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Graph-Theoretic Analysis of Unilateral Electrical Networks Using Signal Flow Graphs and Mason's Gain Formula

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Abstract: ***The transfer function of an electrical circuit can be derived using various analytical techniques. This study explores the use of Signal Flow Graphs (SFGs) and Mason's Gain Formula to evaluate the transmittance of a unilateral electrical network. By constructing the SFG representation of a given circuit and applying reduction rules alongside Mason's method, the paper demonstrates how both approaches yield the same transfer function, thereby validating their accuracy and complementarity. The findings underscore the effectiveness of graph-theoretic modeling in simplifying complex network analyses and highlight its utility in circuit design and optimization.***

Keywords: Signal Flow Graph, Mason's Gain Formula, Unilateral Network, Graph Theory, Transfer Function

1. Introduction

Mathematics plays a vital role in various fields. Graph theory is among the most critical and exciting branches of mathematics. The principles of graph theory are applied across computing, the social sciences, and the natural sciences. Using graph theory, many mathematical problems can be formulated as models to address real-world problems across various fields [1,2,10,11,12]. Various forms of optimization can be addressed using graph theory in Physics, Chemistry, Computer Science, and electrical engineering. It also has close relationships with other branches of mathematics, such as Group Theory, Matrix Theory, and topology [4,9].

In 1736, Leonhard Euler (1707-1783) presented a paper at the Academy of Sciences of St. Petersburg. In this paper, he solved a well-known, long-standing problem, thereby advancing graph theory. Thus, Euler is regarded as the father of Graph Theory [1,2,3]. Euler provided a pictorial representation of the situation that led to the development of the Eulerian graph, which was published in Nature. A.F. Mobius introduced the concepts of the complete and bipartite graphs in 1840 [4,5,6]. Kuratowski developed the concept of the planner graph. The concept of a tree was introduced by Gustav Kirchhoff in 1845. Kirchhoff used graph-theoretic ideas to calculate currents in electrical circuits. In 1852, Thomas Gutherie found the famous four-color problem, which was solved after a century by Kenneth Appel and Wolfgang Haken. In 1856, Thomas. P. Kirkman and William R. Hamilton introduced another critical concept: the Hamiltonian graph [6,7,8]. In 1913, H. Dudeney posed a puzzle. This period is often considered the formal birth of Graph Theory. It remained dormant until the 2nd half of the 19th century. In recent years, graph theory has advanced significantly and has become a field of study in its own right.
This paper aims to evaluate the transfer function of unilateral electrical networks using both Signal Flow Graph (SFG) reduction rules and Mason's Gain Formula, demonstrating their equivalence in analytical circuit modeling.

1.1 Graph Theory and Network Theory

Graph Theory is used to model transport networks, activity networks, and game theory. Using network activity, large numbers of combinatorial problems are solved. Graph theory is used in many popular and successful network applications.

In brief, graph theory [13,14] is used across various research areas for its flexibility in assigning properties to vertices and edges. Various graph-theoretic properties are used to solve problems in these fields.

1.2 Signal Flow Graph (SFG):

The Signal Flow Graph (SFG) is a method to present the internal structure of a system. This representation enhances understanding of how the system functions. It increases clarity and knowledge of the circuit [14, 15]. The SFG provides a clear, simple visual representation of the problem. The Signal Flow Graph is a suitable method for representing the circuit.

The SFG analysis offers a faster and more effective alternative to a complex structure. In network analysis, the SFG is an essential alternative method [20,21]. The key to understanding a circuit is always its physical structure [14]. Since SFG is a comprehensible method, only a small amount of material is required, thereby minimizing cost. Despite this, the method's signal-flow graph still requires extensive research.

Claude Shannon introduced the Signal Flow Graph (SFG), also known as the Mason graph [18,19,20]. Samuel Jefferson Mason, who first coined the term as a specialized flow graph [15,16,17,22]. Initially, the concept of Signal Flow Graph was introduced by Shannon [1942] in the context of analog computers. The most excellent credit is due to Mason [1953] [1956] for the formulation of Signal Flow Graphs, as he demonstrated how to use the technique to solve complex electronic problems in a relatively straightforward manner. The term Signal Flow Graph was initially used in the analysis of electronic problems involving signal flow and system flowcharts.

