International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2024: 7.101

Unsteady MHD Ethanol Boundary Layer Flows at a Three-Dimensional Stagnation Point with Variable Viscosity and Prandtl Number

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ABSTRACT

In this paper the effect of variable viscosity and Prandtl number on an unsteady magnetohydrodynamic (MHD) mixed convective ethanol boundary flow of viscous, incompressible and electrically conducting fluid at a three-dimensional stagnation point have been investigated. The governing partial differential equations have been solved numerically using the finite-difference scheme along with quasilinearization technique. The influence of magnetic parameter on skin-friction coefficients and velocity components in $x-$ and $y-$ directions, Nusselt number and temperature profiles in both accelerating and decelerating flow regimes are analyzed. Numerical results show that magnetic parameter has a significant effect on skin friction (x and y directions) and heat transfer coefficients. The effect of variable viscosity is to increase skin friction coefficient and to decrease heat transfer coefficient.

Keywords: *Magneto hydrodynamic flow, Skin friction, Heat transfer, Mixed Convection, Three-dimensional stagnation point, Variable viscosity and Prandtl number*

1. INTRODUCTION

The influence of temperature-dependent properties in the study of viscous incompressible fluid flow is of great importance in industries and technological fields where temperature is generated. In fluid dynamics, viscosity is the well-known property which is most sensitive to temperature rise. The increase of temperature leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so the heat transfer rate at the wall is also affected. Fluids used in industries such as polymer fluids, engine oil, palm oil have a viscosity that varies rapidly with temperature which normally leads to considerable changes in the structure of the fluid flow. The boundary layer flow considering the effect of variable fluid properties and Prandtl number is studied by Herwig and Wickern [1], Eswara and Nath [2], Jayakumar.et.al [3], Eswara [4]. The applications of hydromagnetic boundary layer flows have great impact in many engineering and industrial processes. Therefore, various investigations have been conducted on the MHD flows and heat transfer in the field of astrophysics, geophysics, electric transformers, power generators, refrigeration coils, pumps, meters, bearing, petroleum production. Several authors [5-7] have investigated magnetic field effects on the boundary layer flow and heat transfer. Ethanol is a liquid alcohol that is produced by the fermentation of wheat, barley, corn and sugar cane. Ethanol is extensively used in variety of products for instance; tinctures, alcoholic drinks, automotive fuel, solvents, polishes, dyes, plastics, and medicines. It is even more being used as a bio-fuel. The disappearing oil can be replaced by the ethanol which has been developed as feasible energy resolution and also it facilitates to reduce the greenhouse effect. Indeed, ethanol is known to be temperature sensitive as far as viscosity and Prandtl number are concerned. The research work regarding the application of ethanol can be found in [8 - 10].

The effects of temperature-dependent viscosity and Prandtl number on an unsteady, three-dimensional boundary layer flow of a viscous, incompressible, electrically conducting fluid (ethanol), in presence of transverse magnetic field have been examined in the present investigation which, to the best of our knowledge, has not been considered so far. Further, both the fluid viscosity and Prandtl number are taken as inverse linear functions of temperature. It is remarked here that Prandtl number is a function of viscosity and as viscosity varies across boundary layer, the Prandtl number varies, too. Since the assumption of constant Prandtl number leads to unrealistic results when viscosity is a strong function of temperature [11, 12], it is proposed to obtain results when both, viscosity and Prandtl number, are assumed to vary with temperature.

2. MATHEMATICAL ANALYSIS

Let us consider the flow of an unsteady, MHD mixed convection laminar incompressible electrically conducting fluid (ethanol) with temperature dependent viscosity (μ) and the Prandtl number ($\Pr$) near the stagnation point of a three-dimensional porous body. We choose a cartesian coordinate system (x, y, z) with the stagnation point as the origin in such a way that the axes are aligned with the plane of symmetry, the $z-$axis being normal to the surface (Fig.1). A transverse magnetic field is applied in $z-$direction perpendicular to the surface of the body and it is assumed that magnetic Reynolds number of the flow is small enough so that the induced magnetic field is negligible.

