

Study on Vertex Degree, Edge Degree, Leap Degree, and Eccentricity of Neutrosophic Graphs

N. K. Raut

Ex. Head, Department of Physics, Sunderrao Solanke, Mahavidyalaya Majalgaon Dist. Beed (M.S.) India

Email: [rautnk87\[at\]gmail.com](mailto:rautnk87[at]gmail.com)

Abstract: A new area of study called neutrosophic graph theory uses neutrosophic logic to expand on traditional graph theory to include ambiguous, indeterminate and uncertain information [1]. Neutrosophic set is simply an extension of intuitionistic fuzzy set and fuzzy set. In this paper vertex degree, edge degree, leap degree and eccentricity of interval-valued neutrosophic line graph, neutrosophic fuzzy magic labeling graph, single-valued pentapartitioned neutrosophic graph, and interval-valued neutrosophic soft graph are studied.

Keywords: Eccentricity, leap degree, line graph, membership, neutrosophic fuzzy graph, non-membership, soft graph, vertex degree

1. Introduction

Graph theory has been successfully used to solve a wide range of problems encountered in diverse fields, such as medical sciences, neural networks, control theory, transportation, clustering analysis, and image capturing and network security [2]. Neutrosophic set is simply an extension of intuitionistic fuzzy set and fuzzy set. A neutrosophic network is an extension of an intuitionistic fuzzy network that offers greater precision, compatibility, and flexibility when organizing the modeling in many real-world applications than an intuitionistic fuzzy graph. Interval-valued neutrosophic soft sets is a generalization of fuzzy soft sets, intuitionistic fuzzy soft sets and neutrosophic soft sets. The concepts of neighbourly pseudo irregular neutrosophic bipolar fuzzy graphs, neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graphs were introduced in [3]. The degree for a vertex $x \in V$ of an inverse fuzzy graph, $\xi^* = (\mu, \rho)$ denoted as $d(x) = \sum_{xy \in E} \rho(xy)$, The order of ξ is $\sum_{x \in V} \mu(x)$, and the size of ξ is $\sum_{xy \in E} \rho(xy)$ [4]. The necessary and sufficient condition for neutrosophic vague line graph to be a line graph were derived in [5]. The degree for $\hat{G} = (\Psi, \Omega)$, single-valued quadripartitioned neutrosophic graph was studied in [6]. Based on the concept of line graphs and anti-fuzzy graphs, the concepts of anti-fuzzy line graphs and some properties of anti-fuzzy line graphs are defined as: $d(w) = (d_T(w), d_C(w), d_U(w), d_N(w))$, where $d_T(w)$ is degree of the truth-membership vertex = sum of the truth membership of all edges those are incident on the vertex, $w = \sum_{u \neq w} T_\Omega(u, w)$, similarly for other vertices [7].

Let $G = (N, M)$ a single-valued neutrosophic graph, and $v \in V$ is a vertex of G . The degree of vertex v is the sum of the truth membership values, the sum of indeterminacy membership values, and the sum of the falsity membership values of all the edges that are adjacent to v [8]. Let $x, y \in V(G)$, then the distance $d(x, y)$ between x and y is defined as the length of any shortest path in G connecting x and y . Then eccentricity is $e(v) = \max\{d(u, v) | \forall u \in V(G)\}$ [9]. Let $G(K, L)$ be a neutrosophic network and the node $k \in V$. The neutrosophic strong degree of a node l is defined as: $d_s(l) = \sum_{k \in N_s(l)} T_L(kl), \sum_{k \in N_s(l)} F_L(kl)$ [10]. Neutrosophic topologized bipartite graph, analyses unreliability, in

bipartite structured graphs by neutrosophic concepts with topologic possessions [11]. Let $N(U)$ be set of all neutrosophic sets on universal set U , E be the set of parameters and $A \subseteq E$. A pair (F, A) is called a neutrosophic soft set over U , where F is a mapping given by $F: A \rightarrow N(U)$ [12-13]. Let $G = (A, B)$ be a single-valued co-neutrosophic graph. Then the degree of a vertex v is defined by $d(v) = [d_T(v), d_I(v), d_F(v)]$, where $d_T(v) = \sum_{u \neq v} T_B(u, v)$, $d_I(v) = \sum_{u \neq v} I_B(u, v)$, and $d_F(v) = \sum_{u \neq v} F_B(u, v)$ [14]. Fuzzy topological indices are not only limited to chemical graphs but also find applications in social network optimization, transportation flow, network modeling, and medical diagnosis [15]. The degree of an edge in an intuitionistic fuzzy graph is defined as $d_\mu(uv) = d_\mu(u) + d_\mu(v) - 2\mu_2(uv)$ and $d_\gamma(uv) = d_\gamma(u) + d_\gamma(v) - 2\gamma_2(uv)$ [16].

Interval-valued neutrosophic line graph:

The line graph of an interval-valued neutrosophic graph $G = (A, B)$, denoted by $L(G) = (C, D)$, is defined over the line graph crisp structure $L(G^*) = (E, E')$ [17].