The significance of this study lies in demonstrating that graph-theoretic approaches are practical and pedagogically valuable alternatives to traditional methods for analyzing electrical networks.

2. Preliminaries (Basic definitions)

Definition 2.1. Node: A point that represents a signal or variable is called a node.

Definition 2.2. Branch: A branch is a line segment that joins two nodes.

Definition 2.3. Graph: A graph is a set of ordered pairs G = (V, E) where V is called the set of vertices (or nodes) of the graph G and the elements of E are called edges.

Definition 2.4. Self-Loop: A self-loop is an edge e where the initial and final vertices are the same.

Definition 2.5. Degree: The degree of the vertex v, denoted as $\deg(v)$ is the number of edges adjacent to v with self-loop counted twice.

Definition 2.6. Vertex Adjacency: For a Grap G, two vertices are said to be adjacent if they have a common edge between them. A vertex v is self-adjacent if there is a self-loop.

Definition 2.7. Edge Adjacency: For a Graph G, two edges are said to be adjacent if they have a common vertex between them.

Definition 2.8. Transmittance: The term "transmittance" denotes the gain from one node to another.

Definition 2.9. Walk: A walk is a finite alternating sequence of vertices and edges such that each edge is incident on the vertices preceding and following it.

Definition 2.10. Path: A path is a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list.

Definition 2.11. Loop: A loop is a closed path that begins and ends at the same vertex, without passing through any node more than once.

Definition 2.12. Electrical network: An electrical network is an interconnection of electrical components. An electrical circuit is a specific type of network [23,24] that forms a closed loop.

Definition 2.13. Circuit: A closed electric network is called a circuit.

Definition 2.14. Loop gain: The gain achieved by following a path that starts and ends at the same node is called loop gain.

Definition 2.15. Non-Touching Loops: Loops that do not share any nodes.

Definition 2.16. Forward Path: A forward path is a sequence that begins at an initial node, continues to another node, and ends at an output node.

Definition 2.17. Feedback path: A feedback path is defined as a path that begins and ends at the same node.

Definition 2.18. Forwarded path gain: The forwarded path gain is the total of gains obtained by tracing a path from an input node to an output node in a signal flow graph.

Definition 2.19. Directed Graph (Digraph): A directed graph, commonly referred to as a digraph, is a type of graph in which each edge has a specific direction.

Definition 2.20. Transfer Function: The transfer function of a system, denoted as T(S), is represented as C(S)/R(S). Here, R(S) means the system's input, illustrated by a single flow graph, while C(S) denotes the system's output, represented by the signal flow graph.

3. Methodology

The proposed methodology is implemented through a multistage process, from modelling the electrical network using graph theory to signal-flow analysis for evaluation. This paper shall first represent the electrical network as a graph. Next, after the network is represented as a graph, optimize it, and develop the SFG of the electrical network to describe the signal transmission behaviour. The nodes and edges of the graph correspond to the components and connections of the original electrical network, and directed edges represent signal flow. Mason's Gain Formula and the reduction rules of SFG applied to yield the network's overall transmittance and, in turn, the transfer function relating the input and output signals.

3.1. Reduction Rule of Signal Flow Graph (SFG)

Rule1:

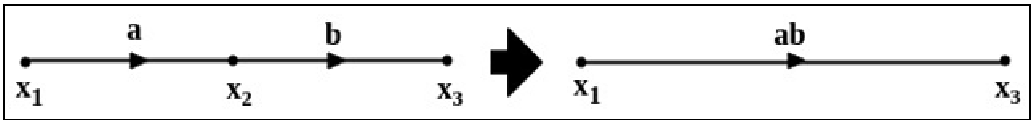

Figure 1

If x_1, x_2 & x_3 are nodes where x_1 is input and x_3 is output with gains a and b. After taken out x_2 We have, $x_2 = ax_1$, $x_3 = bx_2 = abx_1$

Rule2:

