International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2025: 7.089

Closed Neighbourhood and Total Degree-Based Molecular Descriptors of Intuitionistic Fuzzy Graphs

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Abstract: ***An intuitionistic fuzzy set is a generalisation of the notion of a fuzzy set. An intuitionistic fuzzy graph is a kind of fuzzy graph which explains the degree of membership and non-membership of vertices and edges. In this paper closed neighbourhood and total degree-based first, second Zagreb indices, forgotten index, Randic index and harmonic index of intuitionistic fuzzy graphs are studied.***

Keywords: Closed neighbourhood degree, fuzzy graph, intuitionistic fuzzy graph, molecular descriptor, total degree

1. Introduction

A graph without loops and parallel edges is called a simple graph. All graphs considered here are finite, undirected and simple. A topological index is a numerical parameter mathematically derived from the graph structure. Fuzzy set is a newly emerging mathematical frame work to exemplify the phenomenon of uncertainty in real life tribulations. An extension of classical graph theory is fuzzy graph theory. It was created to deal with imprecision and uncertainty and has widespread use in the field of computer science, social networks, system analysis and transport problems [1-5]. A fuzzy set is a generalized form of a crisp set, in which elements have different degrees of membership values. As this crisp set has only two values 0 and 1 (no or yes), it cannot manage uncertain real-world problems. Instead of considering 0 or 1, a fuzzy set allows its elements to have a membership value between 0 and 1 for a better result. However, in some cases, those single membership degree values are unable to handle the vagueness. Most important thing which is to be noted is that, when we have uncertainty regarding either the set of vertices or edges, or both, the model becomes a fuzzy graph.

The fuzzy set theory was developed by L. Zadeh [6] in 1965.A membership function for a fuzzy set A on the universe of discourse X is defined as $\mu_A:X \rightarrow [0,1]$. Here each element of X is mapped to a value between 0 and 1. It is called membership value or degree of membership. It quantifies the degree of membership of the element in X to the fuzzy set A. A set is an unordered collection of different elements.A fuzzy graph is a pair of functions G = (σ, μ), where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, i.e. $\sigma : V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$, for all u, v ∈ V, where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of σ(u) and σ(v), σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E. The underlying crisp graph of a fuzzy graph G = (σ, μ) is denoted by $G^* = (\mu^*, \sigma^*)$ where $\sigma^* = \{v \in V|\sigma(v) > 0\}$ and $\mu^* = \{(u, v) \in V \times V |\mu(u, v) > 0\}$. Here σ(v) and μ(u, v) represent the membership values of the vertex v and edge (u, v) respectively. A fuzzy graph G: (σ, μ) is said to be complete if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$ for all u and v. Neutrosophic graph theory serves as an extension of classical graph theory, specifically focusing on the notion of indeterminacy. The degree of vertex of a fuzzy graph G can be calculated by adding the weights of all corresponding edges to that vertex v.

In recent years scientists and analysts have been using the concept of intuitionistic fuzzy set successfully in various areas such as image processing, robotic system, social network, machine learning, decision making and medical diagnosis recognition etc. The intuitionistic fuzzy set is widely used in real world situations that involve human perception and knowledge which are entirely imprecise and unreliable. Membership functions allow you to quantify linguistic term and represent a fuzzy set graphically. Intuitionistic fuzzy graphs serve as a sophisticated framework for modelling complex and uncertain phenomena across diverse domains, such as decision-making, economics, medicine, computer science. K.T. Atanassov [7] introduced the notion of the intuitionistic fuzzy graph in 1999. An intuitionistic fuzzy graph is a kind of fuzzy graph which explains the degree of membership and non-membership of vertices and edges. In a complete intuitionistic fuzzy graph with at least three vertices, there exists at least one pair of edges whose membership degrees are equal and at least one pair of edges whose non-membership degrees are equal.

