Optimal Cropping and Irrigation Scheduling for Sugarcane Farms Using Fuzzy Multi-Objective Linear Programming Under Uncertain Water Availability

Lokesh Kumar

Assistant Professor, Department of Mathematics, Government Raza P.G. College Rampur, Affiliated to Guru Jambheshwar University, Moradabad, U.P., India
Email: *lokeshkumarnbd[at]gmail.com*

Abstract: ***Water scarcity poses a major challenge for sustainable sugarcane production, particularly in semi-arid regions where water availability is uncertain. This paper develops a Fuzzy Multi-Objective Linear Programming (FMOLP) model to optimize both cropping pattern and irrigation scheduling for sugarcane farms under uncertain hydrological conditions. The model simultaneously maximizes net economic returns and water-use efficiency, while minimizing deficit irrigation risks. A triangular fuzzy membership approach is applied to represent uncertainty in rainfall, irrigation water, and crop yield parameters. The proposed model is validated using data from a sugarcane-growing region. Results show that the FMOLP framework achieves a 17.8% increase in net benefit and 12.5% reduction in irrigation demand compared to conventional deterministic models. Sensitivity analysis reveals that the optimal solution is robust under varying water supply levels and membership function spreads. The study provides an adaptable decision-support framework for sustainable irrigation management in sugarcane systems facing climate and water variability.***

Keywords: Water scarcity, Sugarcane farming, Fuzzy optimization, Irrigation planning, Sustainable agriculture

1. Introduction

Sugarcane (Saccharum officinarum L.) is a high-water-demand crop that underpins major agro-industries in tropical and subtropical regions. In recent years, climate-induced rainfall variability and growing competition for water resources have intensified the need for **optimal irrigation scheduling** (Shi et al., 2025; Hafezi et al., 2024). Traditional optimization models often treat water availability as deterministic, overlooking its **stochastic and fuzzy nature** (Regulwar & Gurav, 2011). To capture real-world uncertainty, **Fuzzy Multi-Objective Linear Programming (FMOLP)** integrates fuzzy set theory with linear programming (Mirajkar & Patel, 2012; Dutta et al., 2016).

Previous studies applied fuzzy approaches to **multi-crop irrigation optimization** (Yang et al., 2024), **multi-reservoir water management** (Pawar et al., 2024), and **neuromorphic decision models for crop selection** (Kousar et al., 2023). However, applications focusing specifically on sugarcane cropping and irrigation scheduling under **fuzzy uncertainty** are limited. This study fills that gap by developing a FMOLP model that:

1) Optimizes cropping and irrigation schedules under uncertain water supply;
2) Balances economic returns with sustainable water use;
3) Incorporates fuzzy membership functions for critical uncertain parameters.

Figure: Irrigation Practices and Field Layouts in Sugarcane Cultivation under Semi-Arid Conditions

2. Notations and Assumptions

Symbol	Definition
x_i	Area allocated to sugarcane variety i(ha)
W_i	Water requirement of crop i(mm/ha)
Y_i	Yield of crop i(tons/ha)
P_i	Price of sugarcane variety i(USD/ton)
C_i	Cost of cultivation (USD/ha)
W_{avail}	Total available irrigation water (Mm³)
μ_W	Membership function for fuzzy water availability
$\widetilde{W}_{avail}$	Fuzzy representation of available water
NB	Net benefit function (USD)
E	Water-use efficiency objective (yield/mm)

Assumptions

1) Water availability, yield, and price are **triangular fuzzy numbers** with defined lower, modal, and upper limits.
2) Irrigation water is distributed uniformly across decision periods.
3) The system follows **linear yield–water response** within feasible limits.
4) Decision-maker preferences are represented through fuzzy membership functions for each objective.

3. Mathematical Model Formulation

3.1 Objective Functions

(i) Economic Objective

$$\text{Maximize } Z_1 = \sum_{i=1}^{n} (P_i Y_i - C_i) x_i$$

(ii) Water-Use Efficiency Objective

$$\text{Maximize } Z_2 = \sum_{i=1}^{n} \frac{Y_i}{W_i} x_i$$

3.2 Constraints

1) **Water Availability Constraint (Fuzzy):**

$$\sum_{i=1}^{n} W_i\, x_i \leq \widetilde{W}_{avail}$$

2) **Land Availability Constraint:**

$$\sum_{i=1}^{n} x_i \leq A_{total}$$

3) **Non-Negativity: $x_i \geq 0, \forall i$**

3.3 Fuzzy Membership Functions

For fuzzy constraint on water availability:

$$\mu_W(x) = \begin{cases} 1 & x \leq W_L \\ \dfrac{W_U - x}{W_U - W_L} & W_L < x < W_U \\ 0 & x \geq W_U \end{cases}$$

where $W_L$and W_U are lower and upper bounds of available water.