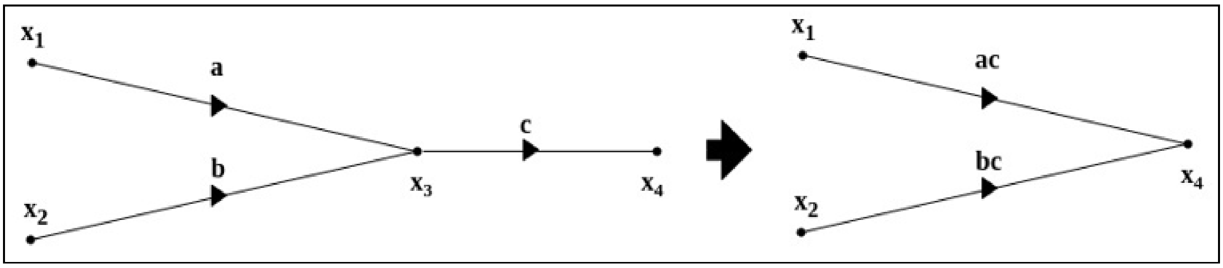

Figure 2

If x_1, x_2, x_3 *and* x_4 are nodes, and a, b, and c are gains of branches. Then $x_3 = ax_1 + bx_2$ and $x_4 = cx_3 = c(ax_1 + bx_2)$. There fore $x_4 = acx_1 + bcx_2$

Rule3: Loop reduction:

(1) Multiloop with touching loops:
For the case that the two loops do share at least one node, as shown in the figure below, the overall transmission is given by

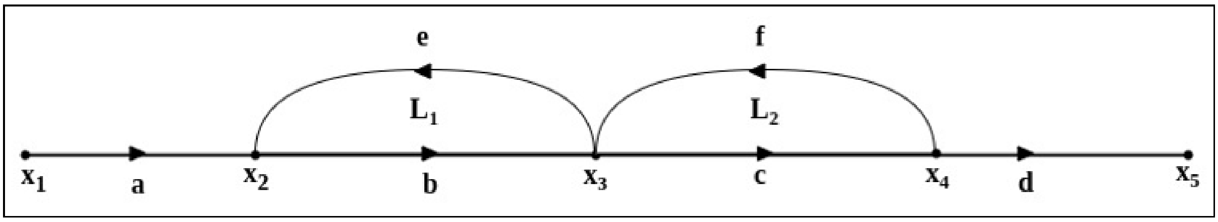

Figure 3

Here, the nodes are denoted as x_1, x_2, x_3, x_4 *and* x_5 and a, b, c, d, e, and f represent the gains of the branches shown in the figure. Then the transfer function between x_1 *and* x_5 We have the following:

$$T_{15} = \frac{x_5}{x_1} = \frac{abcd}{1-be-cf} = \frac{P_{15}}{1-L_1-L_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

3.2 Mason's Gain Formula

An input variable in a signal flow graph is connected to an output variable by the net gain from the input to the output node, also known as the system's overall gain. The overall gain (transfer function) of signal flow graphs may be found using Mason's gain formula.

The transfer function of the system is provided by

$$T(S) = \frac{C(S)}{R(S)} = \sum_{k=1}^{N} \frac{P_k \Delta_k}{\Delta} \quad (1)$$

Where,

- C(s) represents node of output
- R(s) represents node of input
- P_k represents the k^{th} forward path gain
- T represents the gain or transfer function between R(s) and C(s)
- $\Delta = 1 -$ (Sum of individual loop gains) $+$ (sum of two non-touching loops gain) $-$ (sum of three non-touching loops gain)+ (sum of four non-touching loops gain) $- \cdots$
- Δ_k forms by removing the loops that are touching the k^{th} forward path

4. Experimental Setup

4.1 An electrical network illustration:

Graphs can be used to represent electrical networks, since each network node corresponds to a vertex and each branch to an edge. It is straightforward to solve a network to determine the loop currents and node voltages when the circuit is simple. However, as the circuit's difficulty increases, we must switch to an alternative approach. As a result, we can solve the problem more easily by representing the electrical network as a graph. A network's graph is crucial to understanding the circuit. Below is an illustration of the signal flow graph for an electrical network:

4.1.1 Unilateral Circuit

A circuit whose property of the circuit changes with the change of direction of supply voltage or current is called a unilateral network. A diode rectifier is the best example of a unilateral network. Let us consider the following Unilateral electrical circuit and analyze it by calculating the transfer function using a Signal-Flow Graph (SFG).