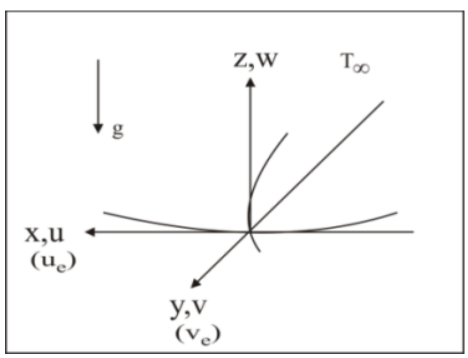

Fig.1. Physical model and coordinate system

The fluid properties except the viscosity and Prandtl number are assumed to be constant. The fluid is assumed to flow with moderate velocities, and the temperature difference between the surface and the free stream is small ($< 40^0 C$). In the range of temperature (T) considered (*i.e.,* $0^0 - 40^0 C$) the variation of both density (ρ) and specific heat (c_p) of ethanol with temperature, is less than 1% (Table I) and hence they are taken as constants. However, since the thermal conductivity (k) and viscosity (μ) [and hence Prandtl number ($\Pr$)] variation with temperature is quite significant, and hence they have been considered as temperature dependent. The viscosity and Prandtl number are assumed to vary as an inverse linear function of temperature viz.,:

$$\mu = \frac{1}{(b_1 + b_2 T)} \tag{1}$$

Paper ID: SR26116105610 DOI: https://dx.doi.org/10.21275/SR26116105610

$$\Pr = \frac{1}{(c_1 + c_2 T)} \tag{2}$$

where

$$\left.\begin{array}{ll} b_1 = 53.804, & b_2 = 1.584 \\ c_1 = 0.0428, & c_2 = 0.0006 \end{array}\right\} \tag{3}$$

The numerical data, used for these correlations, are taken from Vargaftik [13]. The relation (1) and (2) are reasonably good approximations for liquids such as ethanol, particularly for small wall and ambient temperature differences.

Table I: Values of thermo-physical properties of ethanol at different temperatures [13]

Temperature (T) (^{0}C)	Density (ρ) (gr./cm^3)	Specific heat(c_p) (J × 10^7/kg ^{0}K)	Thermal conductivity (k) (erg × 10^5/cm.s-^{0}K)	Viscosity (μ) (gr. × 10^{-2}/ cm-s)	Prandtl number (Pr)
0	0.8037	2.2416	0.1721	1.7730	23.0906
10	0.7981	2.2416	0.1703	1.4662	20.1064
20	0.7901	2.4283	0.1670	1.2003	17.4532
30	0.7821	2.5296	0.1611	1.0035	13.7531
40	0.7741	2.6242	0.1585	0.8340	12.6316

As the fluid is incompressible, the contribution of heating due to compression is very small and it has been neglected. By using the empirical formula (2) the $\Pr$ value of ethanol can be approximated as 17.0 at the temperature $T = 28.7^0 C$. Under these assumptions, the governing equations of the boundary layer flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) - \frac{\sigma B_0^2}{\rho}(u - u_e) + \frac{[g\beta\, x(T - T_\infty)]}{L} \tag{5}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \frac{\partial v_e}{\partial t} + v_e\frac{\partial v_e}{\partial y} + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right) - \frac{\sigma B_0^2}{\rho}(v - v_e) + \frac{[g\beta\, cy(T - T_\infty)]}{L} \tag{6}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{\mu}{\Pr}\frac{\partial T}{\partial z}\right) \tag{7}$$

subject to the initial and boundary conditions

$$\begin{array}{ll} t = 0: & u(x,y,z,t) = u_i(x,y,z), \quad v(x,y,z,t) = v_i(x,y,z) \\ & w(x,y,z,t) = w_i(x,y,z), \; T(x,y,z,t) = T_i(x,y,z) \end{array} \tag{8}$$

$$t > 0: \quad u(x,y,0,t) = 0, \quad v(x,y,0,t) = 0, \quad w(x,y,0,t) = w_w, \quad T(x,y,0,t) = T_w \tag{9}$$
$$u(x,y,\infty,t) = u_e(x,t), \qquad v(x,y,\infty,t) = v_e(y,t), \quad T(x,y,\infty,t) = T_\infty$$

Here u, v and w are the velocity components in $x-, y-$ and $z-$ directions, respectively; T is the fluid temperature; t is time; ρ is fluid density; μ is viscosity; $\Pr$, c_p and σ are Prandtl number, specific heat at constant pressure and electric conductivity respectively. B_0 is the transverse magnetic field; g is acceleration due to gravity; β is thermal expansion coefficient; L is the characteristic length. The subscripts i, w, e and ∞ denote the initial conditions, conditions at the wall, at the edge of the boundary layer and condition at infinity, respectively.

