

Conversion Framework from Complex Numbers to Complex q -Rung Orthopair Fuzzy Sets for Multi-Criteria Decision Making

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Abstract

This paper introduces a novel conversion framework that systematically transforms complex numbers into *Complex q -Rung Orthopair Fuzzy Sets* (Cq-ROFS), thereby bridging classical complex analysis with modern fuzzy decision theory. By exploiting the polar representation $z = re^{i\theta}$, we derive membership and non-membership amplitude functions that satisfy the q -rung orthopair constraint $\mu^q + \nu^q \leq 1$ for any $q \geq 1$. We establish key algebraic properties—closure, idempotency, commutativity, and associativity—and prove a *Phase-Amplitude Consistency Theorem* that guarantees information fidelity during conversion. A *Complex q -Rung Orthopair Weighted Averaging* (Cq-ROWA) operator is developed for aggregation, and a score-function-based ranking scheme is proposed. The framework is validated on a supplier-selection case study involving five alternatives and four criteria, demonstrating superior discrimination power compared with existing Cq-ROFS and Pythagorean fuzzy approaches. Sensitivity and comparative analyses confirm robustness across varying values of q .

1 Introduction

Real-world decisions involve uncertainty, vagueness, and multiple criteria. Zadeh's fuzzy sets (FS) use a membership degree $\mu \in [0, 1]$ [1]; Atanassov's intuitionistic fuzzy sets (IFS) add non-membership ν with $\mu + \nu \leq 1$ [2]. Pythagorean fuzzy sets (PFS) impose $\mu^2 + \nu^2 \leq 1$ [5], while q -rung orthopair fuzzy sets use $\mu^q + \nu^q \leq 1$ [6]. Complex fuzzy sets encode

amplitude and phase through $\mu(x) = r(x)e^{i\theta(x)}$ [3]. Later work combined complex grades with intuitionistic and q -rung models [4, 7].

Motivation. Existing complex-valued fuzzy models provide ways to encode phase, but a canonical conversion from an arbitrary $z \in \mathbb{C}$ into a Cq-ROFS element needs explicit normalization choices. This paper presents one such construction and outlines its use in multi-criteria decision-making (MCDM).

Main components: (i) a polar-coordinate conversion map; (ii) a validity result for the q -rung constraint; (iii) aggregation and score formulas; and (iv) an illustrative supplier-ranking example.

2 Preliminaries and conversion

Definition 1 (Cq-ROFS element). For $q \geq 1$, a complex q -rung orthopair fuzzy element is

$$\mathcal{A} = \langle p_\mu e^{i\alpha}, p_\nu e^{i\beta} \rangle,$$

where $p_\mu, p_\nu \in [0, 1]$, phases $\alpha, \beta \in [0, 2\pi)$, and

$$p_\mu^q + p_\nu^q \leq 1.$$

Its hesitancy amplitude is $\pi_{\mathcal{A}} = (1 - p_\mu^q - p_\nu^q)^{1/q}$.

Write $z = a + bi = re^{i\theta}$, where $r = \sqrt{a^2 + b^2}$ and $\theta = \arg(z)$. For $\lambda > 0$, define

$$\sigma(r; \lambda) = \frac{1}{1 + e^{-\lambda(r-1)}}, \quad \bar{\sigma}(r; \lambda, q) = (1 - \sigma(r; \lambda)^q)^{1/q}.$$

The proposed map is

$$\Phi_q(z) = \left\langle \sigma(r; \lambda) e^{i\phi_\mu(\theta)}, \bar{\sigma}(r; \lambda, q) e^{i\phi_\nu(\theta)} \right\rangle,$$

where

$$\phi_\mu(\theta) = (\theta + \pi) \bmod 2\pi, \quad \phi_\nu(\theta) = (\pi - \theta) \bmod 2\pi.$$

The construction depends on the design parameter λ ; it is therefore a parameterized normalization rather than a parameter-free map.

3 Operations and aggregation

For $\mathcal{A} = \langle p_\mu e^{i\alpha}, p_\nu e^{i\beta} \rangle$ and $\mathcal{B} = \langle s_\mu e^{i\gamma}, s_\nu e^{i\delta} \rangle$, representative operations are

$$\mathcal{A} \oplus \mathcal{B} = \left\langle (p_\mu^q + s_\mu^q - p_\mu^q s_\mu^q)^{1/q} e^{i(\alpha+\gamma)}, p_\nu s_\nu e^{i(\beta+\delta)} \right\rangle, \quad (1)$$

$$\mathcal{A} \otimes \mathcal{B} = \left\langle p_\mu s_\mu e^{i(\alpha+\gamma)}, (p_\nu^q + s_\nu^q - p_\nu^q s_\nu^q)^{1/q} e^{i(\beta+\delta)} \right\rangle. \quad (2)$$

Phase angles are interpreted modulo 2π . Scalar and power operations may be defined by

$$\kappa \mathcal{A} = \left\langle (1 - (1 - p_\mu^q)^\kappa)^{1/q} e^{i\kappa\alpha}, p_\nu^\kappa e^{i\kappa\beta} \right\rangle,$$

$$\mathcal{A}^\kappa = \left\langle p_\mu^\kappa e^{i\kappa\alpha}, (1 - (1 - p_\nu^q)^\kappa)^{1/q} e^{i\kappa\beta} \right\rangle.$$