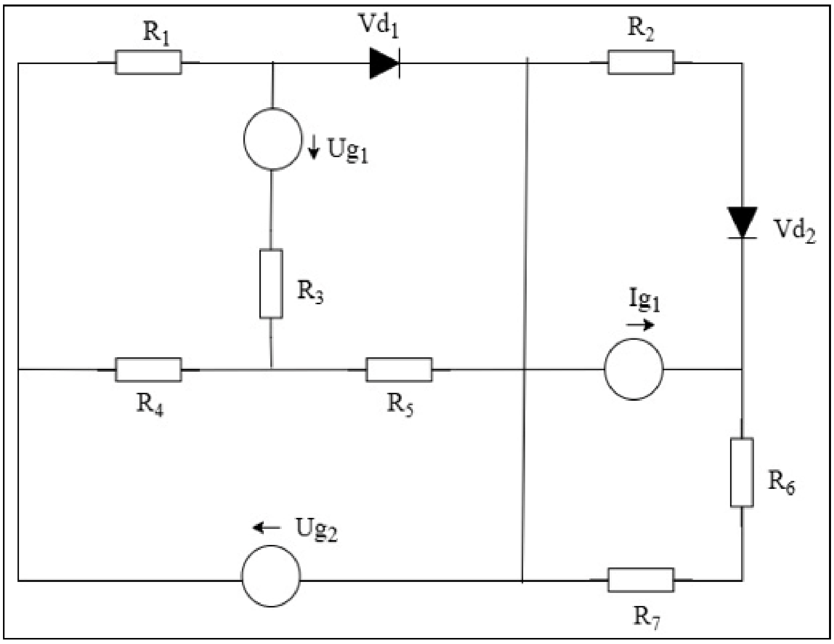

Figure 4

4.2 A graph model representation

A graph model used to represent a network by using the vertices and edges that cover the entire network. Given below is the graph model of the circuit shown above

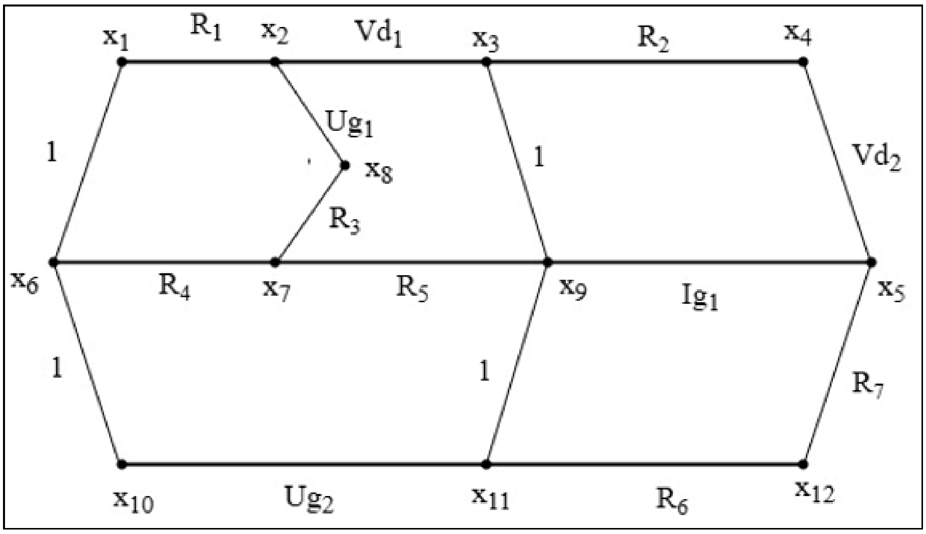

Figure 5

4.3 Matrix representation of graphs:

Matrix representations are among the most essential tools in graph theory. A graph is represented by its adjacency and incidence matrices, which are its two principal forms. For our vertices, $x_i$and x_jThe square matrix with entries 0 or 1 is called the adjacency matrix (eqn. 2) of the graph G, denoted A(G).

$$a_{ij} = \begin{cases} 1 & if \;\; x_i \; and \; x_j \; are \; adjacent \\ 0 & if \;\; x_i \; and \; x_j \; are \;\; not \; adjacent \end{cases} \tag{1}$$

$$\mathbf{A\,(G)} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{2}$$

An incidence matrix denoted by $I(G)$ for the graph G with n vertices (x_j) and m edges (e_i) is defined as a $m \times n$ matrix whose components are given by

$$a_{ij} = \begin{cases} 1 & if \;\; e_i \; is \; incident \; with \; x_j \\ 0 & if \;\; e_i \; is \; not \; incident \; with \; x_j \end{cases} \tag{3}$$

