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Analysis of an Active Electrical Network Using Signal Flow Graphs and Mason's Gain Formula

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Abstract: ***The transfer function of an electrical network can be determined using various techniques, such as nodal analysis, Kirchhoff's laws, and the time constant method. Another method for algebraically determining the transfer function is to use a Signal Flow Graph (SFG). An SFG is a directed, weighted graph consisting of nodes and branches. We will create a signal-flow graph of an active electrical network and analyze it using Mason's Gain Formula and the reduction rules of signal-flow graphs.***

Keywords: Transfer function, Mason's Formula, Signal Flow Graph, Active Electrical Network

1. Introduction

Mathematics plays a crucial role across disciplines, and one of the most significant and fascinating areas is graph theory [1, 2, 3]. Graph theory is considered one of the most important branches of mathematics, with applications in computing, social sciences, and natural sciences. By applying graph theory, many mathematical problems can be formulated as models that help solve real-world issues across disciplines. Additionally, graph theory enables the formulation of a range of optimization problems in physics, chemistry, computer science, and electrical engineering. It also has strong connections with other branches of mathematics, including group theory, matrix theory, and topology.

A signal flow graph (SFG)[4] is a directed, weighted graph where signals are applied to nodes and functions are applied to edges to represent the flow of signals [5] within a system. The transfer function of electrical circuits can be calculated using various techniques, including Kirchhoff's rules [6], two-port network theory [7], nodal analysis [8], and the time constant method [9]. However, these methods can be time-consuming and computationally intensive. In contrast, SFG analysis provides a faster and more effective alternative for analysing complex structures. For network analysis, the SFG is a valuable method. Understanding a circuit fundamentally relies on its physical structure [3,12,13,20]. Because SFG is a clear and straightforward method, it requires minimal materials, thereby reducing costs. However, the signal flow graph method [10,11] remains underdeveloped.

The Signal Flow Graph (SFG) method was developed by J.A. Brzozowski and E.J. McCluskey Jr. in 1963 to characterize sequential circuits using regular expressions [14]. It has been shown that state diagrams of sequential circuits can be accurately interpreted using methodologies derived from signal flow graph theory. After 1990, Dalibor Biolek and Viera Biolkova used signal-flow graphs to explain linear circuits. In their 2017 study [15,16], Feim Ridvan Rasim and Sebastian M. Sattler evaluated the reliability of the SFG method [17-18].

1.1 Electrical Network

An electrical network is an interconnection of electrical components. An electrical circuit is a specific type of network [14,15,16] that forms a closed loop. Electrical networks can be classified into different types based on their behavior and characteristics.

- **Linear Network:** A circuit or network consisting of electrical elements such as resistors, inductors, and capacitors, in which these components remain constant over time regardless of changes in voltage, temperature, or other factors, is referred to as a linear network. In a linear network, Ohm's law and the superposition theorem can be applied to solve mathematical equations.
- **Nonlinear Network:** A circuit with parameters that change over time, temperature, voltage, etc., is called a nonlinear network. Ohm's law and the superposition theorem do not apply to this network.
- **Active Network:** A network that contains one or more energy sources, such as voltage and current sources, is known as an active network.
- **Passive Network:** A circuit without an energy source is called a passive network. It consists of passive components such as resistors, capacitors, and inductors.
- **A circuit whose properties remain unchanged regardless of the direction of the applied** voltage or current is called a bilateral network. A good example of a bilateral network consists solely of resistors and transmission lines.
- **Unilateral Network:** A unilateral network is a circuit whose properties depend on the direction of the applied voltage or current. A diode rectifier serves as the best example of a unilateral network.

2. Preliminary (Basic definitions)

Definition 2.1. Circuit: A circuit is a closed path that begins and ends at the same vertex.

Definition 2.2: Graph: A graph is defined as a set of ordered pairs G (V, E), where V represents the set of vertices (or nodes) and E represents the set of edges. In this context, the elements of V are called vertices, and those of E are called edges.

Definition 2.3 Walk: A walk is a sequence of vertices and edges, where each edge connects the adjacent vertices.

Definition 2.4 Path: A path is a simple graph where the vertices can be arranged such that two vertices are adjacent if and only if they are consecutive in this arrangement.

Definition 2.5 Loop: A loop is a closed path that starts and ends at the same vertex, without visiting any node more than once.

Definition 2.6 Self-Loop: Let G be a graph. A self-loop is an edge that connects a vertex to itself.

Definition 2.7 Gain: The transmittance of the load between two nodes of an edge is known as gain.