1) The vertex set of $L(G)$ (denoted by C):

The vertices of $L(G)$ correspond to the edges of G . Each vertex in C is assigned the exact neutrosophic interval values of its corresponding edge in G .

For every edge $e = (u, v) \in E$, the corresponding vertex $e \in C$ has the membership intervals:

$$C(e) = B(u, v) = \{[T_B^L(u, v), T_B^U(u, v)], [I_B^L(u, v), I_B^U(u, v)], [F_B^L(u, v), F_B^U(u, v)]\}.$$

2) The edge set of $L(G)$ (denoted by D):

An edge exists in $L(G)$ between two vertices e_1 and e_2 if and only if the corresponding edges e_1 and e_2 share a common end-vertex in the original graph G . For any adjacent edges $e_1 = (u, v)$ and $e_2 = (u, v)$ share vertex v , the neutrosophic intervals for connecting edges (e_1, e_2) in D are typically defined using interval bounds based on minimum truth and maximum indeterminacy/falsity of the shared vertex or connection component:

$$\text{Truth membership: } T_D(e_1, e_2) \leq \{\min[T_B^L(e_1), T_B^L(e_2)], \min[T_B^U(e_1), T_B^U(e_2)]\}.$$

Indeterminacy membership: $I_D(e_1, e_2) \geq \{ \max[I_B^L(e_1), I_B^L(e_2)], \max[I_B^U(e_1), I_B^U(e_2)] \}$.
Falsity membership: $F_D(e_1, e_2) \geq \{ \max[F_B^L(e_1), F_B^L(e_2)], \max[F_B^U(e_1), F_B^U(e_2)] \}$.

Neutrosophic Fuzzy Magic Labeling Graph:

A Neutrosophic Fuzzy Magic Labeling Graph (NFMLG) if there exists a magic graph 'M' such that degree of truth-membership function equals $Tr_1(\delta(v_i)) + Tr_2(\delta(v_i), \delta(v_j)) + Tr_1(\delta(v_j))$, indeterminacy $Ind_1(\delta(v_i)) + Ind_2(\delta(v_i), \delta(v_j)) + Ind_1(\delta(v_j))$, falsity $Fal_1(\delta(v_i)) + Fal_2(\delta(v_i), \delta(v_j)) + Fal_1(\delta(v_j))$.
 $M = \{Tr_1(\delta(v_i)) + Tr_2(\delta(v_i), \delta(v_j)) + Tr_1(\delta(v_j)), Ind_1(\delta(v_i)) + Ind_2(\delta(v_i), \delta(v_j)) + Ind_1(\delta(v_j)), Fal_1(\delta(v_i)) + Fal_2(\delta(v_i), \delta(v_j)) + Fal_1(\delta(v_j))\} = \text{constant}$ [18].

The magic truth membership value is represented by $m_{Tr}(G)$, indeterminacy $m_{Ind}(G)$, falsity $m_{Fal}(G)$, the neutrosophic fuzzy magic labeling graph is represented by $M_{m(G)}(G) = (m_{Tr}(G), m_{Ind}(G), m_{Fal}(G))$.

Single-valued pentapartitioned neutrosophic graph:

Single-valued pentapartitioned neutrosophic graphs (SVPN-graph) is defined as:

suppose that $V = \{k_i: i=1, 2, \dots, n\}$ be a fixed set of vertices and $E = \{(k_i, k_j): i, j=1, 2, \dots, n\}$ be the set of edges between the vertices of V [19]. An SVPN-graph of $\hat{G}^* = (V, E)$ is defined by $\hat{G} = (P, Q)$, where

- $T_P: V \rightarrow [0, 1]$, $C_P: V \rightarrow [0, 1]$, $I_P: V \rightarrow [0, 1]$, $U_P: V \rightarrow [0, 1]$, and $F_P: V \rightarrow [0, 1]$ denotes the truth, contradiction, ignorance, unknown, and false membership functions of the vertices $k_i \in V$ respectively, such that $0 \leq T_P(k_i) + C_P(k_i) + I_P(k_i) + U_P(k_i) + F_P(k_i) \leq 5$, $\forall k_i \in V$ ($i=1, 2, \dots, n$).
- $T_Q: E \subseteq V \times V \rightarrow [0, 1]$, $C_Q: E \subseteq V \times V \rightarrow [0, 1]$, $I_Q: E \subseteq V \times V \rightarrow [0, 1]$, $U_Q: E \subseteq V \times V \rightarrow [0, 1]$, and $F_Q: E \subseteq V \times V \rightarrow [0, 1]$, defined by $T_Q(k_i, k_j) \leq \min\{T_P(k_i), T_P(k_j)\}$, $C_Q(k_i, k_j) \leq \min\{C_P(k_i), C_P(k_j)\}$, $I_Q(k_i, k_j) \geq \max\{I_P(k_i), I_P(k_j)\}$, $U_Q(k_i, k_j) \geq \max\{U_P(k_i), U_P(k_j)\}$, and $F_Q(k_i, k_j) \geq \max\{F_P(k_i), F_P(k_j)\}$, denotes the truth, contradiction, ignorance, unknown and false-membership functions from $E \subseteq V \times V$ to $[0, 1]$ respectively,

