mass of neutron star observed so far. While if this is a black hole then it will be the lowest mass of the black hole. Therefore, existence of “mass-gap” faces a crisis depending on the nature of the low mass component of the GW190814 whether it is a neutron star or a black hole.

Detection of black widow pulsar PSR J0952 – 0607 with mass $2.35 \pm 0.17 M_{\odot}$ provides the evidence of second highest massive neutron star [21] Recent observation of another millisecond pulsar PSR J0514 – 4002E [22] offers the companion mass ranging from $2.09 M_{\odot}$ to $2.71 M_{\odot}$ that falls in the mass-gap range. But regarding the nature or type of this compact object is still remain unclear. Analysis of the associated stochastic gravitational wave background suggests that the observed pulsar consists with the primordial black hole i.e. “no confirmed identification of lowest black hole mass”.

Recent theoretical study [1] indicates the idea of absence of Black-hole singularity formation in nature and ensures release of energy (broken of asymptotic freedom of quark due gravitational compactness) which prevents the formation of Black-holes singularity. Based on this idea it is proposed the formation of a new kind of Neutron Star, called as Black Neutron Star, from the star with mass $3.20M_{\odot} < M < 3.22M_{\odot}$ in the lower range “mass-gap”. The importance of our finding

Volume 15 Issue 3, March 2026

Fully Refereed | Open Access | Double Blind Peer Reviewed Journal

www.ijsr.net

is that it will help the astronomers to identify the future discovery of compact objects through binary mergers in the gravitational wave signal to place them in the “mass-gap” region.

2. The Mass-Gap

Neutron Star Internal Structure

Neutron stars are in general very complex compact objects. The standard picture of a neutron star consists of three main regions --- an outer crust, an inner crust and a core [23-25]. The outer most region, the atmosphere, consists of elements ranging between hydrogen and iron depending on the neutron star age, its history. Beneath a thin stellar atmosphere, the low density region is called outer crust. In the outer crust neutrons are confined within the nuclear cluster forming coulomb lattice and electrons form a degenerate Fermi sea i.e. a sea of free electron gas [26,27]. The density of the outer crust extends from the surface density about 10^4 g/cm^3 to the interior where the nuclei become more neutron rich, density is $\sim 4 \times 10^{11} \text{ g/cm}^3$. As one proceeds deeper into the neutron star the next region is the inner crust where some of the energetic neutrons leave the nuclei and form a “neutron drip” the density of which is $\sim (2.7 - 1.5) \times 10^{14} \text{ g/cm}^3$ or nuclear saturation density [28]. At lower this density the matter becomes inhomogeneous in this region.

The neutron star core begin towards deeper interior approximately at densities greater than $(2.7 - 1.5) \times 10^{14} \text{ g/cm}^3$ where the transition into homogeneous nuclear matter is happening [29]. The nuclear matter in the core is composed of neutrons, protons, electrons and muons that maintain the system in β -equilibrium [30,31]. For an average neutron star mass $M \sim 1.4 M_{\odot}$ the nearly pure neutron matter with admixture of protons, electrons is all there in a neutron star core. But for massive neutron star the existence of an exotic inner core is assumed i.e. at high densities the extremely neutron rich uniform matter in the outer core and possibly exotic states of matter such as strange baryons, de-confined quarks may appear in the inner core [32-35]. In our calculation we consider neutron star with a core of nuclear matter without hyperons or exotic particles. The crust contributes only a small fraction of the star mass and radius. But most of the mass and size of a neutron star are accounted for by its core i.e. the liquid core contains charge neutral matter consisting of neutrons, protons and leptons (e^- , μ^-) in β -equilibrium.

3. The Maximum Mass of Neutron Star

Applying the exact solution of Einstein’s equation for a spherical star full of incompressible matter Karl Schwarzschild [36], first indicated the idea of existence of maximum mass (M_{max}) above which the star can’t remain in hydrostatic equilibrium. The existence of a limited mass for degenerate star was first introduced in White Dwarf case. In 1930 Chandrasekhar [37] calculated theoretically that during the sequence of evolution of White Dwarf the gravitational pressure is balanced by the degeneracy pressure of the