The set of non-linear partial differential equations (4)-(7) involving four independent variables x, y, z and t can be reduced to semi-similar partial differential equations involving two independent variables η and t^* by applying the following transformations:

$$\eta = \left(a/\nu\right)^{1/2} z, \quad t^* = at, \quad F = f'(\eta, t^*), \quad S = s'(\eta, t^*), \quad u = ax\phi(t^*)F(\eta, t^*),$$
$$v = by\phi(t^*)S(\eta, t^*), \qquad w = -\sqrt{a\nu}\,(f + cs)\phi(t^*), \quad G(\eta, t^*) = (T - T_\infty)/(T_w - T_\infty),$$
$$u_e = ax\phi(t^*), \qquad v_e = by\phi(t^*), \qquad a = (du_e/dx)_{t^*=0}, \qquad b = (dv_e/dy)_{t^*=0}, \qquad c = b/a \tag{10}$$

Consequently, we find that Eq.(4) is identically satisfied and Eqs. (5)-(7) are, transformed to:

$$(NF')' + (f + cs)F'\phi + (1 - F^2)\phi + \phi^{-1}\phi_{t^*}(1 - F) - F_{t^*} - M(F - 1) + \phi^{-1}\Omega G = 0 \tag{11}$$
$$(NS')' + (f + cs)S'\phi + c(1 - S^2)\phi + \phi^{-1}\phi_{t^*}(1 - S) - S_{t^*} - M(S - 1) + \phi^{-1}\Omega G = 0 \tag{12}$$
$$(N \Pr^{-1} G')' + (f + cs)G'\phi - G_{t^*} = 0 \tag{13}$$

where

$$N = \frac{\mu}{\mu_\infty} \tag{14}$$

$$\Pr = \frac{\mu C_p}{k}; \qquad \Omega = Gr/(\mathrm{Re}_L)^2; \qquad \mathrm{Re}_L = aL^2/\nu; \qquad f = \int_0^\eta F\, d\eta;$$

$$s = \int_0^\eta S\, d\eta \qquad M = \frac{\sigma B_0^2}{\rho a}; \qquad Gr = g\beta(T_w - T_\infty)L^3/\nu^2 \tag{15}$$

The transformed boundary conditions are:

$$\begin{aligned} F = S = 0, \quad G = 1 \quad \text{at} \quad \eta = 0 \\ F = S = 1, \quad G = 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{16}$$

Here N is the dimensionless viscosity ratio; μ_∞ is the viscosity of the free stream (i.e.ethanol) at $T_\infty = 28.7^0 C$, a and b are velocity gradients along x and y directions in the potential flow and $c = b/a$ is their ratio. Most of the three dimensional shapes lie between cylinder ($c = 0$) and a sphere ($c = 1$). Ω is the buoyancy parameter; Gr is the Grashof number; Re_L is the local Reynolds number based on a characteristic length L; $\Pr$ is the Prandtl number; ν is the kinematic viscosity. F and S are the dimensionless velocities; f and s are the dimensionless stream functions; T and G are dimensional and dimensionless temperatures, respectively; η and t^* are the transformed coordinates; M is the magnetic parameter. It has been assumed that the flow is initially $(t^* = 0)$

steady and changes to unsteady for $t^* > 0$. Therefore, the initial conditions for F, S and G at $t^* = 0$ are given by the steady-flow equations, obtained by putting $\phi(t^*) = 1, \quad \phi_{t^*} = F_{t^*} = S_{t^*} = G_{t^*} = 0$ in Eqs. (11)-(13). Here, ϕ is an arbitrary function of t^*, representing the nature of unsteadiness in the inviscid flow and has a continuous first derivative for $t^* \geq 0$. It may be noted that for $T_w > T_\infty$ (i.e. when the wall temperature is greater than the free stream temperature) the buoyancy force, which arises due to temperature differences, will aid the forced flow. Consequently, the buoyancy parameter $\Omega > 0$ for $T_w > T_\infty$ (i.e. for aiding flow).