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Equation (4) displays the adjacency matrix for Fig.5 with the vertex set
$X = \{1,2,3,4,5,6,7,8,9,10,11,12\}$ and the edge set
$E = \{R_1, R_2, R_3, R_4, R_5, R_6, R_7, Vd_1, Vd_2, Ug_1, Ug_2, Ig_1, x_1x_6, x_6x_{10}, x_3x_9, x_9x_{11}\}$.

$$\mathbf{I(G)=} \begin{bmatrix} 1&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&1&0&0&0&0\\ 0&0&0&0&0&1&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&1\\ 0&0&0&0&1&0&0&0&0&0&0&1\\ 0&1&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&1&0\\ 0&0&0&0&1&0&0&0&1&0&0&0\\ 1&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&1&0 \end{bmatrix} \quad (4)$$

From the adjacency and incidence matrices of an electrical network graph, the network loop currents and node voltages can be easily found. The adjacency matrix indicates the direct connection of nodes (components), and the incidence matrix explains the attachment of the nodes to the edges (branches). These matrices enable the formulation and solution of linear systems that model network behavior and satisfy Kirchhoff's laws. This method makes it straightforward to determine the current through the loops and the voltage at various nodes, making the analysis of the electrical network clear and methodical.

4.4 Digraph Representation

A digraph is a graph whose vertices and directed, weighted edges cover the entire network. The digraph model of the circuit shown above is presented below.

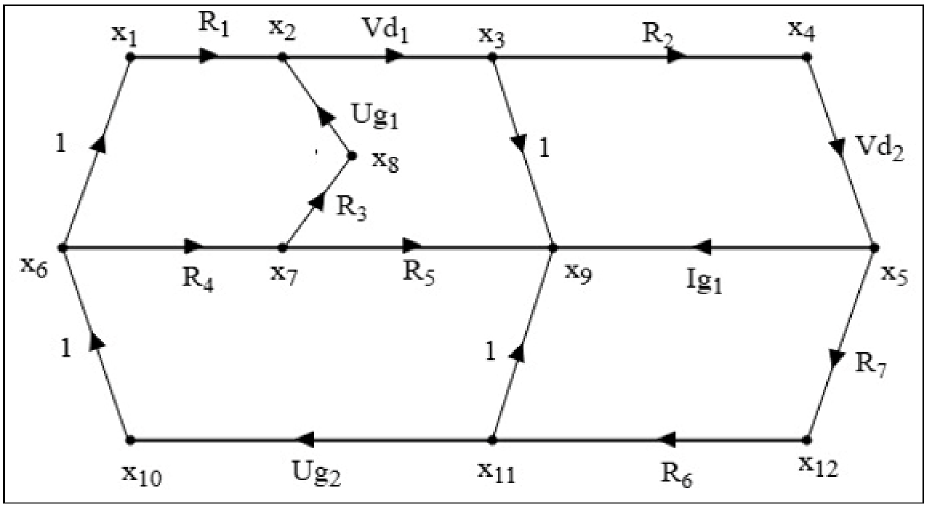

Figure 6

4.5. Signal Flow Graph (SFG) representation

A signal flow graph (SFG) model is used to represent a diagraph by using the vertices and a directed and weighted graph that covers the entire network. The signal flow graph (SFG) model of the network shown above is presented below.

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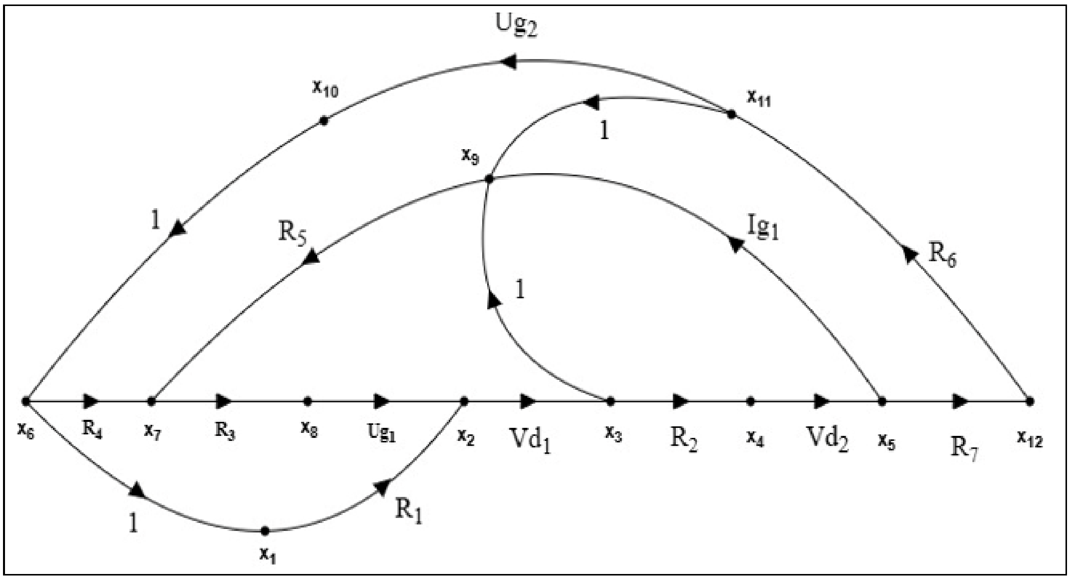