Let X be a non-empty set. An intuitionistic fuzzy set A in X is defined as A= $\{(x, \mu_A(x), \gamma_A(x) /x \in X\}$ which is characterized by a membership function $\mu_A(x):X\rightarrow[0,1]$ and the non-membership function $\gamma_A(x):X\rightarrow[0,1]$ and satisfying the following conditions

1) $0 \leq \mu_A(x)+\gamma_A(x)\leq 1, \forall x \in X$.
2) $0 \leq \mu_A(x)+\gamma_A(x), \Pi_A (x)\leq 1, \forall x \in X$.
3) $\Pi_A (x) = 1 -\mu_A(x)-\gamma_A(x)$

where $\Pi_A (x)$ is called intuitionistic fuzzy index of element x in A, the value denotes a measure of indeterminacy.

An intuitionistic fuzzy graph is of the form G = (V, E), where V= $\{v_1, v_2, \ldots,v_n\}$ such that

1) μ_1: V→ [0,1] and γ_1: V → [0,1] denote the degree of membership and non-membership of the element $v_i \in V$, respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V$, (i = 1, 2, ..., n);
2) E ⊆ V x V, where μ_2: VxV→ [0,1] and γ_2: VxV→ [0,1] are such that $\mu_2(v_i, v_j) \leq \min [\mu_1 (v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \geq \max [\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, (i, j = 1, 2, ...,n) [8-11].

Let G= $((\mu_1, \gamma_1),(\mu_2, \gamma_2))$.The μ-degree of a vertex v_i is $d_\mu[v_{i,}] = \sum_{v_{i,}v_j \in E} \mu_2(v_{i,}v_j)$ and γ-degree of a vertex v_i is $d_\gamma[v_{i,}] = \sum_{v_{i,}v_j \in E} \gamma_2(v_{i,}v_j)$. The neighbourhood of any vertex v is defined as $N(v) = \left(N_\mu(v), N_\gamma(v)\right)$, whre $N_\mu(v) = \{w \in V: \mu_2(v,w) = \mu_1(v)\Lambda\, \mu_1(w)\}$, $N_\gamma(v) = \{w \in V, \gamma_2(v,w) = \gamma_1(v)$ □ $\gamma_1(w)\}$ and N[v]=N(v)U{v}is called closed neighbourhood of v [12-15].

In intuitionistic fuzzy graph G = (V, E), the closed neighbourhood degree of a vertex $v \in V$ is $d_N[v] = (d_{N_\mu}[v], d_{N_\gamma}[v])$, where

1) $d_{N_\mu}[v] = \mu_1(v) + \sum_{w \in N(v)} \mu_1(w)$ is membership component and
2) $d_{N_\gamma}[v] = \gamma_1(v) + \sum_{w \in N(v)} \gamma_1(w)$ is non-membership component.

Each vertex v has membership $\mu_1(v)$ and non-membership $\mu_2(u,v)$ and non-membership $\gamma_1(w)$. By definition, the closed neighbourhood μ degree of each vertex is the sum of the membership values of the vertices and itself and the closed neighbourhood γ degree of each vertex is the sum of the non-membership values of the vertices and itself. Therefore, all the vertices will have the same closed neighbourhood μ-degree and closed neighbourhood γ -degree. For each edge (u, v) has a membership $\gamma_2(u, v)$. The degree of a vertex v is denoted by d(v) =$(d_\mu(v), d_\gamma(v))$ is defined defined as

μ-degree: $d_\mu(v)=\sum_{u \neq v} \mu_2(u,v)$ (sum of membership values of incident edges),

γ -degree: $d_\gamma(v)=\sum_{u \neq v} \gamma_2(u,v)$ (sum of non-membership values of incident edges).

Let G(V, E) be intuitionistic fuzzy graph with G(μ, γ), then total degree of a vertex $v_i \in V$ is defined as:

$t\,d(v_i) = \left[d_\mu(v_i) + \mu(v_i), d_\gamma(v_i) + \gamma(v_i)\right]$, where $d(v_i)$ =$(d_\mu(v_i), d_\gamma(v_i))$.