Such that $0 \leq T_P(k_i) + C_P(k_i) + I_P(k_i) + U_P(k_i) + F_P(k_i) \leq 5$, $\forall (k_i, k_j) \in E$ ($i, j = 1, 2, \dots, n$). Here, P is the SVN vertex set of V and Q is the SVN edge set of E respectively. Therefore $\hat{G} = (P, Q)$ is an SVPN-graph of $\hat{G}^* = (V, E)$ if and only if $T_Q(k_i, k_j) \leq \min\{T_P(k_i), T_P(k_j)\}$; $C_Q(k_i, k_j) \leq \min\{C_P(k_i), C_P(k_j)\}$; $I_Q(k_i, k_j) \geq \max\{I_P(k_i), I_P(k_j)\}$; $U_Q(k_i, k_j) \geq \max\{U_P(k_i), U_P(k_j)\}$; and $F_Q(k_i, k_j) \geq \max\{F_P(k_i), F_P(k_j)\}$. Clearly both P and Q are the SVPN-set over V and E respectively.

Interval-valued neutrosophic soft graph:

Let $G = (V, E)$ be a graph and let A be a set of parameters. A single-valued neutrosophic soft graph is a mathematical structure that integrates soft set theory with single-valued neutrosophic graphs to model parameterized networks contains, uncertain, indeterminate, and inconsistent information [20]. For every parameter $a \in A$, an interval-valued neutrosophic soft graph associates with each vertex

$v \in V$, three intervals: $T_A(v) = [T_A^L(v), T_A^U(v)]$, $I_A(v) = [I_A^L(v), I_A^U(v)]$, $F_A(v) = [F_A^L(v), F_A^U(v)]$, where, $T_A(v)$ is truth-membership interval, $I_A(v)$ -indeterminacy, $F_A(v)$ -falsity.

Thus, $\{T_A^L(v), T_A^U(v), I_A^L(v), I_A^U(v), F_A^L(v), F_A^U(v)\} \in [0, 1]$.

Condition for vertex:

For any vertex $v \in V$, $0 \leq T_A^L(v) \leq T_A^U(v) \leq 1$, $0 \leq I_A^L(v) \leq I_A^U(v) \leq 1$, $0 \leq F_A^L(v) \leq F_A^U(v) \leq 1$.

Edge membership:

For an edge $uv \in E$, $T_A(uv) = [T_A^L(uv), T_A^U(uv)]$, $I_A(uv) = [I_A^L(uv), I_A^U(uv)]$, $F_A(uv) = [F_A^L(uv), F_A^U(uv)]$.

Neutrosophic condition:

$T_A^L(uv) \leq \min[T_A^L(u), T_A^L(v)]$, $T_A^U(uv) \leq \min[T_A^U(u), T_A^U(v)]$, $I_A^L(uv) \geq \max[I_A^L(u), I_A^L(v)]$, $I_A^U(uv) \geq \max[I_A^U(u), I_A^U(v)]$, $F_A^L(uv) \geq \max[F_A^L(u), F_A^L(v)]$, $F_A^U(uv) \geq \max[F_A^U(u), F_A^U(v)]$.

Consider the graph $G = (V, E)$, where V and E is the set of its vertex and edges respectively. Let $NF_G(A, B)$ be the NF-graph. Then the degree of the vertex $v \in V$ is denoted by $d(v)$ and defined as $d(v) = [d_T(v), d_I(v), d_F(v)]$ or $d_G(v) = [d_{T_G}(v), d_{I_G}(v), d_{F_G}(v)]$, where $d_T(v) = \sum_{v \neq w} T_B(v, w)$, $d_I(v) = \sum_{v \neq w} I_B(v, w)$, and $d_F(v) = \sum_{v \neq w} F_B(v, w)$, are the degree of truth membership vertex, degree of indeterminacy, and degree of falsity respectively [21].

In 2012, Ghorbani et al. [22-23] introduced eccentricity based version of Zagreb eccentricity indices as:

$M_1^*(G) = \sum_{uv \in E(G)} (e(v) + e(u))$, $M_1^{**}(G) = \sum_{uv \in E(G)} (e(v))^2$, and $M_2^*(G) = \sum_{uv \in E(G)} (e(v) \times e(u))$, where $e(v)$ is the eccentricity of vertex v .

For a simple connected graph G , first and second Zagreb indices can be defined as follows:

$M_1(G) = \sum_{uv \in E(G)} (d_u + d_v)$ and $M_2(G) = \sum_{uv \in E(G)} d_u \times d_v$, where d_v denotes the degree of vertex v in G .