electrons and the limiting mass is $1.44 M_{\odot}$ which is called Chandrasekhar limit. He treated electrons as an ideal Fermi gas and found this limiting mass when the electrons become ultra-relativistic. In the case of ideal gas of fermions a sequence of neutron stars in which gravity is balanced by the degeneracy pressure of neutron gas and for arbitrarily massive stars there will be some upper limit to the possible size of such a Neutron Star. Assuming the average density inside the Neutron Star which is comparable to that inside heavy atomic nuclei (i. e. $\rho \approx 10^{14} \text{ g.cm}^{-3}$) and using Schwarzschild’s solution Zwicky showed Neutron Star’s maximum mass $\cong 11 M_{\odot}$ [38,39]. But for the degeneracy pressure of an ideal neutron gas which balances the gravity the limiting mass will be $\sim 5.76 M_{\odot}$ [40]. Here, Zwicky pointed out that star’s gravitational mass should be distinguished from the baryon or rest mass (i. e. the sum of baryon masses in the star) because of the astrophysical importance of this difference that represents the amount of the energy released during core collapse of massive stars. Considering a star full of ideal Fermi gas of neutrons Oppenheimer and Volkoff [41] estimated the maximum mass of Neutron Star which is $M_{\text{max}} \cong 0.7 M_{\odot}$. In this calculation presence of protons was included but nuclear forces were ignored. Cameron suggested that nuclear forces should be considered while calculating the maximum mass of Neutron Star. Considering the nuclear forces among the constituents i.e. stiff Equation of State (EOS) Cameron [42] found the maximum mass of Neutron Star $\cong 2.0 M_{\odot}$. On the basis of observation of pulsars with the help of advanced technology and various theoretical model calculations with the microscopic level neutron star matter predict that the maximum mass of spherical, non-rotating neutron star is $M_{\text{max}} \leq 2.5 M_{\odot}$ [43]. The heaviest Neutron Star, observed recently by Antoniadis et al. [44], has a mass of $2.01 \pm 0.04 M_{\odot}$. Using the relativistic Shapiro delay method the measurement of the component mass of the millisecond pulsar J0740+6620 claims that the host Neutron Star may have a mass $2.14^{+0.10}_{-0.09} M_{\odot}$ with the systematic error (68.3% credibility) [45]. Recent analysis of mass distribution models (bimodal distribution) of neutron star suggests the cut-off in Neutron Star’s mass i.e. $M_{\text{max}} = 2.26^{+0.12}_{-0.05} M_{\odot}$ (with 68% credible interval) [46].

4. Hoop Conjecture and Black-hole Formation

The Hoop conjecture, suggested by Thorne [47], asserts that a self-gravitating matter configuration of mass “M” will form an engulfing horizon if its circumference radius “R” = $C/2\pi$ is equal to (or less than) the corresponding Schwarzschild radius $2M$ i.e.

$$2M / R_{\text{H}} = 4\pi M / C \geq 1 \quad \text{for black-hole horizon exists;}$$

(1);

and $2M / R_{\text{H}} = 4\pi M / C < 1$ for horizon-less compact object characterization [48, 49].....(2) where C is the circumference of the smallest ring that can engulf the self-gravitating compact object in all azimuthal directions. The mass “M” is interpreted as the mass contained within the engulfing sphere (not as the asymptotically measured total ADM mass) [50]. For compact objects this conjecture states that a compact

object collapses to form a black-hole when it fits inside a certain critical 2-sphere, a surface of revolution constructed from rotating a circular hoop of a certain critical circumference $C = 2\pi R_H$, where R_H is the radius of the would-be-horizon [51]. For a non-accreting, non-rotating neutron star this R_H is equivalent to Schwarzschild radius.

In this study we consider neutron star as a spherical, static body where radius collapses under its own gravitation. It consists of full of neutrons with a fraction admixture of protons and electrons and the whole system maintains the β -equilibrium. As the crust contributes a small fraction of the star, we can consider the neutron star's core mass and radius as the star mass and radius. Due gravitational forces the radius of the core shrinks with its increasing compactness and mass. We consider the best dense packing (i. e. Kepler's Conjecture) which is approximately 74% of the volume with average density $\pi/3\sqrt{2} \approx 0.740489$ [52]. The unpacked space inside the core is approximately 26 % the volume of the core. Formation of event horizon will be or not has been checked through the validity of hoop conjecture of the ratios $R_H/R(s)$ and $R_H/R(p)$. Event horizon will not appear till the $R(p) > R_H$ i.e. $R_H/R(p)$ is less than 1. While $R_H/R(p)$ is greater than 1 (i.e. $R_H/R(p) > 1$) implies that event horizon appears. The ratio $R_H/R(n)$ provides an indication of formation of event horizon if the star is more packed than $R(p)$ with full of neutrons.

5. Minimum Mass of a stellar Black-Hole

In the theory of general relativity a Black-Hole with any mass could exist. Theoretically, it is predicted that the stellar Black-hole with mass $< 5 M_\odot$ cannot be formed directly by the gravitational collapse of a star. On the basis of observation the detected Black-holes are classified into two categories – (a) Stellar Black-holes ($5M_\odot - 80M_\odot$) and massive Black-holes ($10^6M_\odot - 10^{10}M_\odot$) that are found at the centers of galaxies [53]. Latest detection of an unseen companion with mass of $3.3 M_\odot$ in the rapidly rotating giant binary system J 05215658 + 4359220 [54] indicates that it is either be a low mass Black-hole or an exceedingly massive Neutron Star. As the companion mass lies within the lower mass gap it creates uncertainty in the maximum mass of Neutron Star as well as the lowest mass Black-hole.