The viscosity ratio N, given by the Eqn.(14), is simplified as

$$N = \frac{\mu}{\mu_\infty} = \left(\frac{b_1 + b_2 T_\infty}{b_1 + b_2 T}\right) = \frac{1}{(1 + a_1 G)} \qquad \text{where } a_1 = \left[\frac{b_2 \Delta T_w}{(b_1 + b_2 T_\infty)}\right], \text{ and } \Delta T_w = (T_w - T_\infty) \tag{17}$$

Eqn.(2) becomes

$$\Pr = \frac{1}{(c_1 + c_2 T)} = \frac{1}{(a_2 + a_3 G)} \qquad \text{where } a_2 = (c_1 + c_2 T_\infty) \text{ and } a_3 = c_2 \Delta T_w \tag{18}$$

The physical quantities of practical interest are local skin friction coefficients in the x and y directions and local heat transfer coefficient that are defined as

$$C_{f_y} = \frac{2\tau_x}{\rho(u_e^2)} = \frac{2}{\sqrt{\mathrm{Re}_x}} \phi(t^*) F'_w; \qquad C_{f_y} = \frac{2\tau_y}{\rho(u_e^2)} = \frac{2}{\sqrt{\mathrm{Re}_x}} \left(\frac{v_e}{u_e}\right) \phi(t^*) S'_w \tag{19}$$

$$Nu_x = \frac{-x(T_z)}{(T_w - T_\infty)} = -\sqrt{\mathrm{Re}_x}\, G'_w \tag{20}$$

where $\tau_x = \mu(\partial u / \partial z)_w, \quad \tau_y = \mu(\partial v / \partial z)_w, \quad \mathrm{Re}_x = a x^2 / \nu$.

3. RESULTS AND DISCUSSION

The system of partial differential Eqns. (11), (12) and (13) subjected to boundary conditions (16) have been solved numerically using the finite-difference scheme along with quasilinearization technique to obtain semi-similar solutions. Since the method is described in complete detail in [14, 15], its description is not presented here. In order to validate the accuracy of the numerical method used, the unsteady mixed convection skin friction (F'_w, S'_w) and heat transfer (G'_w) parameter results [Fig.2], have been compared with those of Kumari and Nath [16]. Our results are found to be in very good agreement.

We have considered the following unsteady free stream velocity distribution viz.,

(i) $\phi(t^*) = 1 + \varepsilon t^{*2}$, $\varepsilon = 0.25$ (accelerating free stream velocity distributions)

(ii) $\phi(t^*) = 1 - \varepsilon t^{*2}$, $\varepsilon = 0.25$ (decelerating free stream velocity distributions)

and the numerical results of the effect of variation of viscosity and Prandtl number with temperature on the flow field, skin friction and heat transfer coefficients has been obtained for various values of magnetic parameter (M).

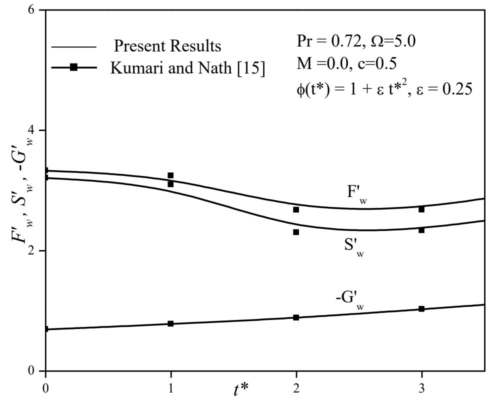

Fig.2. Comparison of skin friction and heat transfer parameter results with unsteady mixed convection flows for $\Pr = 0.7$ when $M = 0.0$ and $N = 1$ with those of [16]

In order to compare the results of constant and variable fluid properties, the governing non-dimensional equations have been solved for $N = 1$ and $\Pr = 17.0$. The variation of skin friction coefficients [$C_{f_x}(\mathrm{Re}_x)^{1/2}$, $C_{f_y}(\mathrm{Re}_x)^{1/2}$] in $x-$and $y-$directions with time (t^*) for both accelerating ($\varepsilon > 0$) and decelerating ($\varepsilon < 0$) flows characterized by $\phi(t^*) = 1 + \varepsilon t^{*2}$, are presented in Fig.3 for different values of magnetic parameter (M) in the presence of variable viscosity [$T_\infty = 28.7^0 C$, $\Delta T_w = 10.0$] and constant viscosity [$N = 1$ and $\Pr = 17.0$]. It is observed that both, $C_{f_x}(\mathrm{Re}_x)^{1/2}$ and $C_{f_y}(\mathrm{Re}_x)^{1/2}$ increase with the increase of M during both accelerating and decelerating flow of the fluid. This is due to the fact that since the free stream velocity is assumed to be accelerating; it imparts additional momentum in the flow filed. Also, the presence of magnetic field exerts viscous drag force on the flow filed which opposes the transport phenomena, and hence an increase in the absolute value of the velocity at the surface, enhances surface shear stress at the wall. Further, for $\varepsilon > 0$, $C_{f_x}(\mathrm{Re}_x)^{1/2}$ [See Fig.3(a)] and $C_{f_y}(\mathrm{Re}_x)^{1/2}$ [See Fig.3(b)] increases monotonically as t^* increases ($0 \le t^* \le 2.0$) whereas opposite trend is observed for $\varepsilon < 0$. Nevertheless, the effect of temperature-dependent viscosity/Prandtl number is to increase both, $C_{f_x}(\mathrm{Re}_x)^{1/2}$ and $C_{f_y}(\mathrm{Re}_x)^{1/2}$ for a preset value of M ($\neq 0$).