The relations for intuitionistic fuzzy graphs are: let A,B and C be intuitionistic fuzzy graphs. Then;
(i) $A \preccurlyeq A$ i.e., A is reflexive relation,
(ii) $A \preccurlyeq B$ and $B \preccurlyeq A$ i.e., symmetric relation,
(iii) $A \preccurlyeq B$ and $B \preccurlyeq C \Rightarrow A \preccurlyeq C$ i.e., transitive relation [16,17].

Different types of products on intuitionistic fuzzy graphs were studied in [18]. Perfectly regular and perfectly edge regular intuitionistic fuzzy graphs were studied in [19]. Co-common neighbourhood of the vertices in graph G is defined and studied in [20]. Laplacian energy of intuitionistic fuzzy graphs was introduced in [21]. The max-min matrix of intuitionistic fuzzy graph structure was introduced by V. Bansal [22]. The notions of subdivisions intuitionistic bipolar fuzzy graph, matrix of bipolar intuitionistic fuzzy graph, and line intuitionistic bipolar fuzzy graph are introduced and studied in [23]. A study on energy in intuitionistic fuzzy graph were done by D. Gunasekaran [24]. The line graph L(G) vertices as the edges of G and two vertices of L(G) are neighbours if and only if the associated edges are incident in G. Jump graph is the complement of the line graph. That is the vertices of jump graphs are neighbours if and only if, the associated edges are not incident in G. Topological polynomials in fuzzy Hamiltonian graph, were computed in [25].

The symbols and notations follow standard conventions as established [26-29]. In this paper first, second Zagreb indices, forgotten index, Randic index and harmonic index in intuitionistic fuzzy graph are investigated by using closed neighbourhood degree and total degree of vertices. The closed neighbourhood degree-based first, second Zagreb indices and forgotten index, Randic index and harmonic index of intuitionistic fuzzy graph can be defined as [30-32]:

1) $M_{1_{IF}}(G) = \left[\sum_{v \in V} d_{N_\mu}(v)^2, \sum_{v \in V} d_{N_\gamma}(v)^2\right]$.
2) $M_{2_{IF}}(G) = [\sum_{uv \in E} d_{N_\mu}(u_i) \times d_{N_\mu}(v_i), \sum_{uv \in E} d_{N_\gamma}(u_j) \times d_{N_\gamma}(v_j)]$.
3) $F_{IF}(G) = [\sum_{u \in V} d_{N_\mu}(v)^3, \sum_{v \in V} d_{N_\gamma}(v)^3]$.
4) $R_{IF}(G) = \sum_{uv \in E} \frac{1}{\sqrt{[(d_{N_\mu}(v_i) \times d_{N_\gamma}(v_i)),(d_{N_\mu}(v_j) \times d_{N_\gamma}(v_j))]}}$.
5) $H_{IF}(G) = \sum_{uv \in E} \frac{2}{[(d_{N_\mu}(v_i) + d_{N_\gamma}(v_i)),(d_{N_\mu}(v_j) + d_{N_\gamma}(v_j))]}$.

The total degree-based molecular descriptors of intuitionistic fuzzy are defined as [33]:

6) $M_{1_{IF}}(G) = \left[\sum_{v \in V} td_\mu(v)^2, \sum_{v \in V} td_\gamma(v)^2\right]$.
7) $M_{2_{IF}}(G) = [\sum_{uv \in E} td_\mu(u_i) \times td_\gamma(v_i), \sum_{uv \in E} td_\mu(u_j) \times td_\gamma(v_j)]$.
8) $F_{IF}(G) = \left[\sum_{v \in V} td_\mu(v)^3, \sum_{v \in V} td_\gamma(v)^3\right]$.
9) $R_{IF}(G) = \sum_{uv \in E} \frac{1}{\sqrt{[(td_\mu(v_i) \times td_\gamma(v_i)),(td_\mu(v_j) \times td_\gamma(v_j))]}}$.
10) $H_{IF}(G) = \sum_{uv \in E} \frac{2}{[(td_\mu(v_i) + td_\gamma(v_i)),(td_\mu(v_j) + td_\gamma(v_j))]}$.