6. Black Neutron Star

During the stellar evolution the onset of hydrogen fusion into helium plays a significant role. Once the supply of hydrogen gets exhausted at the central core of the star then gravitational pressure forces the core to get contracted. Depending on the core mass the gravitational contraction is countered either by electron degeneracy or the initiation of fusion of helium in the core of the star. The upper limit of formation of White Dwarf supported through electron degeneracy, called Chandrasekhar limit, is $1.44M_\odot$ (M_\odot being the mass of the sun). If the mass of the stellar core is more than the Chandrasekhar limit then gravitational contraction causes an increase in core

temperature which is sufficient to enable helium fusion. This helium fusion continues to produce higher atoms and finally to ^{56}Fe . Once the iron is formed in the core, the fusion process is no longer sustainable. In this stage, the abundant neutrons develop neutron degeneracy pressure to counter the gravitational pressure and a hydrostatic equilibrium is established, thus forming a Neutron Star [54,55]. The upper limit of the mass of Neutron Star was first calculated by Tolman, Oppenheimer and Volkoff in 1939, known as TOV limit [41], which is $0.7 M_\odot$. This limit is very low. However, theoretical model calculations and observational estimates put this upper mass in the range from $1.5 M_\odot$ to $3 M_\odot$ [54]. The calculation of neutron degeneracy pressure provides the relationship between mass (M) of the Neutron Star core and its radius (R) as [56]:

$$R(n) = k(M)^{-1/3} \quad (3)$$

Now, if the mass of the core of collapsing star is more than the TOV limit, then what happens the neutron degeneracy pressure cannot produce sufficient counter balancing pressure against the gravitational collapse and ultimately a Black-hole is formed. A Black-hole possesses extremely strong gravitational attraction such that no particle or electromagnetic radiation can escape from it. The boundary of this region of the Black-hole from which no escape is possible is called "Event Horizon". Mathematically, this region is called Schwarzschild Radius. An object with the size of the Schwarzschild radius $R(s)$ is defined as

$$R(s) = 2GM/c^2 \quad (4)$$

where G = Gravitational Constant, c = velocity of light, ' M ' being the mass of the object. So, an object of mass M is smaller than the size of Schwarzschild radius is called Black-hole.

7. Idea of Black Neutron Star

The formation of Neutron Star as the end product of stellar evolution can be understood from the perspective of the spherical packing aspect. During core collapse under extreme gravitational pressure it is most likely that neutrons inside the core which will pack or arrange themselves completely in dense form in the given core space. Regarding dense packing if there are " n " neutrons each of radius " r " then these neutrons will just pack (i.e. fit) themselves inside the spherical space of radius $R(p)$ (called Packing Radius) which can be expressed according to Kepler Conjecture [57- 60] as:

$$R(p) = \{n/0.74\}^{1/3} \cdot r \quad (5)$$

Table 1 represents the typical values of these radii $R(s)$, $R(p)$ and $R(n)$ in Km. We have taken one solar mass i.e. $1M_\odot$ has 1.18×10^{57} neutrons, the realistic core radius of neutron is 0.55 fm and the compactness of a Neutron Star with mass $1.4 M_\odot$, radius 10.7 Km having mass-radius profiles with a maximum mass consistent with observation and common radius in the range (8 – 11) Km [56]. It is observed that the tabulated data shows three distinct core mass ranges –

i) $1.4M_{\odot}$ – $2.65M_{\odot}$, ii) $2.65 M_{\odot}$ – $3.20M_{\odot}$, and (iii) $3.20 M_{\odot}$ and above.

(i) Mass of the star core between $1.4 M_{\odot}$ - $2.65 M_{\odot}$

In this region $R(n) > R(p) > R(s)$. This means that from the pure dynamical point of view, there is still ample space for neutrons to occupy or to move around even after neutron degeneracy pressure has countered the gravity. Therefore, it is a stage of stable and visible Neutron Star formation without any ambiguity. These are normal, detectable Neutron Star.

(ii) Stellar core mass between $2.65 M_{\odot}$ to $3.20 M_{\odot}$

In this case the situation is: Neutron Star radius $R(n)$ is less than the Packing Radius $R(p)$ and Neutron Star radius requires to sustain the gravitational pressure which implies that something must happen in densely packed neutrons such as either neutrons turn into quark or convert into energy. If it happens so then we can no longer be called that this is a normal Neutron Star.