Definition 2.8 Loop gain: The gains achieved by a path starting and ending at the same node are called the loop gain.

Definition 2.9 Non-Touching Loops: Non-touching loops are defined as loops that do not share any nodes.

Definition 2.10 Forward Path: A forward path is a sequence that starts with an initial node, includes another node, and concludes at an output node.

Definition 2.11 Feedback path: A feedback path is a path that starts and finishes at the same node.

Definition 2.12 Forwarded path gain: The forwarded path gain is the sum of gains obtained by tracing a path from an input node to an output node in a signal flow graph.

Definition 2.13: Directed Graph (Digraph): A directed graph, also known as a digraph, is a type of graph in which each edge has a direction.

Definition 2.14: Transfer Function: The transfer function of a system, denoted as T(S), is represented as C(S)/R(S). Here, R(S) refers to the system's input, illustrated by a single flow graph, while C(S) represents the system's output, depicted in a signal flow graph.

3. Methodology

3.1. Mason's Gain Formula (MGF)

The Mason's Gain Formula (MGF) is used to calculate the transfer function of a linear signal flow graph. In this context, let R(S) represent the system's input, while C(S) denotes the system's output, as illustrated in the signal flow graph.

The system's transfer function is given by $T(S) = \frac{C(S)}{R(S)}$

Overall gain $T = T(S) = \sum_{k=1}^{N} \frac{P_k \Delta_k}{\Delta}$

Where,
N: Number of paths from input to output
P_k: Forward path gain of the k[th] forward path
Δ: 1 - (Sum of individual loop gains) + (sum of two non touching loops gain)
- (sum of three non-touching loops gain) + (sum of four non-touching loops gain)
- ……..
Δ_k = 1 - (loops gain which are not touching the k[th] forward path)

3.2 Signal Flow Graph

A Signal Flow Graph (SFG) is a directed, weighted graph that represents the signal flow of a system. It provides a graphical representation of the set.

3.2.1. Reduction Rule for Signal Flow Graphs (SFG): Modifications of Signal Flow Graphs

Rule 1:

(a)

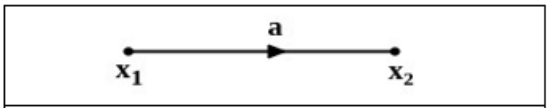


Figure 1

If x_1 is input and x_2 is output and a is gain. Then $x_2 = ax_1$

(b)

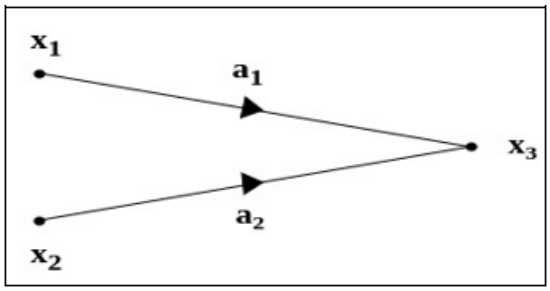


Figure 2

If x_1 and x_2 are input signals with gains a_1 and a_2 respectively, then the output signal x_3 can be written as $x_3 = a_1x_1 + a_2x_2$

Rule 2:

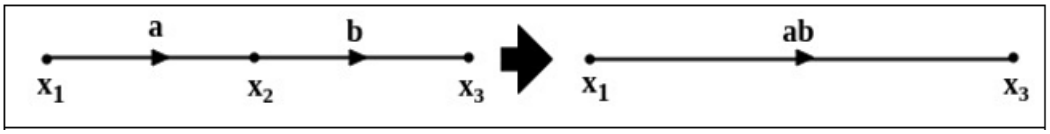


Figure 3

If x_1, x_2 & x_3 are nodes where x_1 is input and x_3 is output with gains a and b. After taken out x_2 We have, $x_2 = ax_1$, $x_3 = bx_2 = abx_1$

Rule 3:

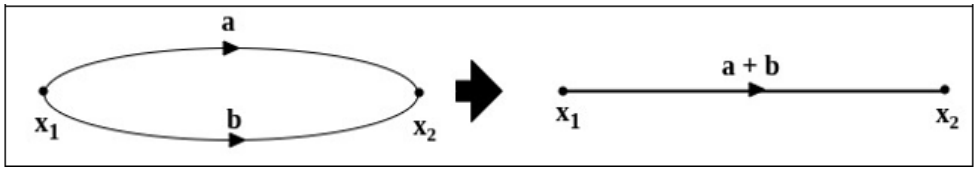


Figure 4

If x_1 is input and x_2 is output with gains a and b of respective branches, then the output

$$x_2 = (a + b)x_1 = ax_1 + bx_1$$

Rule 4:

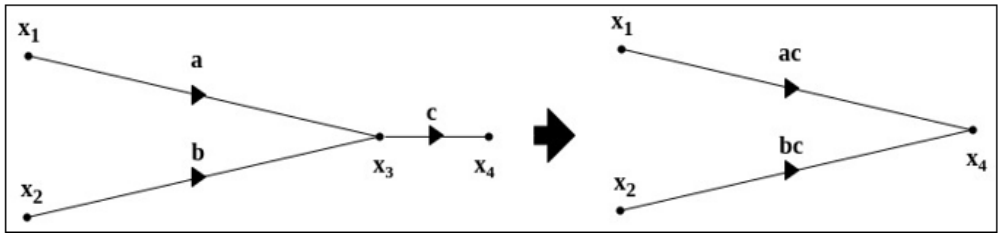


Figure 5

If x_1, x_2, x_3 *and* x_4 The nodes a, b, and c are branch gains. Then $x_3 = ax_1 + bx_2$ and $x_4 = cx_3 = c(ax_1 + bx_2)$. There fore $x_4 = acx_1 + bcx_2$

Rule 5:

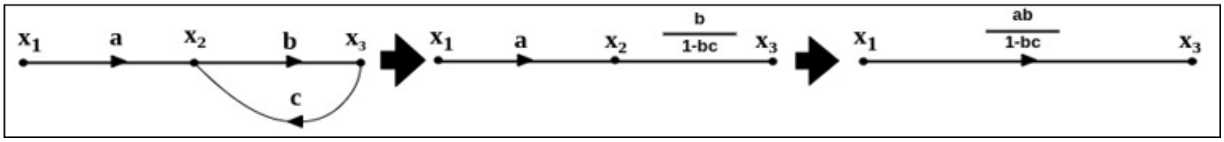


Figure 6

If x_1, x_2 and x_3 are nodes with a, b, and c are gains of branches as shown in the figure, then

$x_2 = ax_1$ and $x_3 = \left(\frac{b}{1-bc}\right)x_2 = \left(\frac{b}{1-bc}\right)ax_1 = \frac{ab}{1-bc}x_1$

Rule6:

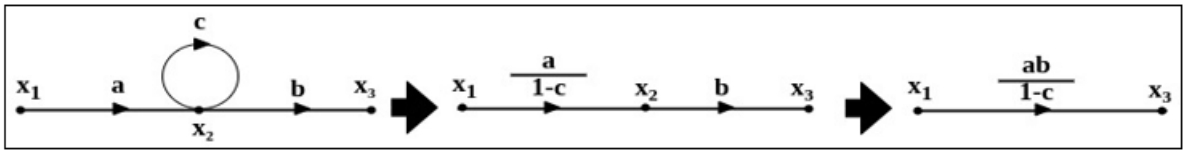


Figure 7

If x_1, x_2 and x_3 are nodes with a, b(reflexive or self-referential), and c gains of branches as shown in the figure, then $x_2 = \frac{a}{1-c}x_1$ and $x_3 = bx_2 = b\frac{a}{1-c}x_1 = \frac{ab}{1-c}x_1$

Rule 7: Loop reduction:

(a) **Signal loop**: The transmission (T_{ij}) from an independent variable x_i (source node) to a dependent variable x_j (internal or sink node) In a signal flow graph containing only one loop and one path, equals

$$T_{ij} = \frac{P_{ij}}{1-L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where P_{ij} is the forward path transmission from x_i and x_j And L is the transmission of the loop.

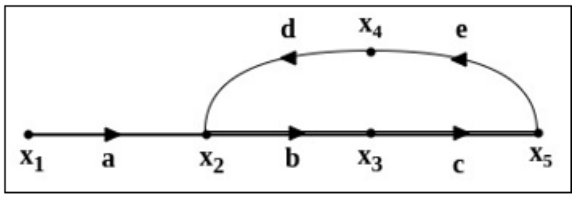


Figure 8

Here, x_1, x_2, x_3, x_5 *and* x_4 are nodes with a, b, c, e, and d are gains of branches as shown in the figure, then we have as follows:

$$T_{15} = \frac{x_5}{x_1} = \frac{P_{15}}{1-L} = \frac{abc}{1-bcde}$$

Where $P_{15}(forward\ path) = abc$, L (loop transmission) = bcde

(b) Multi-loop with non-touching loops: With a cascade of non-touching (loops possessing no common nodes), the

overall transmission is the product of the individual transmissions as given by equation (1) (single loop)

$$T_{in} = \frac{P_{ij}}{1-L_j} \cdot \frac{P_{jk}}{1-L_k} \dots\dots\dots\dots\dots\dots \cdot \frac{P_{(n-1)n}}{1-L_n} \quad \dots\dots (2)$$