In 2017, Naji et al., [24-26] introduced new distance degree-based topological indices depending on the second degree of vertex, called leap Zagreb indices of a graph G .

Let $G = (V, E)$ denote a simple, connected graph with p vertices and q edges. Then

- First leap Zagreb index = $LM_1(G) = \sum_{v \in V(G)} d_2(v)^2$.
- Second leap Zagreb index = $LM_2(G) = \sum_{uv \in E(G)} d_2(u) \times d_2(v)$.
- Third leap Zagreb index = $LM_3(G) = \sum_{v \in V(G)} d(v) d_2(v)$, where $d_2(v)$ is 2-degree of v , defined as the number of vertices which are at distance two from v in G and $\deg(v)$ is degree of v in G , that is the number of edges incident at v in G .

A. Milicevic, S. Nikolic, and N. Trinajstić introduced reformulated the Zagreb indices in 2004 in terms of edge-degree instead of vertex-degree as [27-28]:

$EM_1(G) = \sum_{e \in E(G)} d_e^2$ and $EM_2(G) = \sum_{e \sim f \in E(G)} d_e \times d_f$,

where d_e denotes degree of the edge $e \in E(G)$, which is defined by $d_e = d_u + d_v - 2$ with $e = uv$, and $\forall e, f \in E(G)$, $e \sim f$ means that the edges e and f are adjacent, i.e., they share a common end and vertex in G .

The symbols and notations follow standard conventions as established [29-33]. In this paper vertex degree, edge degree, leap degree and eccentricity of interval-valued neutrosophic line graph, neutrosophic fuzzy magic labeling graph, single valued pentapartitioned neutrosophic graph and interval-valued neutrosophic soft graph are obtained.

2. Materials and Methods

Interval-valued neutrosophic line graph $L(G)$ with 4 vertices, neutrosophic fuzzy magic labeling graph, single-valued pentapartitioned neutrosophic graph, and interval-valued neutrosophic soft graph are shown in figures (1-4) respectively. The number of vertices and edges of the above graphs are observed from these figures are used to vertex degree, edge degree, neutrosophic degree, leap degree, radius, diameter, eccentricity, regularity and total regularity.

3. Results and Discussion

Interval-valued neutrosophic line graph

The interval-valued neutrosophic line graph ($L(G)$) with 4 vertices is shown in figure (1). In line-graph if G has m edges, then its line graph $L(G)$ has m vertices; $|V(L(G))| = |E(G)|$, then degree of a line-graph vertex corresponding to an original edge uv is $d_G(uv) = d_G(u) + d_G(v) - 2$. Therefore $d_{L(G)}(v_1) = 3$, $d_{L(G)}(v_2) = 2$, $d_{L(G)}(v_3) = 3$, $d_{L(G)}(v_4) = 2$, that is $d(e_{12}) = 3$, $d(e_{13}) = 3$, $d(e_{23}) = 4$, $d(e_{24}) = 3$, and $d(e_{34}) = 3$ and the neutrosophic edge degrees are: $d(e_{12}^N) = (1.8, 0.9, 0.9)$, $d(e_{13}^N) = (1.6, 1.0, 1.1)$, $d(e_{23}^N) = (1.6, 1.3, 1.3)$, $d(e_{24}^N) = (1.6, 1.0, 1.0)$, and $d(e_{34}^N) = (1.6, 0.9, 0.9)$. A small degree means that the edge is connected to relatively few other edges and a large edge degree means that edge is located between highly connected vertices. It is useful for identifying important or central links in a network.

Leap degree:

The leap degree of a vertex is generally defined as the sum of the weights of edges joining its vertices at distance two. Hence, $d_2(v_1) = (0.9, 0.8, 1.2)$, $d_2(v_2) = (1.0, 1.1, 1.1)$, $d_2(v_3) = (0.9, 0.1, 1.3)$, and $d_2(v_4) = (0.9, 0.8, 1.2)$.

Degree:

A graph is regular of degree (V_T, V_L, V_F) if every vertex has the same neutrosophic degree. Here, $d^N(v_1) = (1.2, 0.5, 0.6)$, $d^N(v_2) = (1.6, 1.0, 1.1)$, $d^N(v_3) = (1.8, 0.9, 0.8)$, and $d^N(v_4) = (1.0, 0.8, 0.7)$. Since these four triples are different, the graph is irregular. It is also irregular in the underlying crisp graph, because $d(v_1) = 2$, $d(v_2) = 3$, $d(v_3) = 3$, and $d(v_4) = 2$.

The degree matrix is diagonal with vertex degrees on its main diagonal. $D(G) = \text{diag}[(1.2, 0.5, 0.6), (1.6, 1.0, 1.1), (1.8, 0.9, 0.8), (1.0, 0.8, 0.7)]$. The degree matrix does not have equal diagonal entries, confirming the irregularity of the graph.