2. Materials and Methods

The intuitionistic fuzzy graphs with four vertices are shown in figures (1-2). The closed neighbourhood degree is determined by membership and non-membership values of figure 1 and total degree of vertices of figure 2 of intuitionistic fuzzy graphs respectively. The membership and non-membership degree of figure 1 are used in computing closed neighbourhood degree-based and total degree-based (figure 2) molecular descriptors of intuitionistic fuzzy graphs.

3. Results and Discussion

Closed neighbourhood degree-based molecular descriptors

Consider intuitionistic fuzzy graph with four vertices (fig.1): as $v_1(0.3,0.5)$, $v_2(0.1,0.6)$, $v_3(0.4,0.5)$ and $v_4(0.2,0.7)$ with edges $v_1v_2 = (0.1,0.6)$, $v_2v_3 = (0.1,0.7)$, $v_2v_4 = (0.1,0.9)$, $v_3v_4 = (0.1,0.8)$, and $v_4v_1 = (0.2,0.8)$. The molecular descriptors are computed as follows by using membership and non-membership degrees of figure 1.

The neighbourhood of any vertex v is defined as: $N(v) = \left(N_\mu(v), N_\gamma(v)\right)$, here $N(v)$ includes neighbours w where edge strengths match vertex strengths. $N_\mu(v) = \{w \in V: (\mu)_2(v,w) = \mu_1(v) \wedge \mu_1(w)\}$, $N_\gamma(v) = \{w \in V, \gamma_2(v,w) = \gamma_1(v) \vee \gamma_1(w)\}$ and then $N[v]=N(v)\cup\{v\}$ is called closed neighbourhood of v.

Theorem 1. Closed neighbourhood degree-based first Zagreb index of intuitionistic fuzzy graph is (2.85,17.06).

Proof. Using membership and non-membership values of vertices and edges from figure 1, we get closed neighbourhood degree-based first Zagreb index in intuitionistic fuzzy graph. The closed neighbourhood degree of a vertex $v \in V$ is $d_N[v] = (d_{N_\mu}[v], d_{N_\gamma}[v])$, where

1) $d_{N_\mu}[v] = \mu_1(v) + \sum_{v\in N(v)} \mu_1(w)$ is membership component and
2) $d_{N_\gamma}[v] = \gamma_1(v) + \sum_{v\in N(v)} \gamma_1(w)$ is non-membership component.

Closed neighbourhood degree of a vertex $v \in V$:
μ-degree for v_1: $\mu_1(v_2) + \mu_1(v_4)$=0.1+0.2=0.3, $d_{N_\mu}[v] = \mu_1(v_1) + \sum_{v\in N(v)} \mu_1(v_1) = 0.3 + 0.3 = 0.6$,
γ-degree for v_1: $\gamma_1(v_2) + \gamma_1(v_4)$=0.6+0.7=1.3, $d_{N_\gamma}[v] = \gamma_1(v_1) + \sum_{v\in N(v)} \gamma_1(v_1) = 1.3 + 0.5 = 1.8$,
$d_N[v_1] = (d_{N_\mu}[v], d_{N_\gamma}[v]) = (0.6,1.8)$, $d_N[v_2] = (1.0,2.3)$, $d_N[v_3] = (0.7,1.8)$ and $d_N[v_4] = (1.0,2.3)$.

$M_{1_{IF}}(G) = \left[\sum_{v\in V} d_{N_\mu}(v)^2, \sum_{v\in V} d_{N_\gamma}(v)^2\right]$
$= [d_{N_\mu}(v_1)^2, d_{N_\gamma}(v_1)^2] + [d_{N_\mu}(v_2)^2, d_{N_\gamma}(v_2)^2] + [d_{N_\mu}(v_3)^2, d_{N_\gamma}(v_3)^2] + [d_{N_\mu}(v_4)^2, d_{N_\gamma}(v_4)^2]$
=(2.85,17.06).