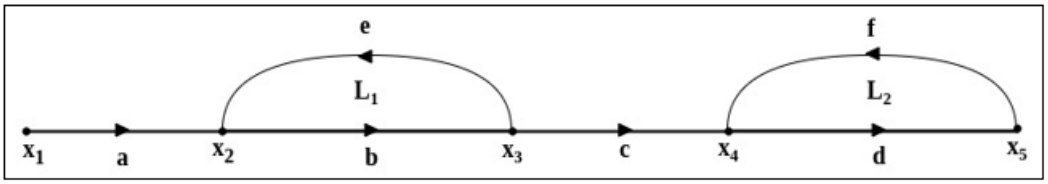


Figure 9

Here, x_1, x_2, x_3, x_4 $and x_5$ are nodes with a, b, c, d, e, and f are gains of branches as shown in the figure, then according to equation (2), the transfer function between x_1 and x_5 we have as follows:

$$T_{15} = \frac{x_5}{x_1} = \frac{P_{13}}{1-L_1}\frac{P_{35}}{1-L_2} = \frac{P_{15}}{1-L_1-L_2+L_1L_2} = \frac{abcd}{1-bc-df-bedf} \quad \dots\dots\dots\dots\dots (3)$$

Where $P_{15}(forward\ path) = abcd$, L_1(Loop transmission) = be, L_2 (Loop transmission) = df

(c) **Multiloop with touching loops:**
For the case that the two loops do share at least one node, as shown in the figure below, the overall transmission is given by

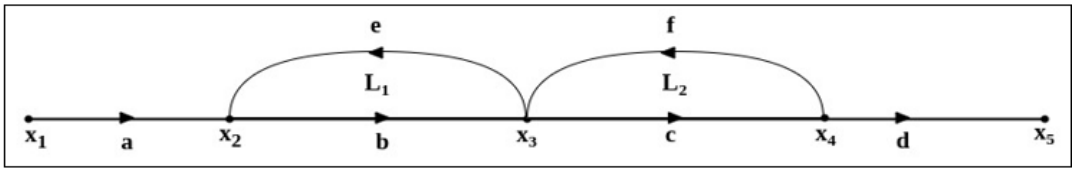


Figure 10

Here, x_1, x_2, x_3, x_4 $and x_5$ Are nodes with a, b, c, d, e, and f gains of branches as shown in the figure, then the transfer function between x_1 $and x_5$we have as follows:

$$T_{15} = \frac{x_5}{x_1} = \frac{abcd}{1-be-cf} = \frac{P_{15}}{1-L_1-L_2} \quad \dots\dots\dots\dots\dots (4)$$

Comparing equation (4) with equation (3) reveals that the cross term L_1L_2 is absent in this case because the two loops in the signal-flow graph intersect at a common node. In fact, determining whether the loops in a multi-loop system share common nodes is crucial for accurately evaluating the corresponding signal flow graphs.

4. An Illustration of an Electrical Network

Graphs can effectively represent electrical networks, where each network node corresponds to a vertex, and each branch corresponds to an edge. Solving a network to determine loop currents and node voltages is straightforward for simple circuits. However, as the circuit complexity increases, we must adopt alternative methods. Using a graph to describe an electrical network [19] greatly simplifies the problem-solving process. Understanding the network graph is essential for analysing the circuit. Below is an illustration of the signal flow graph for an electrical network:

4.1 Active Network with Voltage Source and Current Source

Let us consider the following Active electrical circuit to analyze by calculating the transfer function using Signal Flow Graph (SFG**).**

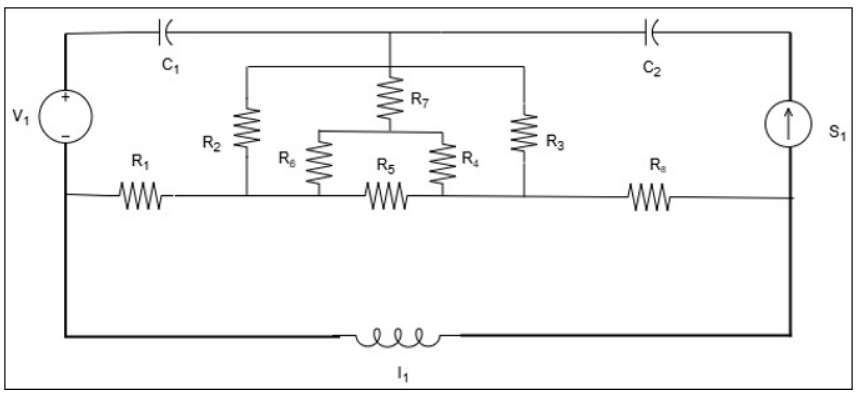


Figure 11

4.1.1 A graph model representation

A graph model represents a network using vertices and edges that cover the entire network. Below is the graph model of the previously displayed circuit.

