Sports Biomechanics: Analyzing the Dynamics of Shuttlecock Speed in Badminton Services Trajectory

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Abstract: ***The badminton serve plays a crucial role in determining rally dynamics and tactical advantage. Due to its feathered structure, the shuttlecock experiences significant aerodynamic drag, leading to rapid deceleration and a steep trajectory. This study presents a mathematical analysis of shuttlecock motion during a low serve by incorporating gravitational and quadratic drag forces. The governing differential equations are solved to determine velocity components, time to peak position, and effective speed at the net. The results demonstrate the influence of initial velocity and service angle on shuttlecock speed, providing practical insights for optimizing serve performance and reducing opponent reaction time.***

Keywords: Badminton biomechanics; Shuttlecock trajectory; Aerodynamic drag; Quadratic drag force; Low serve; Fluid dynamics; Projectile motion; Velocity analysis; Sports physics; Performance optimization.

1. Introduction

Badminton, a fast-paced and dynamic racquet sport, demands exceptional athleticism and mastery of intricate biomechanical principles. The serve, the initial act that sets the tone for the rally, plays a crucial role in dictating the flow of the game. A well-executed serve can put immediate pressure on the opponent, creating tactical advantages and scoring opportunities. Understanding the biomechanical factors that influence the speed and trajectory of the served shuttlecock is paramount to optimizing this crucial aspect of the game. This project ventures into the exciting realm of sports biomechanics to analyse the dynamics of shuttlecock speed across various badminton service angles. Our primary focus is to unveil the intricate relationship between different parameters of service and the resultant velocity profile of the shuttlecock.

Cao et al. [1] studied an analytical model to examine the rotation speed of shuttlecocks. This model is used to measure the rotation behaviour of shuttlecocks. The effect of relative airflow velocity and rotation speed on the moments which act on the shuttlecocks is also analysed. Alam et al. [2] analysed experimentally to determine the aerodynamic properties of a series of shuttlecocks (synthetic and feather made) under a range of wind speeds, and compare their aerodynamic properties. Le Personnic et al. [3] experimentally determine the aerodynamic properties of a series of shuttlecocks (synthetic and feather made) under a range of wind speeds, and compare their aerodynamic properties and built flight trajectories for a synthetic and feather shuttlecocks. Lin et al. [4] made a comparative analysis of the turnover and deformation of various shuttlecocks through observation by high-speed capturing. Bankosz et al. [5] studied the assessment of simple reaction time in badminton players. Chan et al. [6] developed an understanding of the aerodynamic properties of various shuttlecock types and of the responses of different shuttlecock constructions to aerodynamic loading; and to integrate empirical aerodynamic responses into a computational simulation of shuttlecock flight. Greater knowledge of shuttlecock aerodynamics and trajectory prediction has the potential to help players at all skill levels, and also may assist shuttlecock designers in ensuring more consistent flight behaviours. Rusdiana et al. [7] attempted a comparative study of velocity reduction on feather and synthetic shuttlecocks using corrected initial velocity during overhead smash.

The lack of spherical symmetry makes shuttlecocks unique sport balls. In order to better control the shuttlecock throughout the game, the player hits the cork instead of the skirt. She makes use of the fact that a shuttlecock propels the cork forward constantly. The shuttlecock must therefore be flipped following each exchange. Thus, we see the action of a shuttlecock flipping after hitting a racket. This research delves into four key biomechanical factors that significantly impact the speed of a served shuttlecock. These factors include:

- Drag Force
- Gravitational Force
- Service Angle
- Racquet Velocity

By analyzing and quantifying the influence of these factors, this project aims to provide valuable insights for badminton coaches and players alike. This knowledge can empower them to refine service techniques, maximize shuttlecock speed and trajectory, and ultimately gain a competitive edge on the court.

1.1 About the Game: Badminton

In the sport of badminton, players use racquets to strike shuttlecocks over nets. The most popular versions of the game are "doubles," which have two players per side, and "singles," which have one player per side, though it can also be played with larger teams. Formal matches are held on an indoor court that is rectangular in shape; badminton is typically played as a light-hearted outdoor activity in yards or on beaches. A point is earned when the shuttlecock is struck with the racquet and lands in the half of the court occupied by the opposing team. Before the shuttlecock crosses the net, each side may only hit it once. The shuttlecock must hit the floor or ground to end play, or if a fault has been called by the umpire, service judge, or (in their absence) the opposing side.

The shuttlecock is a projectile with feathers or, in non-official competitions, plastic that flies shuttlecock gives the sport its distinctive nature, and in certain languages, e.g. German, the sport is named by reference to this feature i.e. Feder ball, literally feather-ball.