Theorem 2. Closed neighbourhood degree-based second Zagreb index of intuitionistic fuzzy graph is (3.6,21.85).
Proof. Using membership values of vertices and edges from figure 1, we get closed neighbourhood degree-based second Zagreb index of intuitionistic fuzzy graph.
The closed neighbourhood degree of a vertex $v_i \in V$ is $d_N[v_i] = (d_{N_\mu}[v_i], d_{N_\gamma}[v_i])$.
$M_{2_{IF}}(G) = [\sum_{uv\in E} d_{N_\mu}(u_i) \times d_{N_\mu}(v_i), \sum_{uv\in E} d_{N_\gamma}(u_j) \times d_{N_\gamma}(v_j)]$
$= [d_{N_\mu}(u_1) \times d_{N_\mu}(v_1), d_{N_\gamma}(u_2) \times d_{N_\gamma}(v_2)] + [d_{N_\mu}(u_2) \times d_{N_\mu}(v_2), d_{N_\gamma}(u_3) \times d_{N_\gamma}(v_3)] + [d_{N_\mu}(u_3) \times d_{N_\mu}(v_3), d_{N_\gamma}(u_4) \times d_{N_\gamma}(v_4)] + [d_{N_\mu}(u_4) \times d_{N_\mu}(v_4), d_{N_\gamma}(u_1) \times d_{N_\gamma}(v_1)] + [d_{N_\mu}(u_2) \times d_{N_\mu}(v_2), d_{N_\gamma}(u_4) \times d_{N_\gamma}(v_4)] = (0.6 \times 1.0, 1.8 \times 2.3) + (1.0 \times 0.7, 2.3 \times 1.8) + (0.7 \times 1.0, 1.8 \times 2.3) + (1.0 \times 0.6, 2.3 \times 1.8) + (1.0 \times 1.0, 2.3 \times 2.3) = (3.6,21.85)$.

Theorem 3. The forgotten index of intuitionistic fuzzy graph is (2.56,35.99).

Proof. Using membership and non-membership values of vertices from figure 1, we compute closed neighbourhood degree-based forgotten index of intuitionistic fuzzy graph, $d_N[v_1] = (d_{N_\mu}[v_1], d_{N_\gamma}[v_1]) = (0.6,1.8)$, $d_N[v_2] = (1.0,2.3)$, $d_N[v_3] = (0.7,1.8)$ and $d_N[v_4] = (1.0,2.3)$.
$F_{IF}(G) = [\sum_{u\in V} d_{N_\mu}(v)^3, \sum_{v\in V} d_{N_\gamma}(v)^3]$
$= [d_{N_\mu}[v_1]^3, d_{N_\gamma}[v_1]^3] + [d_{N_\mu}[v_2]^3, d_{N_\gamma}[v_2]^3] + [d_{N_\mu}[v_3]^3, d_{N_\gamma}[v_3]^3 + [d_{N_\mu}[v_4]^3, d_{N_\gamma}[v_4]^3$
$= [(0.6)^3, (1.8)^3] + [(1.0)^3, (2.3)^3] + [(0.7)^3, (1.8)^3] + [(1.0)^3, (2.3)^3]$
= (2.56,35.99).
The closed neighbourhood degree-based values of Randic index and harmonic index are given in table 1.

Total degree-based molecular descriptors

Theorem 4. Total degree-based first Zagreb index of intuitionistic fuzzy graph is (6.11,18.01).
Proof. Let G(V, E) be intuitionistic fuzzy graph with $G(\mu, \gamma)$, then total degree of a vertex $v_i \in V$,
$t\ d(v_i) = [d_\mu(v_i) + \mu(v_i), d_\gamma(v_i) + \gamma(v_i)]$, where $d(v_i) = (d_\mu(v_i), d_\gamma(v_i))$,with
μ-degree: $d_\mu(v) = \sum_{u\neq v} \mu_2(u,v)$, γ-degree: $d_\gamma(v) = \sum_{u\neq v} \gamma_2(u,v)$.