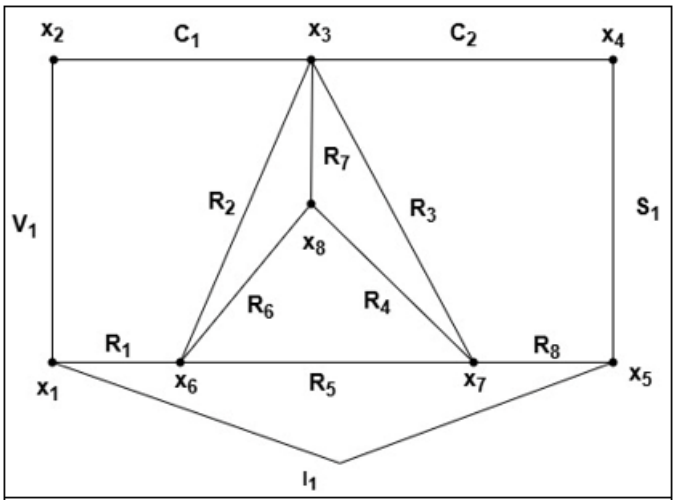


Figure 12

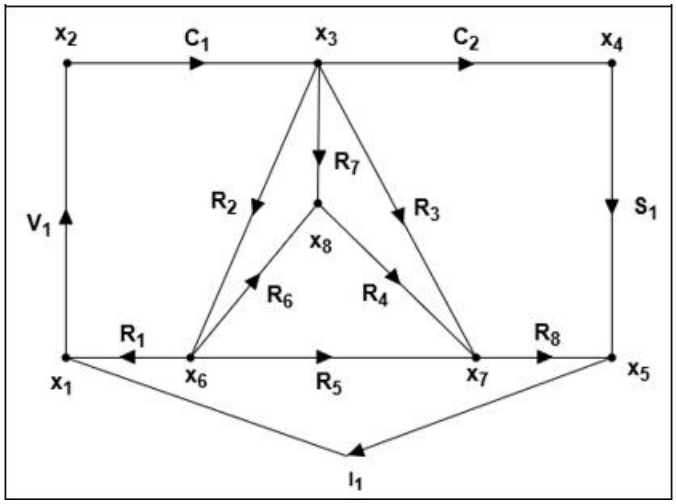


Figure 13

4.1.2 A Digraph model representation: A digraph model represents a graph using vertices and directed weighted edges that cover the entire network. Below is the digraph model of the circuit shown above.

4.1.3 Signal Flow Graph (SFG) model representation: A signal flow graph (SFG) model is used to represent a directed graph, utilizing vertices along with a directed and weighted graph that encompasses the entire network. Below is the SFG model of the network shown above.

