

PDT Presents Novel Concepts with Direct Method to Find Minimum Network Losses and Sharing

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Abstract: *The Power Division Theorem (PDT) is a powerful formula which establishes the novel concepts (1) Operating with equal voltage at all the source nodes (Common Source node Voltage) decides the source currents, network minimum loss and other node voltages in the network (2) Each source current ratio to the total of source currents is the same in the solution for any common source node voltage (CSNV), but other node voltages and minimum loss are different. (3) Therefore, the operating voltage of the Power System is decided by the CSNV (4) All the node voltages decrease when the CSNV is reduced and vice versa. (5) Only selection of (CSNV) is needed in the innovated Novel Direct Load Flow (NDLF) method. The method is derived/proved by the Loss Minimization Lagrangian PDT (LMLPDT) load flow approach presented in this paper (6) The Lagrangian multiplier is equal to the total generation in all the solutions. (7) No need for selection of Slack Bus in the NDLF. The work presented is innovative and surpasses all the laborious existing methods applied with different concepts/approaches in power sharing, tracing, pricing, optimizing, system planning and operations. Also, very effective for Deregulated Power Environment (DPE). Most of the calculations/estimations required in Power System Analysis will be drastically reduced/eliminated. This is achieved by the fundamental PDT innovated by the basic concept of KCL Examples which include standard IEEE-5 Bus system to validate the NDLF method. The PDT must be included in the circuit theory subject of the entire universities / Institutions world over for recognition of such important theorem.*

Keywords: Total Entering Current, Common Source Node Voltage, Network Minimum Loss, Node Voltages, Operating voltage, Deregulated Power Environment, Load flow Equations, Novel Direct Load Flow, Objective function, Lagrangian Multiplication Factor, Proportional Power Sharing Principle (PSP)

1. Introduction

Minimization of Transmission network Losses (Active and Reactive Power) is an very important task in Power System Engineering. An equivalent circuit model has been proposed to split a Power Network for developing series of equivalent acyclic circuits in order to trace the contributions of generators to a specific load [1]. The effectiveness of the method is demonstrated with Power flows in finding losses including emission. It follows Proportional Sharing Principles (PSP). This work was supported by the National Science Foundation of the USA (2021) under Grant ECCS-1508910. (Corresponding author: Caisheng Wang.). Restructured power system is used to allot the transmission losses to its users. Two new algorithms are proposed [2] by using AC power flow calculations, new matrices called outflow-line and inflow-line are created within the algorithms. These matrices are used to allocate the active loss of every transmission line to each generator or load. Virtual power flow tracing with P- δ approximation by applying Superposition Theorem for assigning active power losses at network loads [3]. Geographical Information Systems (GIS) is used for developing different methodologies to track the power flow in dynamic distribution networks of smart grids [4]. Particle Swarm Optimization (PSO) is applied for optimization along with the determination of real power loss allocation in transmission lines with in a deregulated power system. PSP is used for power flow tracing [5]. Investigating and implementing various methods and algorithms applied in Power Flow Tracing to determine the individual generators to specific loads and line flows in a complex power system is presented [6]. The PSP used for tracing and numerical algorithms are applied for implementation. Optimal tracing based loss allocation in the transmission lines is carried out [7] by using (PSO). In all the existing works involve PSP for

finding/minimizing losses and determining the cost allocation with different methods in a deregulated electricity market. PDT is applied along with load flow equations in this work to find directly the minimum line losses. The Lagrangian multiplication factor is used for forming and minimizing the objective function. The presented NDLF is established by the Loss Minimization Lagrangian PDT (LMLPDT) approach. The ratio of each source current to the total of entering current in to the network is used to find its sharing on each load and network losses (PDT). The approach results equal node voltages such way to share the loads for minimizing the network losses and providing proper operating voltages. of other nodes. The Lagrangian method is applied to form the constraints. The presented method establishes new concepts of elimination of slack bus selection, determination of minimum loss for the required operating network node voltages by the proper selection of CSNV. The (LMLPDT) approach proves the method. The input generating-powers (Source powers) scheduling is not the correct approach adopted in all the conventional approaches. The sharing of loads and losses developed basically by the flow of source currents through the network to the load impedances, not by the source powers. The source currents are independent and the powers are dependent on them. Deciding the CSNV is the correct approach for the Power System problems which determines the generator currents and its sharing ratios k_i for minimum loss. A dc circuit and standard IEEE5 bus system are given as examples to validate the novel method. The detailed equations are provided for the purpose of engineers, researchers, staff and students to understand/verify the results.