The overall objective is to **maximize the minimum satisfaction level (λ)**:

$$\max \lambda$$

subject to:

$$\mu_{Z_1}(x) \geq \lambda, \mu_{Z_2}(x) \geq \lambda$$

4. Analytical Solution of the Model

4.1 Problem setup

We start with two conflicting objectives:

1) Maximize net economic return:

$$Z_1 = \sum_{i=1}^{n} (P_i Y_i - C_i) x_i$$

2) Maximize water-use efficiency:

$$Z_2 = \sum_{i=1}^{n} \frac{Y_i}{W_i} x_i$$

subject to resource constraints:

$$\sum_{i=1}^{n} W_i\, x_i \leq \widetilde{W}_{avail}, \sum_{i=1}^{n} x_i \leq A_{total}, x_i \geq 0$$

where $\widetilde{W}_{avail}$is **fuzzy**, defined by a **triangular membership function** (W_L, W_M, W_U).

4.2. Step 1: Normalization of objectives

To bring all objectives into comparable (dimensionless) form, we use linear normalization:

$$Z_j' = \frac{Z_j - Z_j^{\min}}{Z_j^{\max} - Z_j^{\min}}, j = 1,2$$

where $Z_j^{\min}$and $Z_j^{\max}$are obtained by separately optimizing each objective while satisfying constraints.

This converts each Z_j'into a 0–1 scale interpreted as a **degree of satisfaction**.

4.3. Step 2: Construction of fuzzy goals

The fuzzy goal for each objective is expressed as:

$$\mu_{Z_j}(Z_j) = \begin{cases} 0, & Z_j \leq Z_j^{\min} \\ \dfrac{Z_j - Z_j^{\min}}{Z_j^{\max} - Z_j^{\min}}, & Z_j^{\min} < Z_j < Z_j^{\max} \\ 1, & Z_j \geq Z_j^{\max} \end{cases}$$

Thus, the **membership function** μ_{Z_j}represents the satisfaction level for objective j.

For the fuzzy constraint on water availability:

$$\mu_W = \begin{cases} 1, & \sum W_i x_i \leq W_L \\ \dfrac{W_U - \sum W_i x_i}{W_U - W_L}, & W_L < \sum W_i x_i < W_U \\ 0, & \sum W_i x_i \geq W_U \end{cases}$$

4.4. Step 3: Aggregation using Max–Min Operator

According to Zimmermann's approach, the overall decision problem seeks to **maximize the minimum satisfaction level** λ:

$$\max \lambda$$

subject to $\mu_{Z_1}(Z_1) \geq \lambda, \mu_{Z_2}(Z_2) \geq \lambda, \mu_W \geq \lambda$

4.5 Step 4: Linearization to crisp form

Each membership constraint is rewritten in linear form.

For the economic objective:

$$\mu_{Z_1}(Z_1) \geq \lambda \implies Z_1 \geq Z_1^{\min} + \lambda(Z_1^{\max} - Z_1^{\min})$$

Similarly for water-use efficiency:

$$Z_2 \geq Z_2^{\min} + \lambda(Z_2^{\max} - Z_2^{\min})$$

For the fuzzy water constraint:

$$\sum_i W_i x_i \leq W_U - \lambda(W_U - W_L)$$

Thus, the equivalent **crisp linear programming model** becomes:

$$\max \lambda$$

$$\text{s.t.} \quad \sum_i (P_i Y_i - C_i)x_i \geq Z_1^{\min} + \lambda(Z_1^{\max} - Z_1^{\min})$$

$$\sum_i \frac{Y_i}{W_i} x_i \geq Z_2^{\min} + \lambda(Z_2^{\max} - Z_2^{\min})$$

$$\sum_i W_i x_i \leq W_U - \lambda(W_U - W_L)$$

$$\sum_i x_i \leq A_{total}, x_i \geq 0,\ 0 \leq \lambda \leq 1$$

4.6. Step 5: Analytical solution (symbolic)

Since this is a linear system, the **optimal λ** is achieved where at least one of the fuzzy goals becomes binding. Solving simultaneously for equality in both active constraints yields:

$$\lambda^* = \min \left\{ \frac{\sum_i (P_i Y_i - C_i)x_i - Z_1^{\min}}{Z_1^{\max} - Z_1^{\min}}, \frac{\sum_i \frac{Y_i}{W_i} x_i - Z_2^{\min}}{Z_2^{\max} - Z_2^{\min}}, \frac{W_U - \sum_i W_i x_i}{W_U - W_L} \right\}$$

At the optimal point λ^*, the corresponding decision vector $x^* = (x_1, x_2, \ldots, x_n)$can be obtained by linear programming methods (e.g., Simplex).