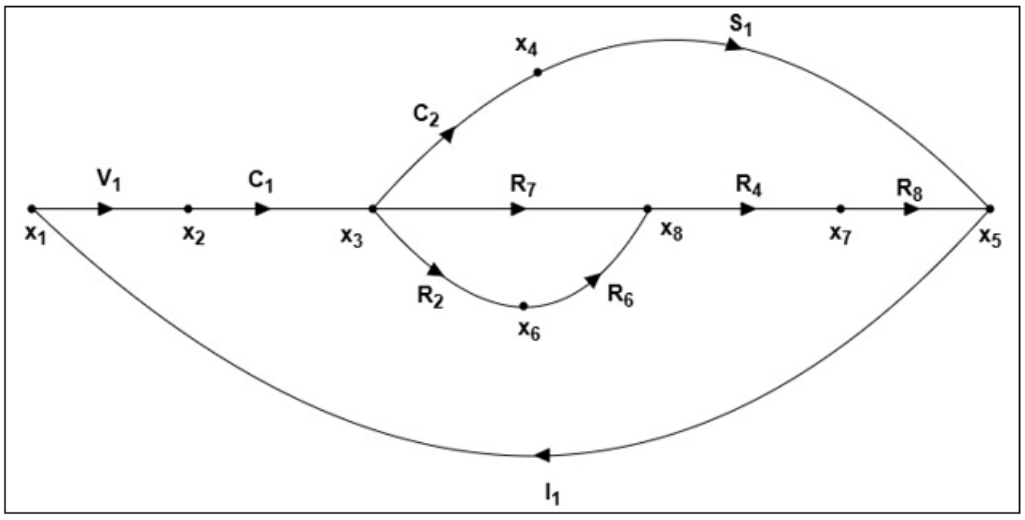


Figure 14

4.1.4 Calculation of Transfer Function by Reduction of the Signal Flow Graph:

Taken out the vertices x_2, x_4, x_6, and x_7 by using rule 2, we have

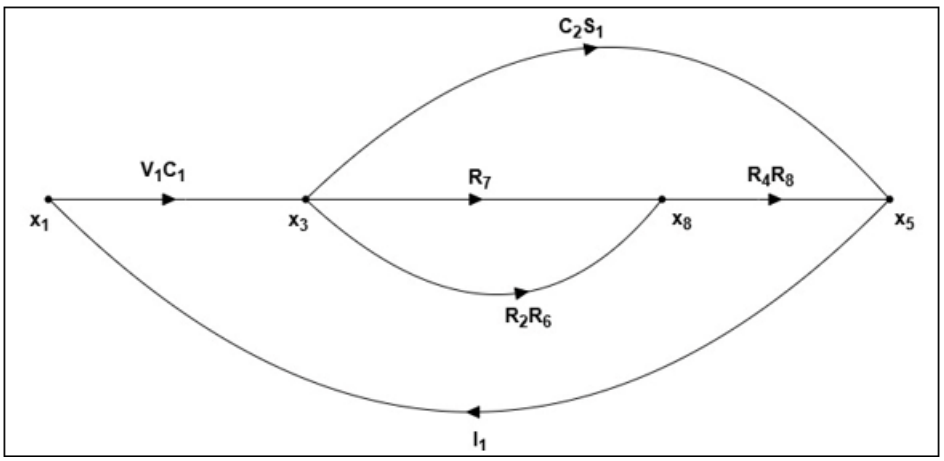


Figure 15

By using rule 3 between the vertices x_3 and x_8 we have

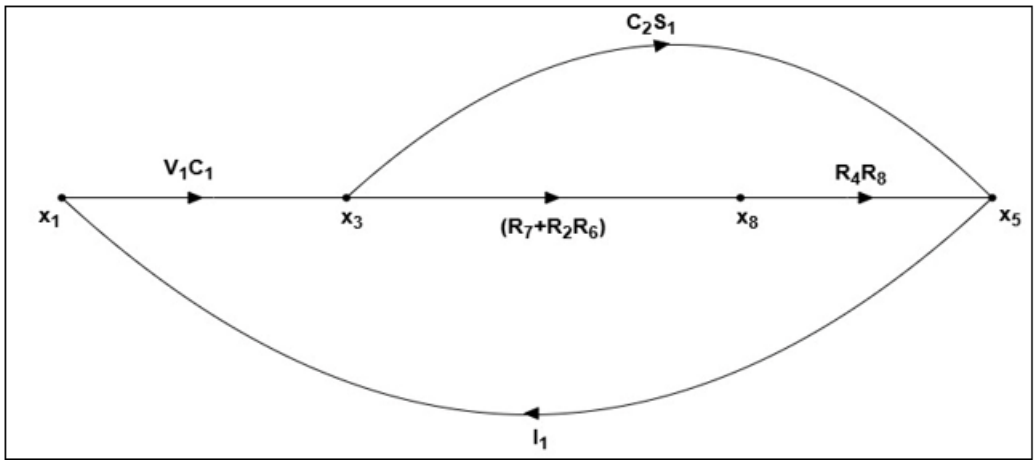


Figure 16

Taken out the vertex x_8 by using rule 2, we have

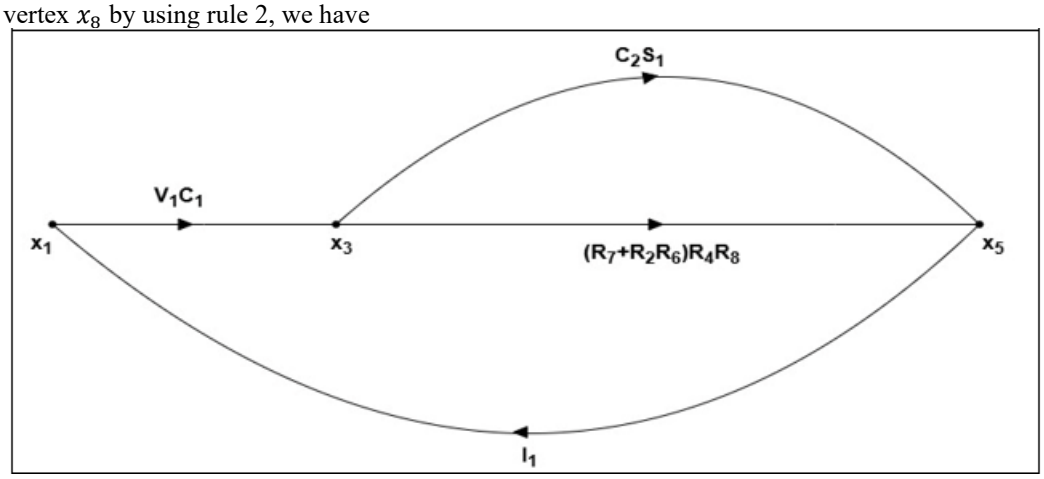


Figure 17

By using rule 3 between the vertices x_3 and x_5 we have

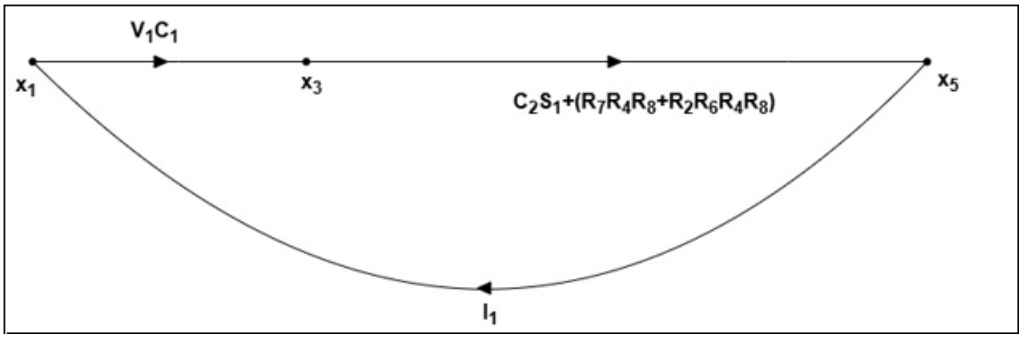


Figure 18