2. Derivation of the Equations for minimizing Transmission Network Loss (LMLPDT) approach:

The Network loss NL is equal to the Total input power to the network minus the total of system load powers. Powers are in complex value. There are n Sources and m Loads.

$$NL = \sum_{i=1}^n S_i - \sum_{j=1}^m L_j \quad \text{-----} \quad (1)$$

$S_i = i^{\text{th}}$ Source Power input and $L_j = j^{\text{th}}$ Load Power

$$NL = \sum_{i=1}^n V_i^* \times I_i - TL \quad TL = \text{Total Load} \quad \text{-----} \quad (2)$$

$V_i = i^{\text{th}}$ Source node Voltage $I_i = i^{\text{th}}$ Source Current

$$\text{Total of Source currents } IT = \sum_{i=1}^n I_i \quad \text{----} \quad (3)$$

$$\text{Total of Load currents} = \text{Total of Source currents} = IT \quad \text{---} \quad (4)$$

$$I_i = k_i \times IT \quad k_i = \frac{I_i}{IT} \quad \text{complex Source current ratio} \quad \text{-----} \quad (5)$$

The Lagrangian objective Function Φ to minimize the Loss NL

$$\Phi = \sum_{i=1}^n V_i^* \times I_i - TL - \lambda \times (\sum_{i=1}^n k_i - 1) \quad \text{-----} \quad (6)$$

$$\Phi = \sum_{i=1}^n V_i^* \times k_i \times IT - TL - \lambda \times (\sum_{i=1}^n k_i - 1) \quad \text{---} \quad (7)$$

Differentiating Φ w.r.t the independent variable k_i and equating to zero for obtaining the condition for minimum NL.

$$\frac{d\Phi}{dk_i} = 0 \quad \text{which results} \quad (\sum_{i=1}^n V_i^* \times IT - \lambda) = 0 \quad \text{---} \quad (8)$$

$$(\sum_{i=1}^n V_i^* \times IT = \lambda) \quad \text{-----} \quad (9)$$

Equation 9 is the Lagrangian Constraint for minimum Loss.

2. Examples

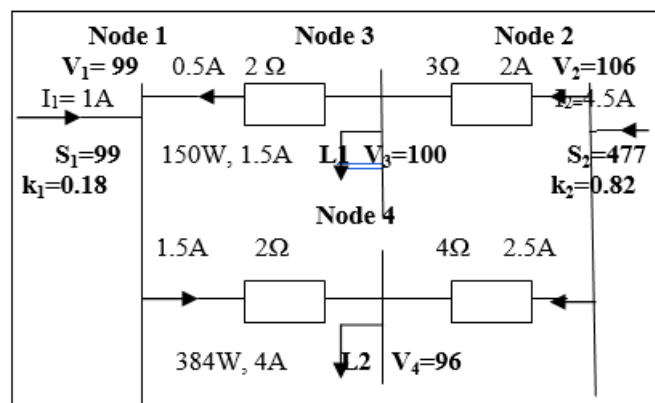


Figure 1

Fig1. is a DC system operating at a particular Point. S_1 source current is 1 ampere and S_2 source current is 4.5 ampere. The total entering current IT is 5.5 ampere. Sharing of Sources calculated by PDT is shown in the Table 1.

Power supplied by Source $S_1 = 99 \times 1 = 99$ W

Power Supplied by Source $S_2 = 106 \times 4.5 = 477$ W

Total Power supplied to the Network = $99 + 477 = 576$ W

Total Load = $L_1 + L_2 = 150 + 384 = 534$ W

Network loss by $i^2R = 1^2 \times 2 + 1.5^2 \times 2 + 2^2 \times 3 + 2.5^2 \times 4 = 42$ W

Network Loss = $576 - 534 = 42$ W.

The loss is not minimum.

Table 1

Sources	Loads	sharing	Loss sharing	Gen.
	L_1	L_2	Total	Total
S_1	27.27	69.82	97.09	7.81
S_2	122.73	314.18	436.91	40.09
Total	150.00	384.00	534.00	42.00

LMLPDT Load Flow Approach

$$\begin{aligned} \text{Node 1} \quad \frac{V_1 - V_3}{R_1} + \frac{V_1 - V_4}{R_2} &= k_1 \cdot IT \quad (1) \\ \text{Node 2} \quad \frac{V_2 - V_3}{R_3} + \frac{V_2 - V_4}{R_4} &= k_2 \cdot IT \quad (2) \\ \text{Node 3} \quad \frac{L_1}{V_3} + \frac{V_3 - V_1}{R_1} + \frac{V_3 - V_2}{R_3} &= 0 \quad (3) \\ \text{Node 4} \quad \frac{L_2}{V_4} + \frac{V_4 - V_1}{R_2} + \frac{V_4 - V_2}{R_4} &= 0 \quad (4) \end{aligned}$$

Lagrangian Constraints

$$V_1 \cdot IT = \lambda \quad (5) \quad V_2 \cdot IT = \lambda \quad (6) \quad k_1 + k_2 = 1 \quad (7)$$

There are seven variables $V_1, V_2, V_3, V_4, k_1, k_2, \lambda$ to be determined. λ (Lambda) is the Lagrangian multiplier. All the seven variables are found by solving the seven equations with initial guess value for the given value of total entering current IT in to the network.