Table1: Comparison of different radii i.e. Schwarzschild Radius $R(s)$, Packing Radius $R(p)$, and Star Radius $R(n)$ of a collapsing star core

Mass M_{\odot}	$R(s)$ Km	$R(p)$ Km	$R(n)$ Km	Validity of Hoop Conjecture	
				$RH/R(p)$	$RH/R(n)$
1.40	4.168888	7.188233	10.9999	0.57455012	0.37545796
2.00	5.955555	8.095732	9.766944	0.728779	0.6040784
2.50	7.44444	8.720863	9.066828	0.8456732	0.81340468
2.55	7.564207	8.778619	9.007176	0.85691164	0.83516745
2.61	7.742188	8.846937	8.937619	0.87030117	0.867471
2.62	7.771852	8.858222	8.926234	0.8725227	0.86587468
2.63	7.801515	8.869477	8.914906	0.8747415	0.87028397
2.64	7.831179	8.880705	8.903636	0.87695743	0.87469885
2.65	7.860842	8.891904	8.903635	0.87917058	0.8791193
2.69	7.979496	8.936419	8.848125	0.88799547	0.89685663
2.70	8.00916	8.947479	8.837188	0.89019484	0.90130477
2.80	8.337778	9.056606	8.730706	0.91204147	0.94608618
2.85	8.486666	9.110196	8.679347	0.922867	0.96867881
2.89	8.605777	9.152619	8.639118	0.9314812	0.9868484
2.90	8.635555	9.163164	8.629176	0.93362949	0.99140396
2.91	8.665333	9.173684	8.619281	0.93577453	0.99596475
2.92	8.695111	9.18418	8.60943	0.93791124	1.00053
2.93	8.724888	9.194653	8.599625	0.94005726	1.005102
2.95	8.784444	9.215526	8.580146	0.94433026	1.01426009
3.00	8.93333	9.267299	8.532211	0.95497074	1.03724573
3.10	9.231111	9.369146	8.439462	0.97607613	1.08359979
3.20	9.528888	9.468825	9.350619	0.99695575	1.13045502
3.21	9.558666	9.478678	8.341939	0.99031661	1.13516771
3.22	9.588444	9.488511	8.333294	1.00110541	1.13988528
3.25	9.677777	9.517887	8.307574	1.00731384	1.15406729
3.30	9.826666	9.564379	8.265403	1.01783916	1.13780096
3.40	10.12444	9.660029	8.183562	1.03829909	1.22562763
3.50	10.42222	9.753822	8.104869	1.05855938	1.27392554
3.55	10.57111	9.800049	8.066382	1.06861705	1.29828959
3.60	10.72001	9.845844	8.029118	1.07862751	1.32268568
3.6078	10.74322	9.852951	8.023328	1.08018506	1.32650816
3.70	10.97552	9.936178	7.956122	1.09851084	1.37189945
3.80	11.27215	10.02489	7.88571	1.11821572	1.42155874
4.00	11.86542	10.19777	7.752029	1.157115	1.52218218
4.50	13.3486	10.60611	7.453573	1.25163646	1.78102502
5.00	14.83177	10.98522	7.196346	1.34271302	2.04965131
5.50	16.37778	11.33982	6.971311	1.43079798	2.32739581
6.00	17.79813	11.67354	6.772021	1.51624959	2.61369538
6.50	19.35556	11.98919	6.593726	1.59936641	2.90811156
7.00	20.84444	12.28905	6.43284	1.68035828	3.21009072
7.50	22.33333	12.57494	6.28658	1.7594518	3.51940165
8.00	23.82222	12.84839	6.152789	1.83680557	3.83565918
8.50	25.31111	13.62867	6.0297	1.91256328	4.15858168
9.00	26.6972	13.36287	5.915905	1.98684903	4.48790168
9.50	28.28889	13.60588	5.810241	2.05977098	4.82337996
10.00	29.77778	13.84051	5.711744	2.13142406	5.1647973

10.50	31.26667	14.06745	5.619603	2.20189225	5.51955204
11.00	32.75556	14.28728	5.533133	2.27125024	5.86467016

Therefore, a visible Neutron Star is possible up to around $2.65 M_{\odot}$. In general, a visible Neutron Star will be in existence between $1.4 M_{\odot}$ to $2.65 M_{\odot}$ and also of radius around $11 - 9$ Km (approx.). This also indicates the upper limit of a visible Neutron Star Mass = $2.65 M_{\odot}$. The important point is that above this mass the Neutron Star radius is less than the Packing Radius i.e. $R(n) < R(p)$ which is not possibly true for all neutrons inside the core. Due to the conversion of neutrons into quarks with releasing energy, the compactness involved among the quarks within the core will change. The situation is as if the quarks are packed within packing radius $R(p)_{\text{quarks}}$ inside the core radius $R(n)$ prior their conversion. Here, gravitational pressure is active that makes more and more compactness among the quarks. Finally, these quarks will be so dense that $R(n)_{\text{quarks}} \approx R(p)_{\text{quarks}}$. This will offer maximum mass of neutron star having the core full of quarks. In the present study this maximum mass is $\approx 2.90 M_{\odot}$. Note that Schwarzschild radius $R(s)$ is almost same as $R(n)$ and this is definable.

In this mass range the Schwarzschild radius is lower than the Packing radius $R(p)$ while it is lower than Neutron Star radius $R(n)$ up to $2.90 M_{\odot}$. At star mass $2.90 M_{\odot}$ $R(s) = R(n)$ and then above it $R(s)$ is greater than $R(n)$.