Total degree:

Total degree is $td(v_i) = d(v_i) + v_i$. Hence $td(v_1) = (4.6, 2.4, 2.5)$, $td(v_2) = (5.6, 3.2, 3.2)$, $td(v_3) = (5.6, 3.2, 3.2)$, and $td(v_4) = (4.4, 2.7, 2.6)$. As total degree of vertices is unequal, the given graph is totally irregular. It is useful when information is incomplete, inconsistent or imprecise such as in medical diagnosis, decision-making systems, and social-networking relationships.

Eccentricity: The eccentricity of a vertex v is $e(v) = \max_{u \in V} d(v, u)$. The eccentricities of the vertices are $e(v_1) = 1$, $e(v_2) = 2$, $e(v_3) = 1$ and $e(v_4) = 2$. Hence diameter $d(G) = \max_{v \in V} e(v) = 2$ and radius $r(G) = \min_{v \in V} e(v) = 1$.

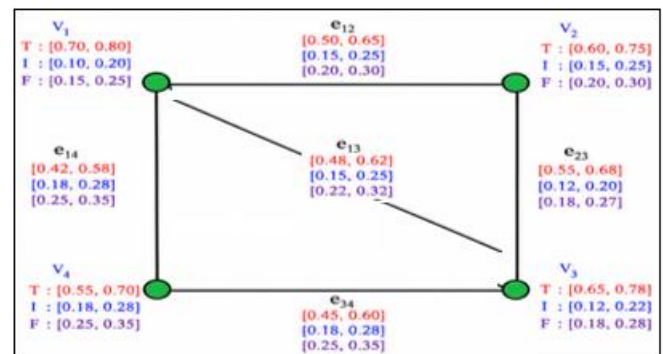


Figure 1: Interval-valued neutrosophic line graph

Neutrosophic Fuzzy Magic Labeling Graph:

Neutrosophic fuzzy magic labeling graph is a specialized mathematical model that assigns values representing truth, indeterminacy, and falsity to both vertices and edges such that their sums yield a constant magic value. Let $M = (T, I, F)$ be a magic number, then magic number for truth is given by $T(M) = T(v_i) + T(v_j) - T(v_i)T(v_j)$.

Consider the path v_1-v_2 , then $T = 0.2 + 0.4 + 0.1 = 0.7$, $I = 0.5 + 0.3 + 0.1 = 0.9$, $F = 0.3 + 0.2 + 0.2 = 0.7 \Rightarrow M = (0.7, 0.9, 0.7)$. Similarly for other paths: v_1-v_3 , v_2-v_3 , v_2-v_4 , v_3-v_4 , the magic number is $M = (0.7, 0.9, 0.7)$. The magic number that stabilizes and validates the specific uncertainty is a wave graph model. This indicates that even in highly variable, non-uniform environments, a system-wide equilibrium or deterministic pattern can still be maintained.

Edge degree: $d(e_{12}) = 3$, $d(e_{13}) = 3$, $d(e_{23}) = 4$, $d(e_{24}) = 3$, and $d(e_{34}) = 3$.

Neutrosophic edge degree: $d^n(e_{12}) = (0.4, 0.5, 0.6)$, $d^n(e_{13}) = (0.5, 0.3, 0.6)$, $d^n(e_{23}) = (0.5, 0.6, 0.6)$, $d^n(e_{24}) = (0.5, 0.3, 0.6)$, and $d^n(e_{34}) = (0.4, 0.5, 0.6)$.

Leap degree: $d_2(v_1) = 1$, $d_2(v_2) = 0$, $d_2(v_3) = 0$, $d_2(v_4) = 1$.

Neutrosophic vertex degree: $\deg(v) = (\sum T(e), \sum I(e), \sum F(e))$.

Hence, $d^N(v_1) = (0.2, 0.3, 0.3)$, $d^N(v_2) = (0.4, 0.4, 0.7)$, $d^N(v_3) = (0.5, 0.4, 0.5)$, $d^N(v_4) = (0.3, 0.3, 0.3)$. Since the degrees of all vertices are different, the graph is irregular.

Total regularity: A neutrosophic fuzzy graph is totally regular if every vertex shares the exact same total degree ($td(v)$).

$td(v_1) = (0.4, 0.8, 0.6)$, $td(v_2) = (0.8, 0.7, 0.9)$, $td(v_3) = (0.8, 0.8, 0.5)$, and $td(v_4) = (0.8, 0.5, 0.7)$.

Hence, $td(v_1) \neq td(v_2) \neq td(v_3) \neq td(v_4)$, shows this graph is not totally regular.

Neutrosophic adjacency matrix $A(G)$: In a neutrosophic fuzzy graph, the entries for adjacent vertices corresponds to their edge membership triplets (T, I, F), non-adjacent vertices and diagonal entries are assigned the zero vector (0,0,0).

$$A(G) = \begin{pmatrix} V1 & V2 & V3 & V4 \\ V1 & (0,0,0) & (0.1,0.1,0.2) & (0.1,0.2,0.1) & (0,0,0) \\ V2 & (0.1,0.1,0.2) & (0,0,0) & (0.2,0.1,0.3) & (0.1,0.2,0.2) \\ V3 & (0.1,0.2,0.1) & (0.2,0.1,0.3) & (0,0,0) & (0.2,0.1,0.1) \\ V4 & (0,0,0) & (0.1,0.2,0.2) & (0.2,0.1,0.1) & (0,0,0) \end{pmatrix}$$

The neutrosophic adjacency matrix serves as a complete algebraic representation of the relational structure, communication strength, and levels of uncertainty across the entire network.