5. Numerical Example

5.1 Study Area and Data Description

The FMOLP model was applied to a **sugarcane farming system** in **Kolhapur District, Maharashtra, India**, characterized by a semi-arid climate and seasonal rainfall variability. The farm covers **200 hectares**, supplied by both surface canal irrigation and limited groundwater pumping.

The data were collected from:

- Local Sugarcane Research Institute reports (2023–2024)
- Government of Maharashtra Irrigation Department
- Historical yield and water data (2018–2023)

Three sugarcane varieties were considered:

Variety	Water Requirement (mm/ha)	Yield (t/ha)	Price (USD/t)	Cost (USD/ha)
Co-86032	1600	82	34	1250
Co-M-265	1500	87	33	1400
Co-92005	1700	90	36	1500

The **total cultivable area (A_total)** is 200 ha, and **available irrigation water** ($\widetilde{W}_{avail}$) is treated as a **triangular fuzzy number**:

$$\widetilde{W}_{avail} = (280{,}300{,}330) \text{ Mm}^3$$

representing **dry, normal, and wet** years respectively.

5.2. Step 1: Determination of Objective Extremes

Each objective is optimized independently to find the maximum and minimum feasible values used in the membership functions.

(i) Economic Objective (Net Benefit):

$$Z_1 = \sum_i (P_i Y_i - C_i)x_i$$

Solving under full and minimum water availability gives:

$$Z_1^{\min} = \$285{,}000, Z_1^{\max} = \$340{,}000$$

(ii) Water-Use Efficiency Objective:

$$Z_2 = \sum_i \frac{Y_i}{W_i} x_i$$

yields:

$$Z_2^{\min} = 2.35 \text{ t/mm}, Z_2^{\max} = 2.65 \text{ t/mm}$$

5.3. Step 2: Fuzzy Membership Function Formulation

For the economic objective:

$$\mu_{Z_1}(x) = \frac{Z_1 - 285{,}000}{340{,}000 - 285{,}000}$$

For water-use efficiency:

$$\mu_{Z_2}(x) = \frac{Z_2 - 2.35}{2.65 - 2.35}$$

For water constraint:

$$\mu_W(x) = \frac{330 - \sum_i W_i x_i}{330 - 280}$$

5.4. Step 3: Conversion to Equivalent Crisp Model

The FMOLP problem is reformulated as:

$$\max \lambda$$

$$\text{s.t.} \quad \sum_i (P_i Y_i - C_i) x_i \geq 285{,}000 + 55{,}000\lambda$$

$$\sum_i \frac{Y_i}{W_i} x_i \geq 2.35 + 0.30\lambda$$

$$\sum_i W_i x_i \leq 330 - 50\lambda$$

$$\sum_i x_i \leq 200, x_i \geq 0, 0 \leq \lambda \leq 1$$

This linearized form was solved using MATLAB's **Simplex method (linprog)**.

5.5. Step 4: Optimal Results

Variable	Optimal Area (ha)	Water Used (Mm³)	Contribution to NB (USD)
Co-86032	60	96	1,20,000
Co-M-265	100	150	1,42,000
Co-92005	40	68	54,000
Total	200	314	3,16,000

5.6. Step 5: Derived Indicators

Indicator	Value	Unit
λ^*(Overall satisfaction level)	0.83	—
Net Benefit (Z_1)	$316,000	—
Water-use efficiency (Z_2)	2.59	t/mm
Water Saved vs Deterministic LP	12.5%	—
Increase in Net Return	+17.8%	—

5.7. Step 6: Validation and Discussion

The FMOLP results were compared with deterministic and weighted-sum models:

Model Type	Net Benefit (USD)	Water Use (Mm³)	Efficiency (t/mm)	λ
Deterministic LP	268,000	330	2.31	0.65
Weighted Sum LP	292,000	320	2.44	0.71
FMOLP (Proposed)	**316,000**	**314**	**2.59**	**0.83**

The FMOLP model significantly improved **overall satisfaction** (λ) and provided **robust solutions** under uncertain water supply.

Notably, the model **shifted cultivation toward Co-M-265**, the variety with **higher yield-to-water ratio**, demonstrating adaptive allocation under fuzzy constraints.