(iii) **Star's Mass greater than $2.90 M_{\odot}$ (and Formation of Black Neutron Star)**

The Schwarzschild radius $R(s)$ up to $3.10 M_{\odot}$ is less than the Packing radius $R(p)$ for the available neutrons in the core. For an object smaller than $3.10 M_{\odot}$, the collapse to Event Horizon is not possible without the neutrons that are actually pushed beyond their extreme possible packing arrangement. At $3.20 M_{\odot}$ $R(p)$ and $R(n)$ both are less than $R(s)$. This means packing radius and neutron core radius both are situated inside the Event horizon area. But Hoop Conjecture condition is valid for $R(p)$ (i.e. $R_H / R(p) < 1$) up to $< 3.20 M_{\odot}$.

This indicates a special situation over neutron star where- in one side it gives the lower mass limit of an invisible star (i.e. popularly known as Black-hole) as $3.20 M_{\odot}$ while on the other side, if the neutrons are pushed towards extreme packing arrangement, then for a star with its mass more than $3.20 M_{\odot}$ and there is still room among neutrons for becoming more compact beyond the Event Horizon although of $R(s) > R(p)$. This means that as the star is large and the gravitational pressure will squeeze the neutron star core to an extent such that it shrinks beyond the Event Horizon but still larger than packing radius (the condition $R_H/R(p) < 1$ is still valid in the mass range $3.20 M_{\odot} < M < 3.22 M_{\odot}$. The fact is depending on the star mass the neutron degeneracy pressure inside the star will sustain gravitational pressure beyond the Event horizon. Not only that

a) *The position of neutrons inside the core keeps getting more*

and more extreme rigid and fixed with reducing positional uncertainty;

b) *The neutrons will become increasingly relativistic, due Heisenberg Uncertainty Principle, to ensure certain stability of star even though its radius is smaller than Event Horizon.*

8. Ni's contribution in the case of Non-Event Horizons and Limitless Neutron Star Maximum Mass

Considering the general static metric for the interior of spherically symmetric neutron star (with usual notations)

$$ds^2 = -e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 d\phi^2 + e^{\nu} dt^2 \quad (6)$$

With the auxiliary function $u = r(1 - e^{-\lambda})/2$

Ni [61] found significant results for Neutron star are:

- 1) The behavior and properties of "normal" neutron star (NS) and "hollow neutron star" (HNS) are similar;
- 2) Like neutron star binaries, the hollow neutron star binaries (i.e. HNS - HNS) would be possible;
- 3) In the Ni's model the neutron star does not have a maximum mass limit and would avoid into a black hole;
- 4) According to the Ni's model a hollow neutron star (HNS) without singularity is a realistic configuration that may exist;
- 5) The solution for hollow neutron star (HNS) supports the star without maximum mass limit;
- 6) Gravitational attraction inside the cavity of a hollow sphere neutron star is outward oriented., not towards the center;
- 7) Dark matter in the form of Bose-Einstein condensate can exist inside the neutron star;
- 8) Ni's model can produce stable compact object having whatever large masses which is valid for sphere with full of neutrons (i.e. neutron star) as well as valid for astrophysical real objects.

This means that the compact object composed predominantly of neutrons i.e. neutron star having inside solid sphere form as well as hollow sphere form can exhibit a realistic scenario of without singularity although it has singularity or Event Horizon implying that a neutron star with its radius near or less than that of the actual radius for its Event Horizon is the realistic configuration that can exist. Due to the effect of singularity or Event Horizon, it is difficult to detect such type of neutron stars. These type neutron stars, can be termed as "**Black Neutron Star**". In other words, a "Black Neutron Star" is such a neutron star whose radius is less than that of its Schwarzschild radius while in the case of normal neutron star radius is greater than the Schwarzschild radius.

(Figure 1)

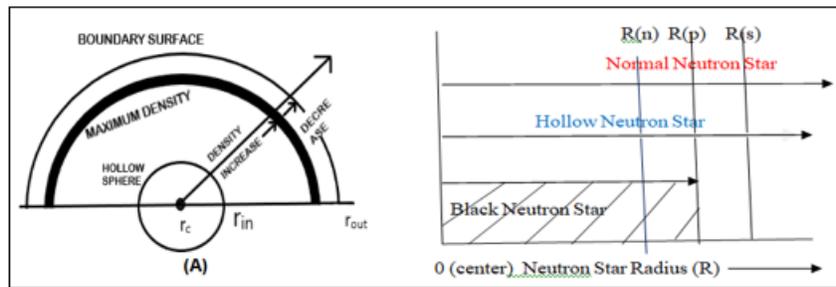


Figure 1: Schematic diagram showing (left) Hollow Neutron Star (adopted from ref. [63]), (right) Black Neutron Star (where $R(n), R(p) < R(s)$). Note – for normal Neutron star $R(n), R(p) > R(s)$