1.2 The Trajectory of the Shuttlecock After Service

Being a cheery body, a shuttlecock produces high aerodynamic drag and a steep flight trajectory. The flight trajectory of a shuttlecock is significantly different from the balls used in most racquet sports due to very high initial speeds that decay rapidly due to high drag produced by the feathered skirt. The parabolic flight trajectory is twisted greatly and thus fall of shuttlecock has much sharper angle than rise. By understanding of trajectory of shuttlecock speed, time, direction and path can be predicted and this information can be helpful for training of players.

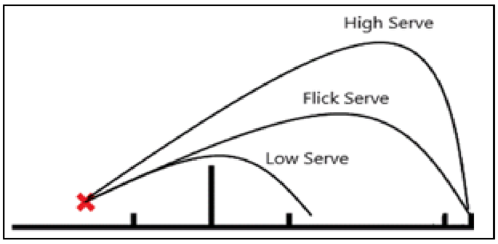

Figure 1: The Serve Trajectory

In this paper, we will basically deal with the low serving of the shuttlecock, and make all our calculation for the same.

It is clearly observable from the above diagram that, until when the shuttlecock reaches the top of the net, the shuttlecock follows a parabolic path and after that due to the effect drag force, the speed decreases significantly and hence the shuttlecock falls down immediately.

So, we can divide the path of the shuttlecock into two parts i.e., first part is the motion of shuttlecock until it reaches the net and the second part is after that until it falls down and hence can find out the velocity at any arbitrary point on both the parts.

2.3 Determination of Velocity in the First Part of Motion

Let us assume that, the mass of the shuttlecock is m, initial velocity of the shuttlecock after hitting it with the racquet be v_i, the angle at which the shuttlecock is shot is θ.

Here, the initial velocity is resolved into two rectangular components, i.e., x-components and y-components.

The initial velocities along the x-axis and y-axis are;

$$v_{(i_x)} = v_i \cos\theta , \; v_{(i_y)} = v_i \sin\theta$$

Now, there are two types of forces acting on the shuttlecock while it is travelling in the air along its trajectory path. So, before proceeding further, it is highly necessary to understand the forces. So, let us have a brief introduction about the forces acting on the shuttlecock:

- **The drag forces**

Drag force for an object moving in air is a mechanical force that acts in the opposite direction to the relative motion object moving through air. The drag force acting due to the air on the shuttlecock is;

$$F_d = bv^n$$

where, b is the coefficient of drag force, v is the velocity of the shuttlecock and n is the order of the drag force.

For the simplicity in our calculation, we have taken the order of the drag force as $n = 1$. So, the drag force acting on the shuttlecock is;

$$F_d = bv$$

- **The gravitational force**

Any pair of objects having mass attract each other. This force of attraction is nothing but the gravitational pull. In similar fashion, the Earth also attract the shuttlecock, while it is travelling in its trajectory and this gravitational force always acts in the downward direction, and the magnitude of gravitational pull experienced by the shuttlecock is;

$$F_g = mg$$

where, m is the mass of the shuttlecock and g is the acceleration due to gravity.

Now, let us consider the motion of the shuttlecock in the first part, i.e., the motion of the shuttlecock until it reaches the net. In this part we will show how the forces are acting on the shuttlecock and how does they affect its motion.

The free body diagram is as follows:

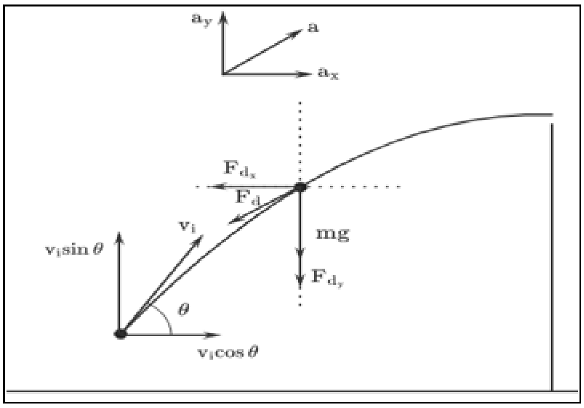

Figure 2: The motion of the shuttlecock in the first part

Consider the shuttlecock at an arbitrary position at time t in its trajectory as shown. The drag force F_d is resolved into two rectangular components i.e., F_{d_x} and F_{d_y}.