The Table 2 shows the network loss is the minimum (24watts) in the solution of LMLPDT equations for the same value of total current $IT=5.5$ A

Table 2

$V_1=101.45, V_2=101.45, V_3=99.65, V_4=96.13, k_1=0.65, k_2=0.35$

$\lambda=558$

$I_1=3.57, I_2=1.93, IT=5.5$ A

Gen.	Loads	sharing	Loss sharing
Sources	L_1	L_2	Total
S_1	97.26	249.00	346.26
S_2	52.74	135.01	187.74
Total	150.00	384.00	534.00

Note that λ is equal to the total generation $S_1 + S_2$ and also all the source node voltages are equal in the solution. This is because of the constraint (9) for Minimum Loss. The source current ratios and the sharing of sources are different from the previous operating point. The loss is minimum. In view of the results, the following equations are used in the Novel Direct Load Flow Method. The equations are solved by deciding V (CSNV). The solution gives the source currents and other node voltages. The loss calculated is the minimum for the (CSNV). The variables are I_1, I_2, V_3 and V_4 .

Novel Direct Load Flow Method for Minimization of Net work Loss

$$\begin{aligned} \text{Node 1} \quad \frac{V - V_3}{R_1} + \frac{V - V_4}{R_2} &= I_1 \quad \text{---} \quad (1) \\ \text{Node 2} \quad \frac{V - V_3}{R_3} + \frac{V - V_4}{R_4} &= I_2 \quad \text{---} \quad (2) \\ \text{Node 3} \quad \frac{L_1}{V_3} + \frac{V_3 - V}{R_1} + \frac{V_3 - V}{R_3} &= 0 \quad \text{---} \quad (3) \\ \text{Node 4} \quad \frac{L_2}{V_4} + \frac{V_4 - V}{R_2} + \frac{V_4 - V}{R_4} &= 0 \quad \text{---} \quad (4) \end{aligned}$$

$V = \text{Common Source Node Voltage (CSNV)}$

The CSNV is set at 101.4538 (Table 2) in the above NDLF equations results the same solution as in Table 2. Note that there is no slack bus and the solution gives the source currents I_1 , I_2 and other node voltage V_3 , V_4 . The minimum loss is 24 watts.

If the total current is decreased from 5.5 to 5.25 in the LMLPDT, node voltages are increased and the source currents are decreased. The minimum loss also decreased from 24 to 21.84. $V_1=105.87$, $V_2=105.87$, $V_3=104.15$, $V_4=100.79$, $k_1=0.65$, $k_2=0.35$, $\lambda=555.84$, $I_1=3.40$, $I_2=1.85$, $IT=5.25$ A. Minimum Loss=21.84. But k_1 and k_2 are the same as in the case of 5.5 A. Set CSNV equal to 105.87 in NDLF. The results are the same as given above. Hence the NDLF (Novel Direct Load Flow method) is validated and proved. Therefore, sharing of sources and minimum loss can be determined directly by the NDLF method without LMLPDT approach.

Example FIG 2. Standard IEEE-5 Bus Power System
The system is operating at Particular Operating Point

All the system line data and the operating voltages, currents and Load powers are shown in the figure.

Slack Bus Power $S_1=1.297-j0.0898$ Base:100 MVA

Generator Bus 2 Power $S_2=0.2-j0.20$

Total Generation= $S_1+S_2=1.497-j0.2898$

Total System Load= $1.45-j0.30$

Network loss = Total Generation – Total system Load
= $S_1+S_2 - (1.45-j0.30)=0.047+j0.0102$

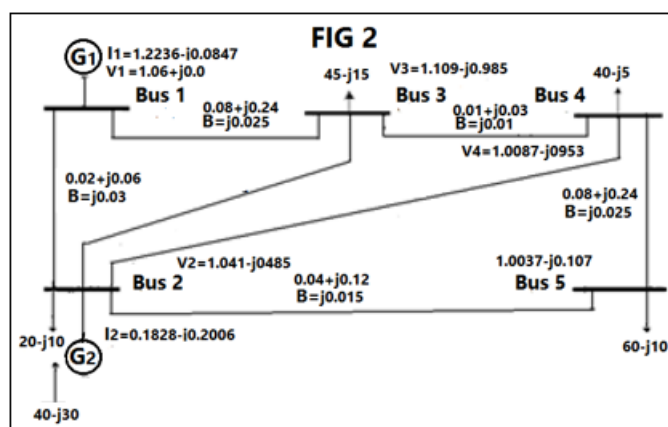


Table 3 displays the sharing of sources on loads and Table 4 shows the calculation of the effect of line capacitances. The values are given in the Table 4.