Thus, we obtain stellar object with mass between $(2.90$ and $3.20) M_{\odot}$ inside the event horizon which is perfectly definable structure (i. e. without singularity). Till date the best known values of normal neutron star maximum mass is $(M_{\text{TOV}})_{\text{max}} \approx 2.2 - 2.90 M_{\odot}$ while for black neutron star, following the Ni's model, it is $2.90 M_{\odot} \lesssim (M_{\text{TOV}})_{\text{BNS}} \lesssim 3.20 M_{\odot}$. i.e. the lowest and highest limits of maximum mass of a black neutron star are $\gtrsim 2.90 M_{\odot}$ and $< 3.20 M_{\odot}$, respectively. It is to be noted that in that case of a black Neutron Star (BNS) it can exist physically although its radius is smaller than that of its Event Horizon in size.

9. Conclusion

Oppenheimer and Volkoff (known as TOV) first estimated the maximum mass of a neutron star (full of neutrons) which is $0.7 M_{\odot}$. Gradual advancement of knowledge on the interior structure of neutron star, based on various types of phase transition, asymptotic degree of freedom, and formation of quarks, etc. both from theoretical modeling and inferring from observations, shows that (a) theoretical values of neutron star maximum mass $(M_{\text{TOV}})_{\text{max}} \approx 3.20 M_{\odot}$ and that of the inferred values from observations as (i) GW190814 (detected by LIGO/VIRGO)- $2.67 M_{\odot}$ and (ii) PSR J0514-4002E — $2.71 M_{\odot}$, if the companion is a neutron star. Till date the confirmed lowest black hole mass is $5.0 M_{\odot}$. Under this situation, “mass-gap” exists by maintaining a gap ($3.20 M_{\odot}$ or $2.67 / 2.71 M_{\odot} \rightarrow 5.0 M_{\odot}$). If the inferred value favors the lowest mass black hole i.e. black hole with its lowest mass $\sim 2.67 M_{\odot} / 2.71 M_{\odot}$ the lowest values cover the gap and “mass-gap” will vanish. So, it can be said that until and unless black hole's lowest mass is confirmed “mass-gap” continues its existence. In other words, mass gap poses a challenge regarding the nature to the scientists- “the need of first confirmation of the lowest mass black hole, and then to say good-bye to mass-gap”?

The term “Black Neutron Star” was first coined by T. Rajesh in his paper published in 2015 [64]. In normal Neutron Star degenerate pressure is very important to support gravitational pressure. In Black Neutron Star, Packing radius $R(p)$ i.e. packing arrangement of neutrons, plays a crucial role in the formation of Black Neutron Stars. The significance of this study is that

- 1) The findings can be useful to resolve the future detectable unseen compact companion object of giant binary system

through Gravitational Waves.

- 2) This study proposes compactness-based constraints on neutron star maximum mass and introduces a transitional ultra-compact configuration within the observed mass gap.
- 3) Analytical estimates suggest a possible range between approximately $2.9 M_{\odot}$ and $3.2 M_{\odot}$. While the approach provides heuristic insight, rigorous relativistic modeling and observational validation are required.
- 4) Future gravitational wave detections and precise mass measurements will be critical in assessing the viability of such configurations.

The author encourages the Gravitational Wave Community to search the “Black Neutron Star” during their observations.

Acknowledgement: The author is greatly indebted to the anonymous referee for valuable comments and suggestions that have improved the quality of the manuscript. He also wishes to thank Prof. H N K Sarma, Dept. of Physics, Manipur University, B K Ganguly, AAI, Mrs. Tapati Parui and specially to Rajarshi Parui for his help in computer works.

Conflict of Interest: The author do not have any conflict of interest.

Data Availability: Data sharing is not applicable as no data sets were analyzed.

References

- [1] T Rajesh: “Gravity causes release of Energy in Neutron Star” Academia.edu . www.academia.edu/34549343(2019)
- [2] W M Farr, N Sravan, et al.: The Mass Distribution of stellar mass blackholes .*Astrophys.J.*, **741**, 103 (2011)
- [3] California Institute of Technology, USA., *Science Daily*, (23rd June, 2020 Issue)
- [4] F Özel, D Psaltis, R Narayan, J E McClintock: The Blackhole mass distribution in the galaxy. *Astrophys. J.* **751**, 1918 (2010)
- [5] B P Abbott, T D Abbott, R Abbott, et al. Binary Black Hole Population Properties Inferred from the First and Second Observing Runs of Advanced LIGO and Advanced Virgo *Astrophys. J.* **882**, L24 (2019)
- [6] L. Kreidberg, C. D. Bailyn, W. M. Farr, V. Kalogera,