Figure 7

5. Calculation of Transfer Function by Reduction of the signal flow graph:

(i) Nodes x_1, x_8, x_4 and x_{10} are taken out by using Rule1, we have

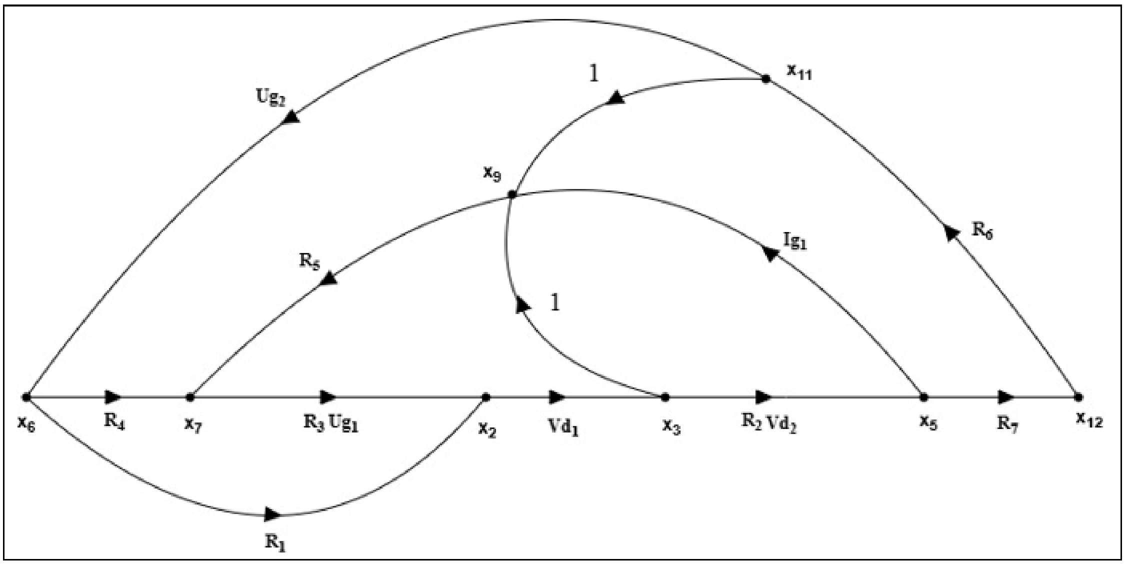

Figure 8

(ii) Using Rule 2 between the nodes x_3 and x_7 by taking out the node x_9 We have the edge with lode. $R_5.1 = R_5$

Using Rule2 between the nodes x_5 and x_7 by taking out the node x_9 We have the edge with lode. $Ig_1.R_5 = R_5Ig_1$

Using Rule2 in between the nodes x_{12} and x_7 by taking out the node x_{11} and x_9 We have the edge with lode. $R_6.1.R_5 = R_5R_6$

Using Rule2 between the nodes x_{12} and x_6 by taking out the node x_{11} We have the edge with lode. $R_6.Ug_2 = R_6Ug_2$