The equation of the motion for the shuttlecock along the x-direction is;

$$-bv_x = m\frac{dv_x}{dt} \tag{1}$$

Similarly, the equation of the motion for the shuttlecock along the y-direction is;

$$-bv_y - mg = m\frac{dv_y}{dt} \tag{2}$$

The initial conditions for this situation are;
At $t = 0$, the initial velocities are;

$$v_x = v_i \cos\theta \tag{3}$$

$$v_y = v_i \sin\theta \tag{4}$$

Let us now solve the above differential equations as follows:
First of all, let us solve the equation (2);

$$-bv_y - mg = m\frac{dv_y}{dt}$$

$$\frac{dv_y}{dt} + \left(\frac{b}{m}\right)v_y = -g$$

$$\frac{dv_y}{dt} = -\left\{\left(\frac{b}{m}\right)v_y + g\right\}$$

$$\frac{dv_y}{\left\{\left(\frac{b}{m}\right)v_y + g\right\}} = -dt$$

Now, integrating both sides;

$$\int \frac{dv_y}{\left\{\left(\frac{b}{m}\right)v_y + g\right\}} = -\int dt$$

Take $\frac{b}{m} = k$, and the equation will be;

$$\int \frac{dv_y}{(kv_y + g)} = -\int dt$$

Assume, $kv_y + g = p \quad \Rightarrow dv_y = \frac{dp}{k}$

$$\int \frac{\left(\frac{dp}{k}\right)}{p} = -\int dt$$

$$\frac{1}{k}\int \frac{dp}{p} = -\int dt$$

$$\frac{1}{k}\ln p = -t + C_1$$

where, C_1 is the constant of integration.

$$\frac{m}{b}\ln\left(\frac{bv_y}{m} + g\right) = -t + C_1$$

Now, considering the initial condition, at $t = 0$,
$v_y = v_i \sin\theta$;

$$C_1 = \frac{m}{b}\ln\left(\frac{bv_i \sin\theta}{m} + g\right)$$

So, the equation becomes;

$$\frac{m}{b}\ln\left(\frac{bv_y}{m} + g\right) = -t + \frac{m}{b}\ln\left(\frac{bv_i \sin\theta}{m} + g\right)$$

$$\frac{m}{b}\ln\left(\frac{bv_y}{m} + g\right) - \frac{m}{b}\ln\left(\frac{bv_i \sin\theta}{m} + g\right) = -t$$

$$\ln\left[\frac{\left(\frac{bv_y}{m} + g\right)}{\left(\frac{bv_i \sin\theta}{m} + g\right)}\right] = -\frac{bt}{m}$$

$$\frac{\left(\frac{bv_y}{m} + g\right)}{\left(\frac{bv_i \sin\theta}{m} + g\right)} = e^{-\frac{bt}{m}}$$

$$\frac{bv_y + mg}{bv_i \sin\theta + mg} = e^{-\frac{bt}{m}}$$

$$bv_y + mg = e^{-\frac{bt}{m}}(bv_i \sin\theta + mg)$$

$$bv_y = e^{-\frac{bt}{m}}(bv_i \sin\theta + mg) - mg$$

$$bv_y = bv_i \sin\theta\, e^{-\frac{bt}{m}} + mge^{-\frac{bt}{m}} - mg$$

$$v_y = v_i \sin\theta\, e^{-\frac{bt}{m}} + \left(e^{-\frac{bt}{m}} - 1\right)\frac{mg}{b}$$

So, the velocity of the shuttlecock along the y-axis at any arbitrary position at time t is given by;

$$v_y = (v_i \sin\theta)e^{-\frac{bt}{m}} + \left(e^{-\frac{bt}{m}} - 1\right)\frac{mg}{b} \tag{5}$$

Now, let us solve equation (1);

$$-bv_x = m\frac{dv_x}{dt}$$

$$\frac{dv_x}{dt} = -\frac{bv_x}{m}$$

$$\frac{dv_x}{v_x} = -\frac{b}{m}dt$$

Now, integrating both sides;

$$\int \frac{dv_x}{v_x} = \int -\frac{b}{m}dt$$

$$\int \frac{dv_x}{v_x} = -\frac{b}{m}\int dt$$

$$\ln v_x = -\frac{bt}{m} + C_2$$

where, C_2 is the constant of integration.