- Mass Measurements of Black holes in X-ray transients: Is there a Mass Gap?: *Astrophys. J.* **757**, 36 (2012)
- [7] Z. Roupas: Secondary component of gravitational-wave signal GW190814 as an anisotropic neutron star. *Astrophys. Space Sci.* **366**, 9R (2021), doi: 10.1007/s10509-021-03919-5
- [8] R Abbott, T D Abbott, S Abraham, et al.: GW190814: Gravitational waves from the coalescence of a 23 solar mass black hole with a 2.6 solar mass compact object. *Astrophys. J. Lett.* **896**, L44 (2020)
- [9] I Tews, D T H Pang, T Dietrich et al.: On the nature of GW190814 and its impact on the understanding of Suprenuclear Matter. *Astrophys. J. Lett.* **908**, L1 (2021)
- [10] F J Fattoyev, C J Horowitz et al: GW190814: Impact of a 2.6 solar mass neutron star on the nuclear equation of state. *Phys. Rev. C* **102**, 065805 (2020)
- [11] K Vattis, I Goldstein, et al: Could the 2.6 solar mass object in GW190814 be a Primordial Black hole? *Phys. Rev. D* **102**, 061301 (R) (2020)
- [12] S Clesse, J Garcia-Bellido: GW190814, GW190521 and GW190814: Three candidate mergers of Primordial black holes from the QCD epoch. *Phys. Dark Universe* **38**, 101111 (2022)
- [13] I Bombaci, A Drago, et al.: Was GW190814 a blackhole-Strange quark star systems. *Phys. Rev. Lett.* **126**, 162702 (2021)
- [14] R K Parui: Unravelling the Mystery of cosmic Baby: Triaxiality and Resolving the companion identification problem of GW190814. *Open Access J. Astronomy* **2**, 000129 (2024)
- [15] V Dexheimer, R O Gomes, T Klahn, S Han, M salinas : GW190814 as a massive rapidly rotating neutron star with exotic degree of freedom. *Phys. Rev. C* **103**, 025808 (2021)
- [16] A V Astashenok, S Capozziello, S D Odintsov, V K Oikonomou: Extended gravity description for the GW190814 supermassive neutron star. *Phys. Lett. B* **811**, 135910 (2020)
- [17] C Ye, M Fishback: Inferring the neutronstar maximum mass and lower Mass-gap in NS-BH systems with spin. *Astrophys. J.* **937**, 73 (2022)
- [18] A Kanakis-Pegios, P S Koliogiannis, C C Moustakidis: Probing the nuclear EOS from the existence of a 2.6 solar mass neutron star: The GW190814 puzzle. *Symmetry* **13**, 183 (2021)
- [19] K Huang, T Hu, Y Zhang, H Shen : The possibility of the secondary object in GW190814 as a Neutron Star. *Astrophys. J.* **904**, 39 (2020)
- [20] J W Moffat: Modified gravity and heavy neutron star in Mass-gap. **Arxiv: 2008.04404 [gr-qc]**
- [21] R W Romani, D Kandel, A V Filippenko et al: *Astrophys. J. Lett.* **934**, L17 (2022)
- [22] Z-C Chen, L Liu: Is PSR J0514-4002E in a PBH-NS binary? *Science China* (2024) **arxiv: 2401.12889 [astro-ph.HE]**
- [23] S. L. Shapiro, S. A. Teukolsky: Blackholes, White Dwarf and Neutronstars: The physics of Compact Objects (Wiley Interscience, NY) (1983)
- [24] P. Haensel, A. Y. Potekhin, D. G. Yakovlev: *Neutron StarI: Equation of State and Structure* (Springer, NY) (2006)
- [25] N. Chamel, P Haensel: Physics of Neutron Star crusts, *Living. Rev. Relativity*, **11**,10 (2008)
- [26] G. Baym, C. Pethick, P. Sutherland: Ground state of Matter at high densities. *Astrophys.J.* **170**, 299 (1971)
- [27] V. R. Pandharipande, D. Pinesand R. A. Smith, An Article on Non-linear Waves in Plasma. *Astrophys. J.* **208**, 550 (1976)
- [28] J. A. Rueda, R. Ruffiniand S. S. Xue, in *AIP Conf. Proceedings CP1205: The Sun, The Star, The Universe and General Relativity* eds: R. Ruffini, G. Vereshchagin (AIP, NY) p. 143 (2010)
- [29] B. K. Sharma, M. Centelles, X. Vinas, M. Balso, G. F. Burgio: Unified equation of states for neutron star on macroscopic basis. *Astron. Astrophys.* **584**, A103 (2015)
- [30] S. Helstrom, Neutron Star Structure and Equation of State, [theory.uchicago.edu /teaching>finalpaper>helstrom\(2020\)](http://theory.uchicago.edu/~teaching>finalpaper>helstrom(2020)
- [31] S. Antic, J. R. Stone, A. W. Thomas: in Proc. HIAS, 2019, *EPJ Web of Conference.* **232**, 03001 (2020)
- [32] K. Hebeler, J. M. Lattimer, C. J. Pethick, A. Schwenk: Equation of State and Neutron Star Properties Constrained by Nuclear Physics and Observation *Astrophys. J.*, **773**, 11 (2013)
- [33] F. Weber: The Many Faces of Neutron Star Interiors. *Acta Phys. Polonica B* **30**, 3149 (1999)
- [34] A. W. Steiner: in XIII Int. Workshop on Hadron Phys., J. Phys. Conf Series **706**, 022001 (2016)
- [35] J. M. Lattimer: in *Xiamen CUSTIPEN Workshop on the EOS of Dense Neutron-rich Matter in the Era of Gravitational Wave Astronomy*, (AIPConf.Proc.2127,020001 (2019)
- [36] K. Schwarzschild: *Sitzungherichteder Kniglich Preussischen Academieder Wissenschaftenzu Berlin, Phys-Math, Klasse*,**424**,1 (1916)
- [37] S. Chandrasekhar: Maximum mass of an Ideal White Dwarf. *Astrophys.J.* **74**, 81(1931)
- [38] F. Zwicky: On Collapsed Neutron Stars. *Astrophys. J.* **88**, 522 (1938)
- [39] F. Zwicky, On the Theory and Observation of Highly Collapsed Stars *Phys.Rev.*,**55**, 726(1939)
- [40] G. Srinivasan, The Maximum Mass of Neutron Stars. *Astron.Astrophys. Rev.* **11**, 67 (2002)
- [41] J. R. Oppenheimer, G.M. Volkoff, On the Massive Neutron Cores. *Phys.Rev.* **55**,374 (1939)
- [42] A. G. W. Cameron, Neutron Stars Models. *Astrophys. J.* **130**, 884 (1959)
- [43] N. Chamel, P. Haensel, et al: On the Mximum Mass of Neutron Stars. *Int.J. Mod. Phys.,E.* **22** 1330018 (2013)
- [44] J. Antoniadis, P. C. C. Freire, et al., A Massive Pulsar in a compact Relativistic Binary. *Science*, **340**,448 (2013)
- [45] H.-T. Cromartie, E. Fonseca, et al: **Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar.** *Nature Astronomy*, **4**,72 (2020)
- [46] D.-S. Shao, S.-P. Tang, et al.: Maximum mass cutoff in the neutron star mass distribution and the prospect of forming supramassive objects in the double neutron star mergers. *Phys. Rev. D* **102**, 063006 (2020)