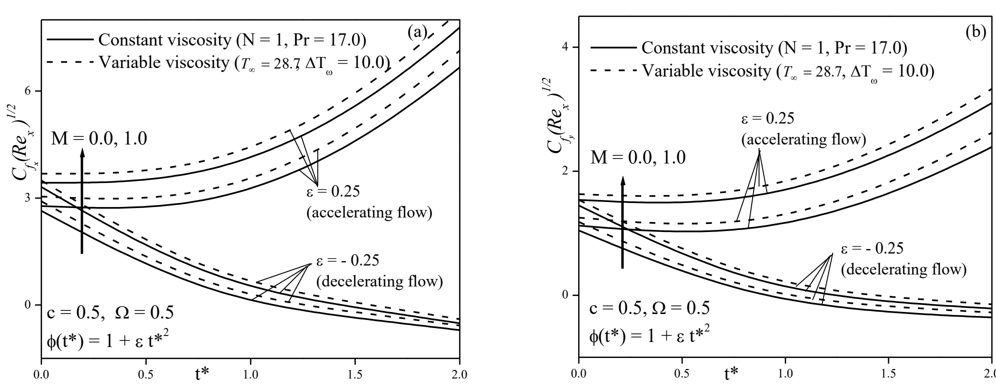

Fig.3. Comparison of variable fluid property results with constant fluid properties results (ethanol) of skin friction coefficients (a) $C_{f\,x}(\mathrm{Re}_x)^{1/2}$ (b) $C_{f\,y}(\mathrm{Re}_x)^{1/2}$ for different magnetic parameter

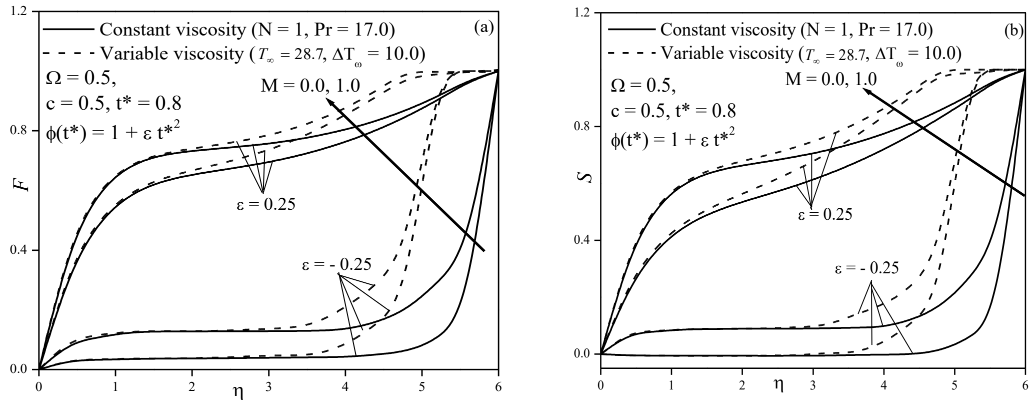

Fig.4. Velocity (x - and y - directions) profiles for different values of M

The velocity profiles in both $x-$direction (F) and $y-$ direction (S) for increasing magnetic field at time $t^* = 0.8$ are presented in Fig.4. It is seen from the figure that, as magnetic field increases the velocity profiles also increases. The velocity profiles for accelerating flow are steeper than those of decelerating flow. The effect of variable viscosity is to increase the velocity when compared to constant viscosity.