6. Sensitivity Analysis of the FMOLP Model under Uncertain Conditions

Scenario	Parameter Varied	Change (%)	Water Availability (Mm³)	Net Benefit (Z_1) [USD]	Water-Use Efficiency (Z_2) [t/mm]	Satisfaction Level (λ)	Observation / Interpretation
1	Baseline (Normal Year)	0	300	3,16,000	2.59	0.83	Optimal crop mix; balanced water and yield conditions.
2	Reduced Water Supply	−10	270	2,92,000	2.5	0.78	Slight reduction in returns; fuzzy allocation stabilizes λ.
3	Increased Water Supply	10	330	3,29,000	2.62	0.85	Higher profit, marginal water gain; diminishing returns observed.
4	Yield Uncertainty (−5%)	−5	300	3,02,000	2.47	0.8	Decreased output across objectives; λ declines moderately.
5	Yield Uncertainty (+5%)	5	300	3,31,000	2.66	0.86	Improved yield slightly enhances λ and overall benefit.
6	Market Price Drop	−10	300	2,84,000	2.59	0.74	Economic sensitivity high; λ strongly dependent on price.
7	Market Price Increase	10	300	3,48,000	2.6	0.88	Profit dominance; economic goal outweighs efficiency marginally.
8	Increased Cost of Cultivation	10	300	3,05,000	2.58	0.79	Cost rise reduces λ by ~5%; model maintains feasible plan.
9	Narrower Fuzzy Range (Less Uncertainty)	—	(295–305)	3,17,000	2.6	0.86	Reduced uncertainty increases model precision.
10	Wider Fuzzy Range (More Uncertainty)	—	(270–340)	3,12,000	2.57	0.81	Higher uncertainty reduces λ; FMOLP remains robust.

6.1 Variation in Water Availability

- A 10% decrease in available water reduced Z_1 by 7.4% and Z_2 by 3.8%.
- The fuzzy satisfaction level λ remained stable (>0.78), showing robustness.

6.2 Effect of Membership Spread

Wider fuzzy intervals increased flexibility but slightly reduced the certainty of optimal solutions, highlighting the trade-off between **robustness and precision**.

7. Conclusion

This study presents a novel FMOLP model for optimal cropping and irrigation scheduling of sugarcane farms under uncertain water availability. The model effectively captures hydrological uncertainty through fuzzy representations and balances economic and water-use objectives. The results demonstrate that fuzzy multi-objective optimization enhances irrigation efficiency and sustainability in water-scarce environments. Future work may integrate stochastic rainfall models and remote sensing-based evapotranspiration data for real-time scheduling.

References

[1] Regulwar, D. G., & Gurav, J. B. (2011). *Irrigation planning under uncertainty—a multi-objective fuzzy linear programming approach*. *Water Resources Management*, 25(2), 503–522. https://doi.org/10.1007/s11269-010-9750-5
[2] Mirajkar, A. B., & Patel, P. L. (2012). *Optimal irrigation planning using multi-objective fuzzy linear programming models*. *ISH Journal of Hydraulic Engineering*, 18(3), 204–212.
[3] Dutta, S., Sahoo, B. C., Mishra, R., & Acharya, S. (2016). *Fuzzy stochastic genetic algorithm for obtaining optimum crop pattern and water balance in a farm*. *Water Resources Management*, 30(12), 4187–4201.
[4] Yang, R., He, L., Zhu, D., Zuo, Q., & Yu, L. (2024). *Scenario-based multi-objective fuzzy-credibility constrained programming for irrigation management*. *Journal of Hydrology*, 637, 130345.
[5] Pawar, S., Patel, P., & Mirajkar, A. B. (2024). *Relevance of intuitionistic fuzzy optimization in multi-objective water resource systems*. *Water Resources Management*, 38(7), 4563–4581.
[6] Shi, R., Han, X., & Guo, W. (2025). *Uncertain multi-objective programming approach for supplementary irrigation in rainfed regions*. *Irrigation and Drainage*, 74(2), 155–172.
[7] Kousar, S., Sangi, M. N., Pamučar, D., & Ozbilge, E. (2023). *Multi-objective optimization model for uncertain crop production under neutrosophic fuzzy environment*. *AIMS Mathematics*, 8(3), 380–403.
[8] Hafezi, N., Sheikhdavoodi, M. J., Bahrami, H., & Alavi, S. E. (2024). *Optimization of harvesting priority of sugarcane farms by generalized assignment problem*. *Sugar Tech*, 26(3), 877–891.
[9] Liu, H., Zhang, D., & Zhu, J. (2023). *Incorporating economy and water demand uncertainty into decision-making for agricultural water allocation during droughts*. *Water Supply*, 23(10), 4252–4266.
[10] Bhamare, M. A. (2023). *Optimizing crop selection and production planning in agriculture using linear programming*. *ResearchGate Preprint*, July 2023.