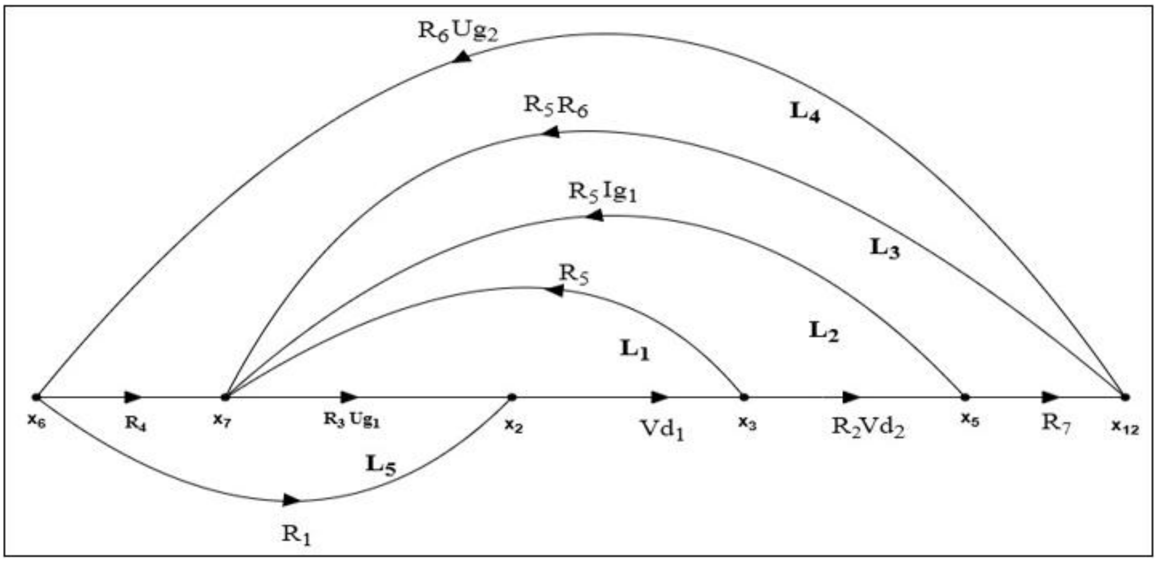

Figure 9

Here, the loops with gains are

$$L_1 = R_3Ug_1Vd_1R_5$$
$$L_2 = R_3Ug_1Vd_1R_2Vd_2R_5Ig_1$$
$$L_3 = R_3Ug_1Vd_1R_2Vd_2R_5Ig_1$$
$$L_4 = R_4R_3Ug_1Vd_1R_2Vd_2R_7R_6Ug_2$$
$$L_5 = R_1Vd_1R_2Vd_2R_7R_6Ug_2$$

Here in this Signal Flow Graph, forward path gains are as follows:

$$F_1 = R_4R_3Ug_1Vd_1R_2Vd_2R_7$$
$$F_2 = R_1Vd_1R_2Vd_2R_7$$

Forwarded path gains between x_6 and x_{12} are

$$P_{6,12} = F_1 + F_2 = R_4R_3Ug_1Vd_1R_2Vd_2R_7 + R_1Vd_1R_2Vd_2R_7$$

Therefore, Transfer Function $T(S) = \frac{C(S)}{R(S)} = \frac{P_{6,12}}{1-L_1-L_2-L_3-L_4-L_5}$

$$T(S) = \frac{R_4R_3Ug_1Vd_1R_2Vd_2R_7 + R_1Vd_1R_2Vd_2R_7}{(1 - R_3Ug_1Vd_1R_5 - R_3Ug_1Vd_1R_2Vd_2R_5Ig_1 - R_3Ug_1Vd_1R_2Vd_2R_5Ig_1 - R_4R_3Ug_1Vd_1R_2Vd_2R_7R_6Ug_2 - R_1Vd_1R_2Vd_2R_7R_6Ug_2)}$$

…………………………………………………………………………. (A)

6. Calculation of Transfer Function by using Mason's formula from Signal Flow Graph (SFG) representation

Mason's Gain Formula (MGF): The Mason's Gain Formula (MGF) is a formula for calculating the transfer function of a linear signal flow graph. Let R(S) denote the system input in the signal-flow graph model. C(S) represents the system output, as shown in the signal flow graph.