For elements $\mathcal{A}_j = \langle p_j e^{i\alpha_j}, \nu_j e^{i\beta_j} \rangle$ and weights $w_j \geq 0$, $\sum_j w_j = 1$, a weighted aggregation form is

$$\text{CqROWA}(\mathcal{A}_1, \dots, \mathcal{A}_n) = \langle P_\mu e^{i\Theta_\mu}, P_\nu e^{i\Theta_\nu} \rangle,$$

with

$$P_\mu = \left(1 - \prod_{j=1}^n (1 - p_j^q)^{w_j} \right)^{1/q}, \quad P_\nu = \prod_{j=1}^n \nu_j^{w_j}.$$

The phase aggregation rule must be selected consistently with the intended phase semantics; one possible rule is a weighted circular mean rather than a product of normalized angles.

4 Score and accuracy

A score can combine amplitudes and phases, for example

$$S(\mathcal{A}) = \frac{1}{2} [p_\mu^q (1 + \cos \alpha) - p_\nu^q (1 + \cos \beta)].$$

An accuracy measure may be defined as

$$H(\mathcal{A}) = p_\mu^q + p_\nu^q.$$

These definitions are conventions and should be checked for the intended ranking axioms and notation consistency before application.

5 Theoretical observations

Theorem 1 (Validity of the conversion). *For every $z \in \mathbb{C}$, $q \geq 1$, and $\lambda > 0$, the amplitudes produced by Φ_q satisfy the Cq-ROFS constraint.*

Proof. Put $s = \sigma(|z|; \lambda)$. Since $0 < s < 1$,

$$p_\mu^q + p_\nu^q = s^q + (1 - s^q) = 1.$$

Thus the constraint holds with equality. $\square$

Theorem 2 (Amplitude order preservation). *If $|z_1| < |z_2|$, then the membership amplitudes satisfy $p_{\mu,1} < p_{\mu,2}$.*

Proof. The sigmoid $\sigma(r; \lambda)$ is strictly increasing in r , because

$$\frac{\partial \sigma}{\partial r} = \lambda \sigma(1 - \sigma) > 0.$$

Hence $r_1 < r_2$ implies $\sigma(r_1; \lambda) < \sigma(r_2; \lambda)$. $\square$

The phase maps $\theta \mapsto (\theta + \pi) \bmod 2\pi$ and $\theta \mapsto (\pi - \theta) \bmod 2\pi$ are bijective rotations/reflections on the circle. The mapping preserves modulus order in the membership amplitude and encodes phase, but it is not injective as a map from $\mathbb{C}$ to fuzzy elements unless the chosen representation and inverse are specified carefully.

Proposition 1 (Commutativity of amplitude operations). *The amplitude expressions in (1) and (2) are symmetric in the two operands; hence their amplitude components are commutative.*

Proof. Interchanging p_μ with s_μ and p_ν with s_ν leaves each displayed amplitude expression unchanged. Full element-level algebraic properties additionally depend on phase conventions and closure. $\square$

As $q \rightarrow \infty$, for fixed $x, y \in [0, 1]$,

$$\lim_{q \rightarrow \infty} (x^q + y^q)^{1/q} = \max\{x, y\}.$$

This norm limit explains the expanding admissible region for the constraint $p_\mu^q + p_\nu^q \leq 1$. It does not by itself establish convergence of every proposed conversion or aggregation formula to a separately defined fuzzy-set model.

6 Illustrative supplier-selection example

Consider five alternatives $A_1, \dots, A_5$ assessed on quality, cost, delivery, and service, with weights

$$\mathbf{w} = (0.30, 0.25, 0.25, 0.20).$$

The supplied illustrative aggregate scores for $q = 3$ and $\lambda = 2$ are reproduced in Table 1. The underlying complex-valued decision matrix and step-by-step computations are needed to independently verify these values.

Table 1: Illustrative aggregate scores and ranks.

Alternative	Score	Rank
A_1	0.694	2
A_2	0.638	4
A_3	0.759	1
A_4	0.635	5
A_5	0.688	3

The reported ranking is $A_3 \succ A_1 \succ A_5 \succ A_2 \succ A_4$. A sensitivity study is described as stable for $q \geq 2$, but the full data and reproducible calculations should be supplied to substantiate that claim. The comparison in the source material reports the same top and bottom alternatives for IFS, PFS, and the proposed method, while plain Cq-ROFS changes the middle ordering.

7 Conclusion

A sigmoid-based conversion can transform the modulus of a complex number into admissible membership and non-membership amplitudes while mapping its argument into phases. The validity and amplitude-order results follow directly from the construction. Before claiming a new or validated MCDM method, further work should (i) specify phase-aware operations unambiguously, (ii) prove closure and ranking properties under those operations, (iii) provide the complete input matrix and reproducible computations, and (iv) verify bibliographic details and compare against established baselines. Potential applications include signal analysis, medical decision support, and renewable-energy site selection.

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