Eccentricity: $e(v_1) = \max(0.4, 0.4, 0.8) = 0.8$, $e(v_2) = \max(0.4, 0.6, 0.5) = 0.6$, $e(v_3) = \max(0.4, 0.6, 0.4) = 0.6$, and $e(v_4) = \max(0.8, 0.5, 0.4) = 0.8$.

Radius: $r(G) = \min\{e(v_1), e(v_2), e(v_3), e(v_4)\} = \min(0.8, 0.6, 0.6, 0.8) = 0.6$.

Diameter: $d(G) = \max\{e(v_1), e(v_2), e(v_3), e(v_4)\} = \max(0.8, 0.6, 0.6, 0.8) = 0.8$.

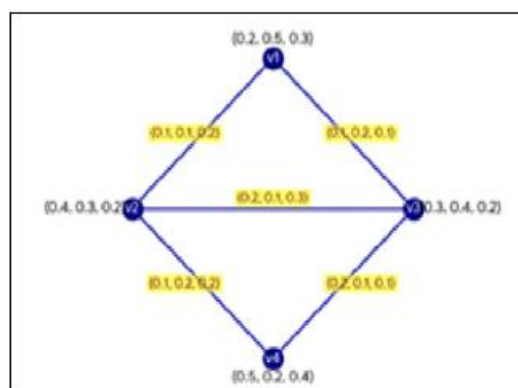


Figure 2: Neutrosophic fuzzy magic labeling graph.

Single-valued pentapartitioned neutrosophic graph:

For any edge e_{ij} connecting vertices v_i and v_j the defining properties in single-valued pentapartitioned neutrosophic graph are: $T(e_{ij}) \leq \min\{T(v_i), T(v_j)\}$, $C(e_{ij}) \leq \min\{C(v_i), C(v_j)\}$, $I(e_{ij}) \geq \max\{I(v_i), I(v_j)\}$, $U(e_{ij}) \geq \max\{U(v_i), U(v_j)\}$, $F(e_{ij}) \geq \max\{F(v_i), F(v_j)\}$.

The conditions can be verified as:

e_{12} : $T: 0.3 \leq \min\{0.4, 0.3\} = 0.3$, $C: 0.4 \leq \min\{0.5, 0.5\} = 0.5$, $I: 0.5 \geq \max\{0.3, 0.4\} = 0.4$, $U: 0.4 \geq \max\{0.4, 0.4\} = 0.4$, $F: 0.6 \geq \max\{0.4, 0.5\} = 0.5$.

e_{23} : $T: 0.2 \leq \min\{0.3, 0.4\} = 0.3$, $C: 0.4 \leq \min\{0.5, 0.5\} = 0.5$, $I: 0.7 \geq \max\{0.4, 0.5\} = 0.5$, $U: 0.8 \geq \max\{0.4, 0.6\} = 0.6$, $F: 0.6 \geq \max\{0.5, 0.5\} = 0.5$.

e_{34} : $T: 0.2 \leq \min\{0.4, 0.2\} = 0.2$, $C: 0.3 \leq \min\{0.5, 0.5\} = 0.5$, $I: 0.8 \geq \max\{0.5, 0.4\} = 0.5$, $U: 0.9 \geq \max\{0.6, 0.3\} = 0.6$, $F: 0.9 \geq \max\{0.5, 0.5\} = 0.5$.

e_{45} : $T: 0.1 \leq \min\{0.2, 0.2\} = 0.2$, $C: 0.5 \leq \min\{0.5, 0.5\} = 0.5$, $I: 0.6 \geq \max\{0.4, 0.4\} = 0.4$, $U: 0.7 \geq \max\{0.3, 0.3\} = 0.3$, $F: 0.8 \geq \max\{0.5, 0.6\} = 0.6$.

e_{51} : $T: 0.2 \leq \min\{0.2, 0.4\} = 0.2$, $C: 0.2 \leq \min\{0.5, 0.5\} = 0.5$, $I: 0.5 \geq \max\{0.4, 0.3\} = 0.4$, $U: 0.5 \geq \max\{0.3, 0.4\} = 0.4$, $F: 0.8 \geq \max\{0.6, 0.4\} = 0.6$.

These conditions are satisfied for single-valued pentapartitioned neutrosophic graph (fig. 3).

Degree:

Vertex degree: $d(v_1) = 2$, $d(v_2) = 2$, $d(v_3) = 2$, $d(v_4) = 2$, and $d(v_5) = 2$.