Consider intuitionistic fuzzy graph represented in figure 2 to compute total degree-based first Zagreb index. Then μ-degree and γ-degree are:
$d_\mu(v_1) = 0.25 + 0.2 + 0.3 = 0.75$ and $\mu(v_1) = 0.3$, $t\ d_\mu(v_1)$ =0.75+0.3=1.05. $d_\gamma(v_1) = 0.45 + 0.8 + 0.25 = 1.50$ and $\gamma(v_1) = 0.6$, $td_\gamma(v_1) = 1.50 + 0.6 = 2.1$,
$(t\ d_\mu(v_1), t\ d_\gamma(v_1)) = (1.05,2.1)$, $(t\ d_\mu(v_2), t\ d_\gamma(v_2)) = (1.63,1.75)$, $\left(td_\mu(v_3), td_\gamma(v_3)\right) = (0.73,2.87)$ and $(td_\mu(v_4), td_\gamma(v_4)) = (1.35,1.52)$.
$M_{1_{IF}}(G) = [\sum_{v\in V} td_\mu(v)^2, \sum_{v\in V} td_\gamma(v)^2]$
$= [td_\mu(v_1)^2, td_\gamma(v_1)^2] + [td_\mu(v_2)^2, td_\gamma(v_2)^2] + [td_\mu(v_3)^2, td_\gamma(v_3)^2] + [td_\mu(v_4)^2, td_\gamma(v_4)^2]$
$= [(1.05)^2, (2.1)^2] + [(1.63)^2, (1.75)^2] + [(0.73)^2, (2.87)^2] + [(1.35)^2, (1.52)^2]$
=(6.11,18.01).

Theorem 5. Total degree-based Randic index of intuitionistic fuzzy graph is $\frac{1}{\sqrt{(8.27,24.94)}}$.

Proof. By using figure 2 we have, μ-degree: $d_\mu(v) = \sum_{u\neq v} \mu_2(u,v)$, γ-degree: $d_\gamma(v) = \sum_{u\neq v} \gamma_2(u,v)$.
Total degree $t\ d(v_i) = [(d_\mu(v_i) + \mu(v_i)), (d_\gamma(v_i) + \gamma(v_i))]$, where $d(v_i) = (d_\mu(v_i), d_\gamma(v_i))$.

$[(t\quad d_\mu(v_{1i}) \times td_\gamma(v_{2i})), (td_\mu d(v_{1j}) \times td_\gamma(v_{2j}))] = (1.05 \times 1.63, 2.1 \times 1.75)$,
$[(t\quad d_\mu(v_{2i}) \times td_\gamma(v_{3i})), (td_\mu(v_{2j}) \times td_\gamma(v_{3j}))] = (1.63 \times 0.73, 1.75 \times 2.87)$,
$[(td_\mu(v_{3i}) \times td_\gamma(v_{4i})), (td(v_{3j}) \times td_\gamma(v_{4j}))] = (0.73 \times 1.35, 2.87 \times 1.52)$,
$[(td(v_{4i}) \times td_\gamma(v_{1i})), (td_\mu(v_{4j}) \times td_\gamma(v_{1j}))] = (1.35 \times 1.05, 1.52 \times 2.1)$,
$\left[\left(td_\mu d(v_{2i}) \times td_\gamma(v_{4i})\right), \left(td_\mu(v_{2j}) \times td_\gamma(v_{4j})\right)\right] = (1.63 \times 1.35, 1.75 \times 1.52)$,
$[(td_\mu(v_{1i}) \times td_\gamma(v_{3i})), (td_\mu(v_{1j}) \times td_\gamma(v_{3j}))] = (1.05 \times 0.73, 2.1 \times 2.87)$,