- [47] K. S. Thorne, in *Magic without Magic: John Archibald Wheeler*, ed: J. Klauder (Freeman, San Francisco, USA) (1972)
- [48] J. P. Leon: Examples against the hoop conjecture of Black holes. *Gen. Relativity and Grav.*, **19**, 289 (1987)
- [49] W.B. Bonnor, *Phys. Lett*, **A99**, 424 (1983)
- [50] S. Hod, Lower bound on the compactness of isotropic ultra-compact objects *Euro. Phys. J. C* **80**, 982 (2020)
- [51] A. Addazi, A. Marciano, and N Yunes, *Euro. Phys.J.*, C80,36 (2020)
- [52] J. Kepler, "A New Year's Gift or on the Six cornered snowflake (Latin), *Francofurti ad Moenum apud Godtfredum Tampach* (1611)
- [53] S R Kulkarni, Lecture Notes- "How to find stellar Black-holes" (2019)
- [54] I. Bombai: The Maximum mass of a Neutron Star. *Astron.Astrophys.*, 305, 871 (1996)
- [55] B. W. Carroll, D. A. Ostlie, *Physics of Dense Matter*, Addison-Wesley (2006)
- [56] J. Timlin, "Neutron Degeneracy Pressure", Quantum Mechanics II, (2013)
- [57] N.J.A. Sloan, *Documenta Mathematica*, **3**,387 (1998)
- [58] T. C. Hales, *Kepler Conjecture*, front.math.ucdavis.edu/math.MG/9811078(1998)
- [59] T.C. Hales, The Sphere Packing Problem. *J. Computational. Applied Maths.*,**44**, 41 (1992)
- [60] W. C. Chen, J. Piekarewicz, Compactness of Neutron Stars. *Phys.Rev. Lett.*, **115**, 161101 (2015)
- [61] J. Ni: Solutions without a Maximum mass limit of the general relativistic field equations for Neutron Star. *Science China.* **54**, 1304 (2011)
- [62] R K Parui: On the Existence of a Hollow Neutron Star. *Int. Astron. Astrophys. Res. J.* **5**, 119 (2023)
- [63] R K Parui: Hollow Neutron Star – Existence of Bose-Einstein Condensate Dark Matter in its core: A possibility. *American J. Planetary. Space Sci.* **2**, 120 (2023)
- [64] T. Rajesh, Existence of Black Neutron Star. *Int. J. Astron.Astrophys.*,**5**,11 (2015)