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On Leap Randic Index of Some Graphs

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Abstract: ***For a graph $G = (V, E)$ the second degree of a vertex v in a graph G is the number of its second neighbors, that is the number of vertices in G having distance 2 to v. In this manuscript, we are introduces the Leap Reciprocal Randic Index $LRRI(G)$ of graph G and Leap Randic Index, $LRI(G)$ of a graph G. The exact values of $LRRI(G)$ and LRI(G) for some well-known graph classes are obtained.***

Keywords: Leap index, leap reciprocal randic index, and leap randic index

Mathematics Subject Classification} 05C05, 05C07, 05C35

1. Introduction

In the current chemical and mathematical literature, a large number of vertex degree based graph invariants have been studied. Among them, the first and second Zagreb indices are, by far, the most extensively investigated [11]. In this paper, we are concerned only with simple graphs, i.e., finite graphs having no loops, multiple or directed edges. Let G = (V, E) be such a graph with vertex set $V(G)$ and edge set $E(G)$. As usual, we denote the number of vertices and edges in a graph G by n and m, respectively. The distance $d_G(u, v)$ between any two vertices u and v of a graph G is equal to the length of (number of edges in) a shortest path connecting them. For a vertex $v \in V(G)$ and a positive integer k, the open k-neighborhood of v in G is denoted by $N_k(v)$ and is defined as $N_k(v)$ = u ∈ V(G): $d_G(u, v)$ = k. The k-distance degree of a vertex v in G is denoted by $d_k(v)$ and is defined as the number of k-neighbors of the vertex v in G, i.e., $d_k(v) = |N_k(v)|$. It is clear that $d_1(v) = d(v)$ for every $v \in V(G)$, [16].

A topological index of a graph is an invariant number calculated from a graph usually representing a molecule and has applications in chemistry. The Zagreb indices have been introduced by Gutman and Trinajstic, [12, 5, 17, 20] in 1972 and elaborated in [2, 3, 4, 7]. They are defined by

$$M_1(G) = \sum_{v\in V(G)} {d_1}^2(v) \text{ and}$$
$$M_2(G) = \sum_{uv\in E(G)} d_1(u)d_1(v)$$

For properties of these two Zagreb indices, see [6, 1, 8, 9, 10, 14, 15, 13, 19, 21, 22] and the papers therein.

In 2017, Naji et. al., [18], had introduced three new distance-degree-based topological indices depending on the second degrees of vertices (number of their second neighbors), and are so-called leap Zagreb index of graph G and are respectively defined by

$$LM_2(G) = \sum_{uv\in E(G)} d_2(u)d_2(v)$$

In this manuscript, motivated by leap Zagreb index and the authors introduced the leap reciprocal Randic index, and leap Randic index of a graph G which is defined as the sum of products of both the second degree of every vertex in G. The exact values of some well-known graphs are obtained.

Definition 1.1: For a graph $G = (V, E)$, the leap reciprocal Randic index of G is defined by

$$LRRI(G) = \sum_{u,v\in V(G)} \sqrt{d_2(u).d_2(v)} \ldots\ldots\ldots\ldots\ (1)$$

Where $d_2(v) = |\{u \in V(G): d(u, v) = 2\}|$.

Definition 1.2. For a graph $G = (V, E)$, the leap Randic index of G is defined by

$$LRI(G) = \sum_{u,v\in V(G)} \frac{1}{\sqrt{d_2(u).d_2(v)}} \ldots\ldots\ldots\ldots\ (2)$$

Where $d_2(v) = |\{u \in V(G): d(u, v) = 2\}|$.

2. Main Results

Theorem 2.1. For the cycle C_n with $n \geq 4$ vertices, the leap Randic index is given by

$$LRI(C_n) = \begin{cases} n + 1.\ if\ n\ is\ even, \\ \frac{2n+3}{2}, if\ n\ is\ odd. \end{cases}$$

Proof: We compute the required result by using definition 2. The cycle C_n with $n \geq 4$ vertices. Here $d_2(v_i) = 2$, Now find leap Randic index of cycle graph.

Case I: If n is even with $C_n \geq 4$, by the Definition 2, we have

$$LRI(C_n) = \sum_{n=1}^{2(n+1)} \frac{1}{\sqrt{2.2}}$$
$$= 2(n+1).\frac{1}{2}$$
$$\therefore \quad LRI(C_n) = n + 1, \text{ if } n \geq 1.$$

Case II: If n is odd with C_n with $n \geq 5$, now by the definition 2, we have

$$LRI(C_n) = \sum_{n=1}^{2n+3} \frac{1}{\sqrt{2.2}}$$

$$\therefore \quad LRI(C_n) = \frac{2n+3}{2}, \quad \text{if } n \geq 1.$$

Theorem 2.2. Let C_n be a cycle graph with $n \geq 4$ vertices, its leap reciprocal Randic index is given by

$$LRRI(C_n) = 2(n+3).$$

Proof: We compute the required result by using definition is 1. Let C_n be a cycle graph with, $n \geq 4$ vertices. Here $d_2(v_i) = 2$. Now find leap reciprocal Randic index of cycle graph, we have

$$LRRI(C_n) = \sum_{n=1}^{n+3} \sqrt{2.2}$$

$$\therefore \quad LRRI(C_n) = 2(n+3), \text{ if } n \geq 1.$$

Theorem 2.3. If S_n be a star graph with $n \geq 3$ then find leap Randic index is given by

$$LRI(S_n) = \frac{n+1}{n}.$$

Proof: We compute the required result by using definition is 2. Let S_n be a star graph with, $n \geq 3$ vertices. Here $d_2(v_0) = 0,\ d_2(v_i) = n$

Where $i = 1, 2, 3, \cdots n$, and $n = 1, 2, 3, \ldots n$. Now find leap Randic index of star graph, we have,

$$LRI(S_n) = 0 + 2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \ldots$$

$$\therefore \quad LRI(S_n) = \frac{n+1}{n}, \text{ if } n \geq 1.$$

Theorem 2.4. If K_{nn} be a complete graph with $n \geq 2$, then find leap Randic index is given by

$$LRI(K_{nn}) = \frac{2(n+1)}{n}.$$

Proof: We compute the required result by using definition is 2. Let K_{nn} be a complete graph with $n \geq 2$,vertices. Now,

$$LRI(K_{nn}) = 4 + \frac{6}{2} + \frac{8}{3} + \ldots$$

$$\therefore \quad LRI(K_{nn}) = \frac{2(n+1)}{n}. \text{ if } n \geq 1.$$

3. Conclusion

In this manuscript, we have defined the leap reciprocal Randic index and leap Randic index and we have Computed the exact values of these graphs cycle graph, star graph, and complete graph.

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