The system's transfer function is given by $T(S) = \frac{C(S)}{R(S)}$

Overall gain $T = T(S) = \sum_{k=1}^{N} \frac{P_k \Delta_k}{\Delta}$

Where,
N = Number of paths from input to output
P_k= Forward path gain of the kth forward path
Δ = 1 - (Sum of individual loop gains) + (sum of two non touching loops gain)
- (sum of three non-touching loops gain) + (sum of four non-touching loops gain)
- …………………………
Δ_k = 1 - (loops gain which are not touching the kth forward path)

First, from Figure 9, we need to identify the forwarder's path gains. There are two forward paths in this problem, and the path gains are as follows between the nodes. x_6 and x_{12}

$$F_1 = R_4R_3Ug_1Vd_1R_2Vd_2R_7$$
$$F_2 = R_1Vd_1R_2Vd_2R_7$$

Secondly, here we have five individual loops, and their gains are as follows from the diagram in Figure 9

$$L_1 = R_3Ug_1Vd_1R_5$$
$$L_2 = R_3Ug_1Vd_1R_2Vd_2R_5Ig_1$$
$$L_3 = R_3Ug_1Vd_1R_2Vd_2R_5Ig_1$$
$$L_4 = R_4R_3Ug_1Vd_1R_2Vd_2R_7R_6Ug_2$$
$$L_5 = R_1Vd_1R_2Vd_2R_7R_6Ug_2$$

Thirdly, we must identify the non-touching-loop gain. There are no non-touching loops in this problem.

Δ= 1 −(sum of individual loops gains) +(sum of two non-touching loops gain) -(sum of 3 non-touching loops gain) +…………………

$$\Delta = 1 - (R_3Ug_1Vd_1R_5 + R_3Ug_1Vd_1R_2Vd_2R_5Ig_1 + R_3Ug_1Vd_1R_2Vd_2R_5Ig_1 + R_4R_3Ug_1Vd_1R_2Vd_2R_7R_6Ug_2 + R_1Vd_1R_2Vd_2R_7R_6Ug_2)$$

Now $\Delta_1 = 1 -$(loops gain, which does not touch the forwarded path F_1)

$$= 1 - 0$$
$$= 1$$

$\Delta_2 = 1 -$(loops gain, which does not touch the forwarded path F_2)

$$= 1 - 0$$
$$= 1$$

Therefore, Transfer Function $T(S) = \frac{C(S)}{R(S)} = \sum_{k=1}^{N} \frac{P_k \Delta_k}{\Delta}$

$$= \frac{F_1\Delta_1 + F_2\Delta_2}{\Delta}$$

$$= \frac{R_4R_3Ug_1Vd_1R_2Vd_2R_7 + R_1Vd_1R_2Vd_2R_7}{1 - (R_3Ug_1Vd_1R_5 + R_3Ug_1Vd_1R_2Vd_2R_5Ig_1 + R_3Ug_1Vd_1R_2Vd_2R_5Ig_1 + R_4R_3Ug_1Vd_1R_2Vd_2R_7R_6Ug_2 + R_1Vd_1R_2Vd_2R_7R_6Ug_2)}$$

………………………………………..(B)

We have observed that equations (A) and (B) are equal.

It is observed that the transfer function of the same electrical circuit can be consistently calculated using both the reduction method and Mason's Gain Formula. In this paper, from the same Electrical circuit (i.e., Figure 9), we find the same transfer function between the two nodes. x_6 and x_{12} by using the reduction of signal flow graph, i.e., equation (A), and by using Mason's gain formula, i.e., equation (B), where both these two equations are equal.

7. Transfer Function (TF) in Electric Circuit

In an electrical system, the Transfer Function (TF) is the ratio of output to input. The Transfer Function (TF) provides the circuit's gain, its frequency response, stability, the range of input values for which the output remains stable, and its behavior under different types of input (AC or DC). It also specifies key features. We have determined the Transfer Function (TF) of a system (circuit) and can use it to determine many of its parameters. One can use this Transfer Function (TF) to analyze the system's behavior for any input. Another thing is that they allow the whole system to be represented by **a single edge, i.e., a one-edge (I-edge)** between any two nodes. It facilitates the study of complex systems.

8. Conclusion

This study, using SFG models and their reduction rules and Mason's Gain Formula, is able to determine the transmittance of the given complex network in a very accurate way and thus put forward the circuit design theory with a novel and systematic approach to network analysis. Our results indicate significant improvements in performance and reliability, thereby yielding practical benefits for the techniques in real-world applications. Subsequent research should examine hybrid optimization methods and real-world applications to further validate and refine these innovations, yielding robust, scalable solutions for advanced circuit design.

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