Now, considering the initial condition, at $t = 0$,
$v_x = v_i \cos\theta$;

$$\ln(v_i \cos\theta) = -\frac{b \times 0}{m} + C_2$$

$$C_2 = \ln(v_i \cos\theta)$$

So, the equation becomes;

$$\ln v_x = -\frac{bt}{m} + \ln(v_i \cos\theta)$$

$$\ln v_x - \ln(v_i \cos\theta) = -\frac{bt}{m}$$

$$\ln\left[\frac{v_x}{v_i \cos\theta}\right] = -\frac{bt}{m}$$

$$\frac{v_x}{v_i \cos\theta} = e^{-\frac{bt}{m}}$$

$$v_x = (v_i \cos\theta)e^{-\frac{bt}{m}}$$

So, the velocity of the shuttlecock along the x-axis at any arbitrary position at time t is given by;

$$v_x = (v_i \cos\theta)e^{-\frac{bt}{m}} \tag{6}$$

So, the speed of the shuttlecock at any arbitrary position before reaching the net at any time t is given by;

$$v_x = (v_i \cos\theta)e^{-\frac{bt}{m}}$$

$$v_y = (v_i \sin\theta)e^{-\frac{bt}{m}} + \left(e^{-\frac{bt}{m}} - 1\right)\frac{mg}{b}$$

When this speed is higher, it will be difficult for the opponent to react to the service and hence the player can have a good

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start in the game. Now, when the opponent gets the less reaction time, it will be again difficult for her to react to the service. So, let us try to calculate the time taken for the shuttlecock to reach up to the net using the above speeds.

1.4 Determination of the Time Taken to Reach the Peak

Now, at the peak position, the speed of the shuttlecock is only towards the horizontal direction and the speed along the vertical direction is zero. This is because, before reaching the peak position, the speed along horizontal direction is upwards and after the peak position, the vertical speed is in downward direction. That means, the shuttlecock changes the direction of the vertical speed at the peak position and hence becomes zero at the top position.

Let, the time taken for the shuttlecock to reach the net be t_{net}.

Now, making the vertical velocity at the peak position zero, we have;

$$[v_y]_{t_{net}} = 0$$
$$(v_i \sin\theta)e^{-\frac{bt_{net}}{m}} + \left(e^{-\frac{bt_{net}}{m}} - 1\right)\frac{mg}{b} = 0$$
$$(v_i \sin\theta)e^{-\frac{bt_{net}}{m}} + \frac{mg}{b}e^{-\frac{bt_{net}}{m}} - \frac{mg}{b} = 0$$
$$e^{-\frac{bt_{net}}{m}}\left(v_i \sin\theta + \frac{mg}{b}\right) = \frac{mg}{b}$$
$$e^{\frac{bt_{net}}{m}} = \frac{\left(v_i \sin\theta + \frac{mg}{b}\right)}{\frac{mg}{b}}$$
$$e^{\frac{bt_{net}}{m}} = 1 + \frac{bv_i \sin\theta}{mg}$$

Taking natural logarithm on both sides of the equation, we get;

$$\ln\left[e^{\frac{bt_{net}}{m}}\right] = \ln\left[1 + \frac{bv_i \sin\theta}{mg}\right]$$
$$\frac{bt_{net}}{m} = \ln\left[1 + \frac{bv_i \sin\theta}{mg}\right]$$

$$t_{net} = \frac{m}{b}\ln\left[1 + \frac{bv_i \sin\theta}{mg}\right] \qquad (7)$$

So, the time taken for the shuttlecock to reach up to the net in terms of the mass of the shuttlecock (m), drag coefficient (b), initial velocity (v_i), and angle of projection (θ) is given by;

$$t_{net} = \frac{m}{b}\ln\left[1 + \frac{bv_i \sin\theta}{mg}\right]$$

Now, by using this time, we can calculate, the horizontal speed of the moving shuttlecock at the peak position just above the net, which will be the effective speed of the shuttlecock at the top position. So, in next chapter, we will try to calculate the horizontal velocity of the shuttlecock at the peak position.

1.5 Speed of the Shuttlecock at the Peak Position

At time t_{net}, the shuttlecock is assumed to be at the peak position just above the net. So, the horizontal velocity of the shuttlecock at this peak position can be calculated by substituting t_{net} in the equation (6). So, the horizontal speed of the shuttlecock at the peak position is;

$$[v_x]_{t_{net}} = (v_i \cos\theta)e^{-\frac{bt_{net}}{m}}$$
$$[v_x]_{t_{net}} = (v_i \cos\theta)e^{-\frac{b\times\frac{m}{b}\ln\left[1+\frac{bv_i \sin\theta}{mg}\right]}{m}}$$
$$[v_x]_{t_{net}} = (v_i \cos\theta)e^{-\frac{m\ln\left[1+\frac{bv_i \sin\theta}{mg}\right]}{m}}$$
$$[v_x]_{t_{net}} = (v_i \cos\theta)e^{-\ln\left[1+\frac{bv_i \sin\theta}{mg}\right]}$$
$$[v_x]_{t_{net}} = (v_i \cos\theta)e^{\ln\left[1+\frac{bv_i \sin\theta}{mg}\right]^{-1}}$$
$$[v_x]_{t_{net}} = (v_i \cos\theta)\left[1 + \frac{bv_i \sin\theta}{mg}\right]^{-1}$$
$$[v_x]_{t_{net}} = (v_i \cos\theta)\left[\frac{mg + bv_i \sin\theta}{mg}\right]^{-1}$$
$$[v_x]_{t_{net}} = (v_i \cos\theta)\left[\frac{mg}{mg + bv_i \sin\theta}\right]$$
$$[v_x]_{t_{net}} = \frac{mgv_i \cos\theta}{mg + bv_i \sin\theta}$$