The effect of magnetic parameter (M) on heat transfer coefficient [$Nu_x(\mathrm{Re}_x)^{-1/2}$] and temperature profile (G) are exhibited in Fig.5 and Fig.6 when flow is both accelerating and decelerating. In case of accelerating flow [Fig.5(a)], it is observed that $Nu_x(\mathrm{Re}_x)^{-1/2}$ increases as M increases from 0.0 to 1.0, this clearly shows that due to enhanced Lorentz force there is an increase in the absolute value of the temperature gradient at the surface, enhancing the rate of heat transfer at the wall. In case of decelerating flow [Fig.5(b)] as M increases, $Nu_x(\mathrm{Re}_x)^{-1/2}$ decreases for constant fluid properties and increase for variable fluid properties. Also, it is observed that in the absence of magnetic field (M) the effect of variable viscosity is to decrease heat transfer coefficient [$Nu_x(\mathrm{Re}_x)^{-1/2}$], whereas when M = 1.0 it enhances $Nu_x(\mathrm{Re}_x)^{-1/2}$. As time t^* increases $Nu_x(\mathrm{Re}_x)^{-1/2}$ increase monotonically, but after certain time it becomes constant [Fig.5(b)]. The temperature profiles (G) for accelerating flow [Fig.6(a)] show that thermal boundary thickness decrease as M increases from 0.0 to 1.0. Fig.6(b) reveals that during decelerating flow, the thickness of thermal boundary layer increases as M increases in case of constant viscosity, while opposite trend is observed in case of variable viscosity.

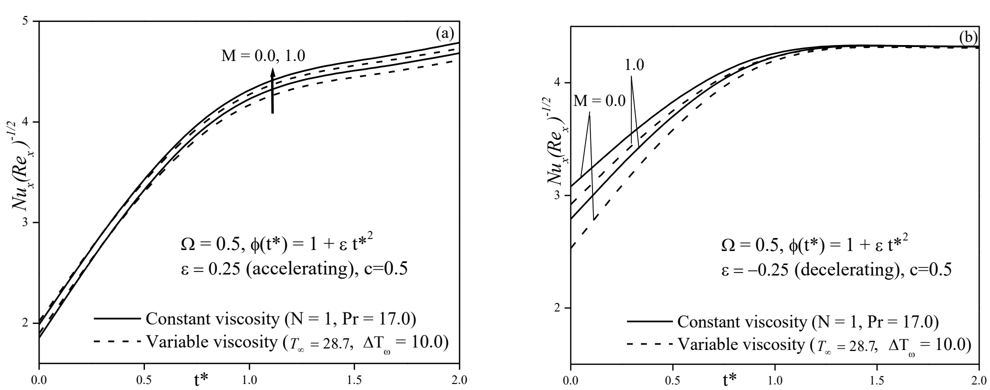

Fig.5. Variation of heat transfer coefficient with unsteady parameter for different values of M (a) accelerating and (b) decelerating flow

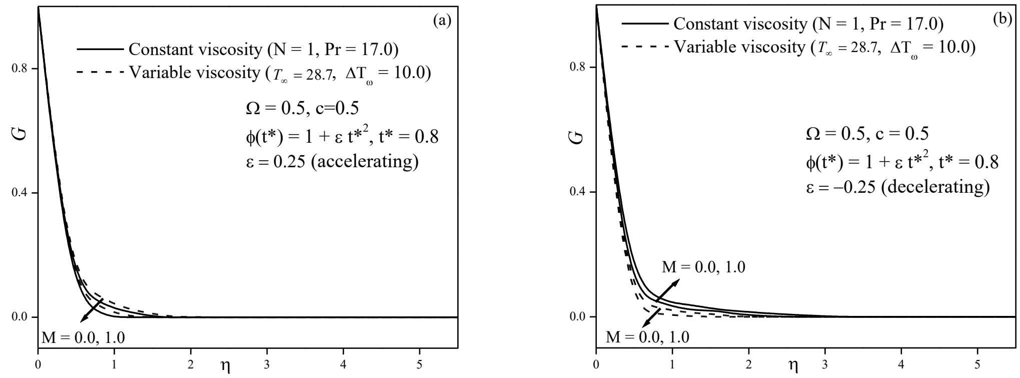

Fig.6. Temperature profiles for various values of M (a) accelerating flow (b) decelerating flow

4. CONCLUSIONS

The problem of unsteady MHD mixed convective ethanol boundary layer flows at a three-dimensional stagnation point with variable viscosity and Prandtl number has been considered. The results indicate that the magnetic field increases both skin friction (x and y directions) and heat transfer coefficients. Consideration of the impact of variable viscosity and Prandtl number facilitates to evade the considerable errors in the calculation of skin friction and heat transfer coefficients at the wall.

***Acknowledgment*:** The author is thankful to the reviewer for their valuable comments and suggestions.

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