Taken out the vertex x_3 by using rule 2, we have

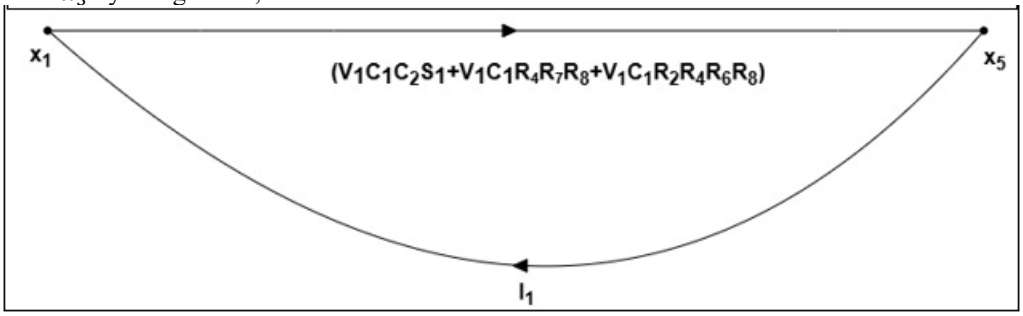


Figure 19

By using rule 3 between the vertices x_1 and x_5 we have

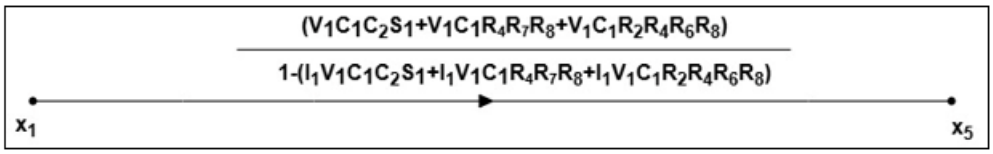


Figure 20

$T_{15} = \left\{\frac{V_1C_1C_2S_1+V_1C_1R_4R_7R_8+V_1C_1R_2R_4R_6R_8}{1-(I_1V_1C_1C_2S_1+I_1V_1C_1R_4R_7R_8+I_1V_1C_1R_2R_4R_6R_8)}\right\}$ ……. (A)

4.2 Calculation of Transfer function by using Mason's gain formula from Signal Flow Graph (SFG) representation(D):

Alternatively, from Fig.14, we have to calculate the transfer function between nodes $x_1$and x_5 By using Mason's gain formula.