Neutrosophic degree: $d^n(v_1) = (0.5, 0.6, 1.0, 0.9, 0.4)$, $d^n(v_2) = (0.5, 0.8, 1.2, 1.2, 1.2)$, $d^n(v_3) = (0.4, 0.7, 1.5, 1.7, 1.5)$, $d^n(v_4) = (0.3, 0.8, 1.4, 1.6, 1.7)$, and $d^n(v_5) = (0.3, 0.7, 1.1, 1.2, 1.6)$.

Since the degrees of all vertices are different, the graph is not regular.

Total degree: The total degree is represented by the formula $td(v_i) = d(v_i) + v_i$, thus

$td(v_1) = (0.9, 1.1, 1.3, 1.3, 1.8)$, $td(v_2) = (0.8, 1.3, 1.6, 1.5, 1.5)$, $td(v_3) = (0.8, 1.2, 2.0, 2.3, 2.0)$, $td(v_4) = (0.5, 1.3, 1.8, 1.9, 2.2)$, and $td(v_5) = (0.5, 1.2, 1.5, 1.5, 2.1)$.

Since the total degrees of all vertices are different, the graph is not totally regular.

Edge degree:

Edge degree: $d(e_{12}) = 4$, $d(e_{23}) = 4$, $d(e_{34}) = 4$, $d(e_{45}) = 4$, and $d(e_{51}) = 4$.

Neutrosophic edge degree: $d^n(e_{12}) = (1.0, 1.4, 2.2, 2.1, 2.6)$, $d^n(e_{23}) = (0.9, 1.5, 2.7, 2.9, 2.7)$, $d^n(e_{34}) = (0.7, 1.5, 2.9, 3.3, 3.2)$, $d^n(e_{45}) = (0.6, 1.5, 2.5, 2.8, 3.3)$, and $d^n(e_{51}) = (0.8, 1.3, 2.1, 2.1, 3.0)$.

Eccentricity: Every vertex has an eccentricity of 2.

Radius: $r(G) = \min\{e(v_1), e(v_2), e(v_3), e(v_4), e(v_5)\} = 2$.

Diameter: $d(G) = \max\{e(v_1), e(v_2), e(v_3), e(v_4), e(v_5)\} = 2$.

Neutrosophic eccentricity for all vertices can be obtained from distance matrix as: $e^n(v_1) = 2$, $e^n(v_2) = 2$, $e^n(v_3) = 2$, $e^n(v_4) = 2$, and $e^n(v_5) = 2$.

Leap degree: $d_2(v_1) = 4$, $d_2(v_2) = 4$, $d_2(v_3) = 4$, $d_2(v_4) = 4$, and $d_2(v_5) = 4$.

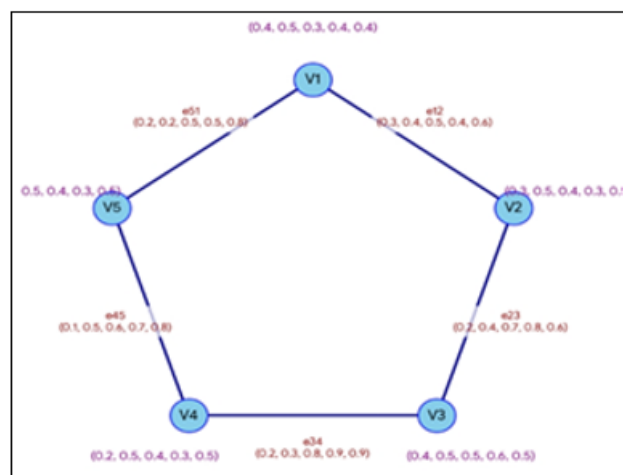


Figure 3: Single-valued pentapartitioned neutrosophic graph.

Interval-valued neutrosophic soft graph:

There are four vertices v_1, v_2, v_3, v_4 and five edges $e_{12}, e_{13}, e_{23}, e_{24}$, and e_{34} in figure 4. The condition is $0 \leq T_A(V) + I_A(V) + F_A(V) \leq 3$.

Degree:

Vertex degree: $d(v_1) = 3$, $d(v_2) = 2$, $d(v_3) = 3$, $d(v_4) = 2$.

Neutrosophic degree: $d^n(v_1) = (1.2, 0.5, 0.6)$, $d^n(v_2) = (1.6, 1.0, 1.1)$, $d^n(v_3) = (0.5, 0.4, 0.3)$, and $d^n(v_4) = (1.0, 0.8, 0.7)$.

Neutrosophic edge degree: $d(e_{12}^N) = (1.4, 0.4, 0.6)$, $d(e_{13}^N) = (1.5, 0.3, 0.3)$, $d(e_{23}^N) = (1.3, 0.5, 0.5)$, $d(e_{24}^N) = (1.1, 0.7, 0.7)$, and $d(e_{34}^N) = (1.2, 0.6, 0.4)$.

Since the degrees of all vertices are different, the graph is not regular.