Then total degree-based Randic index

$$R_{IF}\quad(G) = \sum_{uv \in E} \frac{1}{\sqrt{[(td_\mu(v_i) \times td_\gamma(v_i)), (td_\mu(v_j) \times td_\gamma(v_j))]}}$$

$$= \frac{1}{\sqrt{[(d_\mu(v_{1i}) \times td_\gamma(v_{2i})), (td_\mu(v_{1j}) \times td_\gamma(v_{2j}))]}} + \frac{1}{\sqrt{[(td_\mu(v_{2i}) \times td_\gamma(v_{3i})), (td_\mu(v_{2j}) \times td_\gamma(v_{3j}))]}} + \frac{1}{\sqrt{[(td_\mu(v_{3i}) \times td_\gamma(v_{4i})), (td_\mu(v_{3j}) \times td_\gamma(v_{4j}))]}} + \frac{1}{\sqrt{[(td_\mu(v_{4i}) \times td_\gamma(v_{1i})), (td_\mu(v_{4j}) \times td_\gamma(v_{1j}))]}} + \frac{1}{\sqrt{[(td_\mu(v_{2i}) \times td_\gamma(v_{4i})), (td_\mu(v_{2j}) \times td_\gamma(v_{4j}))]}} + \frac{1}{\sqrt{[(td_\mu(v_{1i}) \times td_\gamma(v_{3i})), (td_\mu(v_{1j}) \times td_\gamma d(v_{3j}))]}} = \frac{1}{\sqrt{(8.27, 24.94)}}.$$

Theorem 6. Total degree-based harmonic index of intuitionistic fuzzy graph is $\frac{2}{(14.25, 24.72)}$.

Proof. By using figure 2 we have, μ-degree: $d_\mu(v) = \sum_{u \neq v} \mu_2(u, v)$, γ-degree: $d_\gamma(v) = \sum_{u \neq v} \gamma_2(u, v)$.
Total degree $t\ d(v_i) = [(d_\mu(v_i) + \mu(v_i)), (d_\gamma(v_i) + \gamma(v_i))]$, where $d(v_i) = (d_\mu(v_i), d_\gamma(v_i))$.
Then total degrees for edges (u,v) are: $[(t\,d_\mu(v_{1i}) + td_\gamma(v_{2i})), (td_\mu(v_{1j}) + td_\gamma(v_{2j}))] = (1.05 + 1.63, 2.1 + 1.75)$, $[(t\quad d_\mu(v_{2i}) + td_\gamma(v_{3i})), (td_\mu(v_{2j}) + td_\gamma(v_{3j}))] = (1.63 + 0.73, 1.75 + 2.87)$, $[(td_\mu(v_{3i}) + td_\gamma(v_{4i})), (td_\mu(v_{3j}) \times td_\gamma(v_{4j}))] = (0.73 + 1.35, 2.87 + 1.52)$, $[(td_\mu(v_{4i}) + td_\gamma(v_{1i})), (td_\mu(v_{4j}) + td_\gamma(v_{1j}))] = (1.35 + 1.05, 1.52 + 2.1)$, $[(td_\mu(v_{2i}) + td_\gamma(v_{4i})), (td_\mu(v_{2j}) + td_\gamma(v_{4j}))] = (1.63 + 1.35, 1.75 + 1.52)$, $\left[\left(td_\mu(v_{1i}) + td_\gamma(v_{3i})\right), \left(td_\mu(v_{1j}) + td_\gamma(v_{3j})\right)\right] = (1.05 + 0.73, 2.1 + 2.87)$.

