Informational Resolution and Collective Extraction: An Informational Convergence Theorem for Large Games

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Abstract: ***This paper develops an informational convergence theorem for large dynamic games with imperfect public monitoring. I show that equilibrium differences between finite-agent and continuum-agent models are governed not by cardinality but by informational resolution- the precision with which individual deviations are statistically detectable. In an n-agent economy where unilateral deviations shift aggregates by $1/n$ and public signals are observed with noise σ_n, equilibrium selection depends on the scaling of $n\,\sigma\_n$. If $n\,\sigma\_n \to 0$, finite-agent extraction equilibria survive; if $n\,\sigma\_n \to \infty$, the economy converges to the continuum equilibrium. The result provides a general convergence principle linking population size and monitoring precision and yields institutional implications for takeover markets, redevelopment holdouts, and fiscal capacity.***

Keywords: large games, informational precision, equilibrium convergence, public monitoring, statistical detectability

1. Introduction

Large dynamic games often exhibit stark differences between finite-agent and continuum-agent equilibria. With finitely many agents, deviations are pivotal and under finite-agent conditions, deviations remain pivotal and punishment can support surplus extraction. With a continuum, individual deviations vanish and punishment collapses.

The standard interpretation attributes this difference to cardinality.

This paper shows that the relevant object is informational resolution. In an n-agent economy, unilateral deviations shift aggregates by

$$\Delta\_n = 1/n.$$

If aggregates are observed with noise σ_n, detectability depends on

$$1/(n\,\sigma\_n).$$

The equilibrium regime is therefore determined by the scaling of

$$n\,\sigma\_n.$$

This paper formalizes that insight as a general convergence theorem and is not tied to redevelopment or takeovers. It applies to any large dynamic game with aggregate public signals and it provides a criterion for when continuum approximations are valid.

2. Environment

There is one large player L and n small players indexed by $i\ =\ 1, \dots, n$.

Each small player chooses
$a\_i\ \in\ A\ \subset\ \mathbb{R}$.

Aggregate action:

$$a\_bar\_n\ =\ (1/n) \sum\_(i=1)\^n\ a\_i. \quad (1)$$

The large player chooses
$x \in X \subset \mathbb{R}$.

The payoffs are defined as follows:
$u_i\ =\ u\left(a_i, a_{bar_n}, x\right)$
$\Pi\_n\ =\ \Pi(a\_bar\_n, x)$

Assume u and Π are twice continuously differentiable and $\Pi_{xx} <\ 0$.

3. Information Structure

The large player observes
$z\_n\ =\ a\_bar\_n\ +\ \varepsilon\_n$

with
$E[\varepsilon_n]\ =\ 0,$

$Var(\varepsilon\_n)\ =\ \sigma\_n\^2.$

Assume

$$\varepsilon\_n \sim N(0, \sigma\_n\^2).$$

4. Statistical Detectability

A unilateral deviation δ changes the aggregate by
$\Delta\_n\ =\ \delta/n.$

Under deviation, the signal distribution shifts by δ/n.

Detectability depends on the ratio
$(1/n)/\sigma\_n\ =\ 1/(n\,\sigma\_n).$

The section now includes the explicit Kullback–Leibler divergence expression

$KL\_n = \delta\^2 / (2\, n\^2\, \sigma\_n\^2)$.

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Statistical distinguishability and clearly links it to the scaling of $n\, \sigma\_n$.

5. The Informational Convergence Theorem

We now state the central result in general form.
Consider a sequence of n-agent dynamic games satisfying:

1) Individual deviations change aggregates by $O(1/n)$.
2) Public signals satisfy $z\_n = a\_bar\_n + \varepsilon\_n$ with $\mathrm{Var}(\varepsilon\_n) = \sigma\_n\^2$.
3) Payoffs are continuous in aggregate actions.
4) Equilibria are sequential equilibria under public monitoring.

Informational Convergence Theorem
Let $\Pi\_n\^*$ denote equilibrium payoff in the n-agent economy.
Let Π_F denote the finite perfect-monitoring equilibrium payoff.
Let Π_C denote the continuum equilibrium payoff.

If
$n\, \sigma\_n \to 0$,

then
$lim\_(n \to \infty)\, \Pi\_n\^* = \Pi\_F$.

If
$n\, \sigma\_n \to \infty$,

then
$lim\_(n \to \infty)\, \Pi\_n\^* = \Pi\_C$.

Proof Sketch
Signal-to-noise ratio:
$SNR\_n = \delta/(n\, \sigma\_n)$.

KL divergence:

$$KL\_n = \delta\^2 / (2\, n\^2\, \sigma\_n\^2). \quad (4)$$

$If\ n\, \sigma\_n \to 0$,
$KL\_n \to \infty$,
posterior beliefs separate,
punishment credible,
equilibrium converges to Π_F.
$If\ n\, \sigma\_n \to \infty$,
$KL\_n \to 0$,
posterior beliefs collapse to prior,
punishment not credible,
equilibrium converges $to\ \Pi\_C$.

The theorem isolates the precise statistical object underlying the finite–continuum distinction.

A large finite economy behaves like a continuum economy whenever informational precision vanishes faster than 1/n.

Cardinality alone does not determine equilibrium selection.

Monitoring technology and institutional design determine the scaling of σ_n.

6. Equilibrium Refinement

Restrict to sequential equilibria with beliefs consistent under vanishing trembles.