Now, at the peak position, the speed of the shuttlecock along the vertical direction is zero and the speed of the shuttlecock along the horizontal direction is as calculated above. Thus, the net speed of the shuttlecock is only due to the horizontal speed.

Thus, the total speed of the shuttlecock at the peak position just above the net is

$$v_{net} = \frac{mgv_i \cos\theta}{mg + bv_i \sin\theta} \qquad (8)$$

Now, this speed can be determined by substituting the variable in the equation, with the exact measured values and this can be greatly helpful in improving the performance of a sportsperson. So, let us see how this speed of the shuttlecock helps the player in her game:

- **Improving Accuracy**: By understanding the speed of the shuttlecock, players can improve their shot accuracy. They can adjust their swing and timing based on the speed of the shuttlecock to hit it more accurately.
- **Enhancing Strategy**: Knowledge of the shuttlecock's speed can help players strategize their game better. For instance, they can decide whether to play a defensive or offensive shot based on the speed of the incoming shuttlecock.
- **Predicting Opponent's Moves**: The speed of the shuttlecock can give clues about an opponent's next move. A fast shuttlecock might indicate an aggressive play, while a slow one could suggest a defensive strategy.
- **Training and Skill Development**: Coaches can use the speed data to help players develop their skills. By practicing at different speeds, players can become adept at handling a variety of shots.
- **Performance Analysis**: Lastly, the speed of the shuttlecock can be used for performance analysis. It can provide insights into a player's strengths and weaknesses, helping them to focus on areas that need improvement.

2. Results and Findings

During a low service of the shuttlecock in a badminton match, the velocity of the shuttlecock at any arbitrary position at time t before reaching the net is calculated as;

$$v_x = (v_i \cos\theta)e^{-\frac{bt}{m}}$$
$$v_y = (v_i \sin\theta)e^{-\frac{bt}{m}} + \left(e^{-\frac{bt}{m}} - 1\right)\frac{mg}{b}$$

Here, v_x represents the velocity of the shuttlecock along the horizontal direction and v_y represents the velocity of the shuttlecock along the vertical direction at an arbitrary position at time t.

Also, the time taken for the shuttlecock to reach the peak position, i.e., just above the net is;

$$t = \frac{m}{b}\ln\left[1 + \frac{bv_i \sin\theta}{mg}\right]$$

This speed can be minimized in order to maximize the performance of a player and hence can be greatly used while playing badminton.

Again, the net speed of the shuttlecock, when it is at the peak position, i.e., just above the net is;

$$v = \frac{mgv_i \cos\theta}{mg + bv_i \sin\theta}$$

This speed can be calculated by using the suitable values for the shuttlecock and hence can help the player to strategize the game of badminton.

3. Conclusion

A good service in badminton is like the opening move in a chess game. It sets the tone and can significantly influence the course of the rally. It's the first strike, the initial step in the dance that is a badminton rally. A well-executed serve can put you in a position of control, allowing you to dictate the pace of the game. So, it is highly necessary to have the knowledge about the speed of the shuttlecock which was discussed earlier in this paper.

The speed of the shuttlecock at the topmost position, just above the net, in a low service type is calculated as:

$$v_{net} = \frac{mgv_i \cos\theta}{mg + bv_i \sin\theta} \quad (9)$$

Although, there are many more factors that affect the speed of the shuttlecock like the temperature of the surrounding, the type of shuttlecock (feather or synthetic), air density, altitude of the position, tension in the badminton string etc., this equation gives an initial idea to calculate the speed of the shuttlecock by ignoring some of the above-mentioned factors.

Along with this the average time taken for the shuttlecock to cross the net is determined by the equation:

$$t = \frac{m}{b}\ln\left[1 + \frac{bv_i \sin\theta}{mg}\right]$$

By reducing this time, the player can reduce the reaction time for the opponent and hence it will become more difficult for the opponent to react in such a short span of time and hence can give more points to the player and gives better opportunity to improve her game.

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