Then total degree-based harmonic index

$$H_{IF}(G) = \sum_{v \in V} \frac{2}{[(td_\mu(v_i) + td_\gamma(v_i)), (td_\mu(v_j) + td_\gamma(v_j))]} = \frac{2}{[(td_\mu(v_{1i}) + td_\gamma(v_{2i})), (td_\mu(v_{1j}) + td_\gamma(v_{2j}))]} + \frac{2}{[(td_\mu(v_{2i}) + td_\gamma(v_{3i})), (td_\mu(v_{2j}) + td_\gamma(v_{3j}))]} + \frac{2}{[(td_\mu(v_{3i}) \times td_\gamma(v_{4i})), (td_\mu(v_{3j}) \times td_\gamma(v_{4j}))]} + \frac{2}{[(td_\mu(v_{4i}) + td_\gamma(v_{1i})), (td_\mu(v_{4j}) + td_\gamma(v_{1j}))]} + \frac{2}{[(td_\mu(v_{2i}) + td_\gamma(v_{4i})), (td_\mu(v_{2j}) + td_\gamma(v_{4j}))]} + \frac{2}{[(td_\mu(v_{1i}) + td(v_{3i})), (td_\mu(v_{1j}) + td(v_{3j}))]} = \frac{2}{(1.05+1.63, 2.1+1.75)} + \frac{2}{(1.63+0.73, 1.75+2.87)} + \frac{2}{(0.73+1.35, 2.87+1.52)} + \frac{2}{(1.35+1.05, 1.52+2.1)} + \frac{2}{(1.63+1.35, 1.75+1.52)} + \frac{2}{(1.05+0.73, 2.1+2.87)} = \frac{2}{(14.25, 24.72)}.$$

The computed values of total degree-based second Zagreb index and forgotten index are given in table 1.

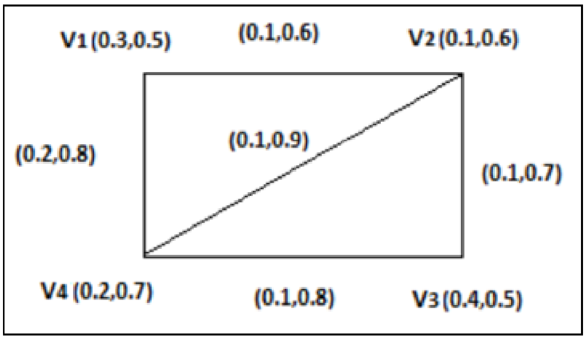

Figure 1: Intuitionistic fuzzy graph with four vertices

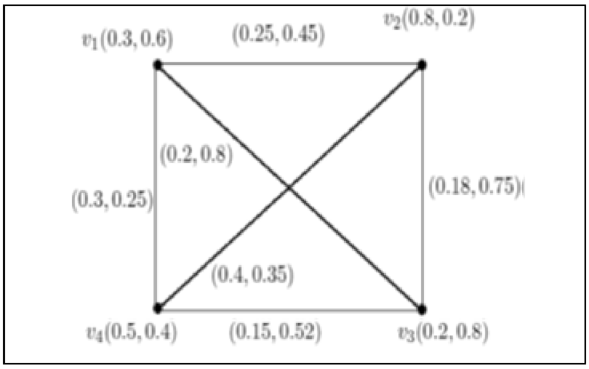

Figure 2: Intuitionistic fuzzy graph with four vertices and six edges.

4. Conclusion

In this study we have computed the closed neighbourhood degree-based and total degree-based first, second Zagreb indices, forgotten index, Randic index and harmonic index of intuitionistic fuzzy graph. These molecular descriptors values are in pairs due to membership and non-membership components of intuitionistic fuzzy graph.

Table 1: Molecular descriptors of intuitionistic fuzzy graph

Molecular descriptor	Closed neighbourhood degree-based $R_{IF}(G)$ (fig.1)	Closed neighbourhood degree-based $H_{IF}(G)$ (fig.1)	Total degree-based $M_{2_{IF}}(G)$ (fig.2)	Total degree-based $F_{IF}(G)$ (fig.2)
Intuitionistic fuzzy graph	$\frac{1}{\sqrt{(3.6,21.85)}}$	$\frac{2}{(8.6,21.00)}$	(8.27,24.94)	(8.337,41.77)

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