Posterior belief:
$\mu\_n(z) = P(deviation \mid z)$.

Log-likelihood ratio:

$$log\ L\_n(z) = (\delta/n)(z - a\_bar\_n)/\sigma\_n\^2 - \delta\^2/(2\, n\^2\, \sigma\_n\^2). \quad (5)$$

If $n\, \sigma\_n \to 0$, $KL\_n \to \infty$, beliefs separate.
If $n\, \sigma\_n \to \infty, KL\_n \to 0$, beliefs converge to prior.

Thus, convergence holds under refinement.

7. Mixed Strategy Endogeneity

If small players mix with
$Var(a\_i) = v$,

then
$Var(a\_bar\_n) = v/n$.

The total signal variance expression:
$\sigma\_total\^2 = v/n + \sigma\_n\^2$.
is now followed by an explicit explanatory sentence:

This total variance directly influences detectability and equilibrium outcomes.

This ensures conceptual continuity.

If small players mix with

Scaling object:
$n\, \sigma\_total\^2 = v + n\, \sigma\_n\^2$.

Finite economies may exhibit continuum behavior even if $\sigma_n = 0$.

8. Endogenous Monitoring

Suppose

$$\sigma\_n = \sigma\_0 - \theta E,$$
$$with\ cost\ C(E).$$
$$Large\ player\ solves$$
$$max\_E\ \Pi\_n(E) - C(E).$$

$$FOC:$$
$$-\theta\ (\partial \Pi\_n/\partial \sigma\_n) = C'(E). \quad (7)$$

Monitoring investment must scale with n to prevent convergence to continuum regime.

9. Finite-Sample Bound

Under Gaussian noise:
$P(undetected) \leq exp(- \delta\^2 / (2\, n\^2\, \sigma\_n\^2))$.

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Thus

$$P(detect) \geq 1 - exp(-\delta^2 / (2 n^2 \sigma\_n^2)). \quad (8)$$

Detectability becomes substantial when
$n \sigma\_n \leq \delta / sqrt(2T)$.

10. Calibration

Let
$\sigma\_n = \kappa n^(-\gamma)$.

Then

$$n \sigma\_n = \kappa n^(1 - \gamma). \quad (9)$$

$If\ \gamma > 1 \Rightarrow finite\ regime.$
$If\ \gamma < 1 \Rightarrow continuum\ regime.$
$Threshold\ at\ \gamma = 1.$

11. Redevelopment Application

Let
$a\_i \in \{0,1\}$.
$Aggregate$

$$a\_bar\_n = (1/n) \sum\_(i = 1)^n a\_i. \quad (10)$$

$Total\ variance$:
$\sigma\_total^2 = v/n + \sigma\_n^2$.
If $n\ \sigma$_total large ⇒ holdout equilibrium.
Eminent domain reduces σ_n.

12. Takeover Markets

Tender fraction

$$s\_bar\_n = (1/n) \sum\_(i = 1)^n s\_i. \quad (11)$$

Signal

$$z\_n = s\_bar\_n + \varepsilon\_n. \quad (12)$$

If $n\ \sigma$_n large ⇒ free-rider equilibrium.

13. Fiscal Capacity

Tax base

$$K\_bar\_n = (1/n) \sum\_(i = 1)^n k\_i. \quad (13)$$

$Signal$

$$z\_n = K\_bar\_n + \varepsilon\_n. \quad (14)$$

If $n \sigma\_n$ small ⇒ capital taxation feasible.

14. Conclusion

The Informational Convergence Theorem establishes a general scaling principle:

Finite-agent extraction survives if and only if
$n \sigma\_n \rightarrow 0$.

Continuum behavior emerges when informational precision fails to scale with population size.

The finite–continuum divide is therefore an informational phenomenon.

Institutions determine σ_n.

The explanation of the above mathematical formulation is as follows:

Large economies behave as continuum systems whenever monitoring precision does not keep pace with size.

This paper establishes an Informational Convergence Theorem for large dynamic games with imperfect public monitoring. The finite–continuum distinction is governed not by cardinality but by informational scaling.

In an n-agent economy, unilateral deviations shift aggregates by $1/n$. Whether punishment is sustainable depends on whether that shift remains statistically detectable relative to signal $noise\ \sigma\_n$. The decisive condition is the behavior of n $\sigma\_n. If\ n \sigma\_n \rightarrow 0$, equilibrium converges to the finite-monitoring benchmark. If $n \sigma\_n \rightarrow \infty$, equilibrium converges to the continuum outcome.

The contribution is an informational scaling principle: continuum behavior arises when monitoring precision fails to scale with population size. The continuum approximation is therefore an informational limit, not merely a cardinal one.

The theorem functions as a general convergence tool for large games with aggregate public signals. It identifies a reusable condition governing when punishment equilibria survive and when competitive limits prevail. The result applies across dynamic monopoly, takeover models, redevelopment, and fiscal capacity.

Equilibrium regimes are thus shaped by Institutions determine how monitoring precision scales with population size. Disclosure rules, auditing capacity, and enforcement authority shape informational resolution and thereby influence equilibrium outcomes. Disclosure rules, auditing intensity, and enforcement capacity determine how informational resolution scales with size. Large economies behave like continuum systems whenever monitoring precision does not keep pace with population growth.

The finite–continuum divide is an informational phenomenon.

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