Edge degree:

$d(e_{12}) = (1.8, 0.9, 0.9)$, $d(e_{13}) = (1.6, 1.0, 1.0)$, $d(e_{23}) = (2.2, 1.3, 1.3)$, $d(e_{24}) = (1.8, 0.9, 0.9)$, and $d(e_{34}) = (1.8, 0.9, 0.9)$.

Total degree: Total degree is given by $td(v_i) = d(v_i) + v_i$, hence $td(v_1) = (2.0, 0.6, 0.8)$, $td(v_2) = (2.2, 1.3, 1.5)$, $td(v_3) = (2.5, 1.1, 0.9)$, and $td(v_4) = (1.5, 1.2, 1.0)$. Since the total degrees of all vertices are different, the graph is not totally regular.

Leap degree: $d_2(v_1) = 1$, $d_2(v_2) = 0$, $d_2(v_3) = 0$, and $d_2(v_4) = 1$.

Eccentricity: $e(v_1) = 2$, $e(v_2) = 1$, $e(v_3) = 1$, and $e(v_4) = 2$.

Neutrosophic eccentricity: $e^n(v_1) = (1.2, 0.2, 0.5)$, $e^n(v_2) = (0.6, 0.3, 0.4)$, $e^n(v_3) = (0.7, 0.2, 0.3)$, and $e^n(v_4) = (1.2, 0.2, 0.5)$. Neutrosophic diameter: $d^n(G) = (1.2, 0.2, 0.5)$ and neutrosophic radius: $r^n(G) = (0.6, 0.3, 0.4)$. The conditions can be verified as:

e_{12} : $0.5 \leq \min\{0.8, 0.6\} \leq 0.6, 0.3 \geq \max\{0.1, 0.3\} \geq 0.3, 0.4 \geq \max\{0.2, 0.4\} \geq 0.4$.

e_{13} : $0.7 \leq \min\{0.8, 0.7\} \leq 0.7, 0.2 \geq \max\{0.1, 0.2\} \geq 0.2, 0.2 \geq \max\{0.2, 0.1\} \geq 0.2$.

e_{23} : $0.6 \leq \min\{0.6, 0.7\} \leq 0.6, 0.3 \geq \max\{0.4, 0.1\} \geq 0.4, 0.3 \geq \max\{0.3, 0.2\} \geq 0.3$.

e_{24} : $0.5 \leq \min\{0.6, 0.5\} \leq 0.5, 0.4 \geq \max\{0.3, 0.4\} \geq 0.4, 0.3 \geq \max\{0.4, 0.3\} \geq 0.4$.

e_{34} : $0.5 \leq \min\{0.7, 0.5\} \leq 0.5, 0.4 \geq \max\{0.2, 0.4\} \geq 0.4, 0.3 \geq \max\{0.1, 0.3\} \geq 0.3$.

The structure is a valid graph, this specific of neutrosophic number does not strictly satisfy the defining conditions for a standard single-valued neutrosophic soft graph. Based on these properties this graph is a valid soft set structure but it is an irregular and strongly connected (due to the bridge) graph that fails for the standard defining conditions for a perfect interval-valued neutrosophic soft graph due to falsity mismatch on vertex v_2 .

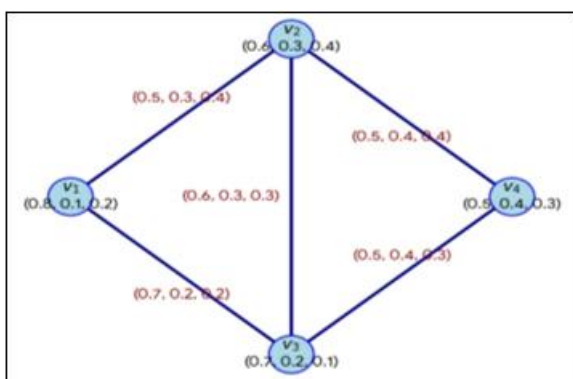


Figure 4: Interval-valued neutrosophic soft graph

4. Conclusion

Different types of degrees, regularity and total regularity of interval-valued neutrosophic line graph, neutrosophic fuzzy magic labeling graph, single-valued pentapartitioned neutrosophic graph, and interval-valued neutrosophic soft graph are studied. Neutrosophically: neutrosophic fuzzy magic labelling graph resembles a diamond or kite shape with the top triangle (v_1, v_2, v_3) and a bottom tail (v_4) with edges $e_{12}, e_{23}, e_{13}, e_{24}$ and e_{34} . The most useful application of neutrosophic fuzzy magic labelling graph is in optimization and decision-making under uncertainty. Single-valued pentapartitioned neutrosophic graph uses 5 components (truth, contradiction, ignorance, unknown and falsity). The most comprehensive option, supports the necessary definitions for degree, size and order, and its 5-component structure is designed for multi-dimensional complex analysis. Provides the highest resolution by effectively breaking down the concept of uncertainty. It captures ignorance and unknown states crucial for modelling multi-faceted human relationships (e.g. disagreement, conflict, lack of information).

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