We know Mason's formula for the Transfer function, $T(S) = \frac{1}{\Delta}\sum_k F_k\Delta_k$

Where T(S) = Transfer function of the system
F_k= Forward path gain of k[th] forward path
Δ= 1- (sum of individual loops gains) + (sum of two non-touching loops gain) – (sum of 3 non-touching loops gain) +…….
Δ_K=1 – (loop gain, which is not touching the k[th] forward path)

From **Fig.17,** we have in this particular problem, there are three forward paths, and they are as follows:

$$F_1 = V_1C_1R_7R_4R_8 = V_1C_1R_4R_7R_8$$
$$F_2 = V_1C_1R_2R_6R_4R_8 = V_1C_1R_2R_4R_6R_8$$
$$F_3 = V_1C_1C_2S_1$$

Secondly, here there are three individual loops, and their gains are as follows:
1[st] loop gain $L_1 = I_1V_1C_1C_2S_1$
2[nd] loop gain $L_2 = I_1V_1C_1R_7R_4R_8$
3[rd] loop gain $L_3 = I_1V_1C_1R_2R_6R_4R_8$

Thirdly, there are no non-touching loops in this problem.
Therefore, $\Delta = 1 - (I_1V_1C_1C_2S_1 + I_1V_1C_1R_7R_4R_8 + I_1V_1C_1R_2R_6R_4R_8)$

Now, $\Delta_1 = 1 - 0 = 1$
$\Delta_2 = 1 - 0 = 1$
$\Delta_3 = 1 - 0 = 1$

Therefore, $T_{15} = \frac{F_1\Delta_1+F_2\Delta_2+F_3\Delta_3}{\Delta}$

$$= \frac{V_1C_1R_4R_7R_8+V_1C_1R_2R_4R_6R_8+V_1C_1C_2S_1}{1-(I_1V_1C_1C_2S_1+I_1V_1C_1R_7R_4R_8+I_1V_1C_1R_2R_6R_4R_8)} \text{……(B)}$$

We have observed that equations (A) and (B) are equal.

Observation: We have observed that from the same electrical circuit, we can calculate the transfer function, i.e., transmittance between any two nodes by using the method of reduction rules of Signal Flow Graph and Mason's gain formula. Here in this paper, from the same Electrical circuit, i.e., from Fig. 11, we can find the same transfer function between two nodes x_1 and x_5 by using the reduction of the signal flow graph, i.e., equation (A), and by using Mason's gain formula, i.e., equation (B), where both these two equations are equal.

5. Conclusion

In an electrical system, the Transfer Function (TF) is the ratio of output to input. The Transfer Function (TF) provides the circuit's gain, its frequency response, stability, the range of input values for which the output remains stable, and its behavior under different types of input (AC or DC). It also specifies key features. We have determined the Transfer Function (TF) of a system (circuit) and can use it to determine many of its parameters. One can use this Transfer Function (TF) to analyze its behavior for any input. Another thing is that they allow the whole system to be represented by a single edge between any two nodes. It facilitates the study of complex systems.

The Signal Flow Graph (SFG) is one of the most effective tools for analyzing electrical networks. These methods are an alternative approach to traditional methods such as Kirchhoff's rules and nodal analysis. Sometimes it may be somewhat difficult to calculate the transfer function using the reduction of signal-flow graphs, but Mason's Gain Formula is useful for calculating complex network circuits. In particular, Mason's Gain Formula provides a systematic, algebraic method for calculating the transfer function, which is particularly useful in situations where traditional techniques are impractical. Signal flow graphs and their reduction provide a graphical representation of the analysis, enhancing visualization and facilitating a deeper understanding of the network's behavior.

This paper tried to contribute to the field of electrical network analysis by introducing the application of Mason's Gain formula and Signal Flow Graph (SFG) in determining the transfer function of such a network. The signal flow graph is crucial for electrical network analysis. From the graphical depiction of the electrical network, we constructed a Signal-Flow Graph, which we then used to determine the transmittance (transfer function) between the vertices. To this end, we must once again select the gains for the forwarding path and their respective loops. A sharp edge, known as a one-edge (I-edge), can be used to depict the electrical network's transfer function (transmittance) between the two edges. Mason's Gain Formula provides a systematic and algebraic method for calculating the transfer function. A faster and more efficient network analysis process is achieved by applying the Reduction Rules of the Signal Flow Graph. It offers excellent potential and a variety of academic options. The Signal Flow Graph approach still necessitates much research.

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