Table 3: Sharing of Loads calculated by PDT

Loads/ (Columns) Generators (Rows)	Bus 3 Load 1 S_{01}	Bus 4 Load 2 S_{02}	Bus 5 Load 3 S_{03}
Bus1 Generator 1	0.3981- j0.0768	0.3445+ j0.0023	0.5196- j0.177
Bus2 Generator 2	0.0519- j0.0732	0.0555- j0.0523	0.0804- j0.0823

Sum of loss in each line = $0.047-j0.1438$

Table 4: Effect of line charging capacitances

$B11 \times V_1^2$	$B22 \times V_2^2$	$B33 \times V_3^2$	$B44 \times V_4^2$	$B55 \times V_5^2$	Total
j0.0309	j0.0462	j0.0283	j0.0283	j0.0204	j0.154

Network loss=Sum of loss in each line + effect of capacitance
 $0.047-j0.01438+j0.154=0.047+j0.0102$

The network loss is not minimum for the Total Current in to the System network= $IT=I_1+I_2=1.4064-j0.2853$. The following equations must be solved to find the minimum loss at this operating point. In the power system (AC) networks, the total entering current must be included as one of the variable. Equation 6 is included. This is because of adjusting the real and reactive part of the IT for convergence. IT value is converged very nearer to the initial value given. Some of the networks without the equation the solution may be obtained. Equation 6 is needed in the IEEE 5 Bus network for LMLPDT approach given below.

LMLPDT Load Flow Approach

$$\begin{aligned} & \frac{V_1 - V_2}{Z_{12}} + \frac{V_1 - V_3}{Z_{13}} + B_{11} V_1 = k_1 IT \quad (1) \\ & \frac{V_2 - V_1}{Z_{12}} + \frac{V_2 - V_3}{Z_{23}} + \frac{V_2 - V_4}{Z_{24}} + \frac{V_2 - V_5}{Z_{25}} + B_{22} V_2 = k_2 IT \quad (2) \\ & \frac{V_3 - V_1}{Z_{13}} + \frac{V_3 - V_2}{Z_{23}} + \frac{V_3 - V_4}{Z_{34}} + B_{33} V_3 + \frac{S_{01}}{V_3} = 0 \quad (3) \\ & \frac{V_4 - V_2}{Z_{24}} + \frac{V_4 - V_3}{Z_{34}} + \frac{V_4 - V_5}{Z_{45}} + B_{44} V_4 + \frac{S_{02}}{V_4} = 0 \quad (4) \\ & \frac{V_5 - V_2}{Z_{25}} + \frac{V_5 - V_4}{Z_{45}} + B_{55} V_5 + \frac{S_{03}}{V_5} = 0 \quad (5) \end{aligned}$$

Node 1
Node 2
Node 3
Node 4
Node 5

Lagrangian Constraints

$$\begin{aligned} & k_1 IT + k_2 IT = 1 \quad (6) \\ & \frac{V_1}{IT} - S_{01} - S_{02} - S_{03} = \lambda \quad (7) \\ & \frac{V_2}{IT} - S_{01} - S_{02} - S_{03} = \lambda \quad (8) \\ & K_1 + K_2 = 1 \quad (9) \end{aligned}$$

There are nine variables V_1 , V_2 , V_3 , V_4 , V_5 , k_1 , k_2 , IT and λ to be determined by solving the nine load flow equations. The Table 5 shows the network loss is the minimum ($0.0312+j0.0516$) in the solution for the total current $IT=1.4338-j0.2405$ A. This value is slightly differ from the given initial IT value $1.4064-j0.2853$.

Power supplied by Source $S_1=0.2663-j0.0268$

Power Supplied by Source $S_2=1.2149-j0.2216$

Total Generation = $1.4812-j0.2484$

Network loss = Total Generation – Total system Load
= $S_1+S_2 - (1.45-j0.30)=0.0312+j0.0516$

The effect of line charging capacitances calculated as in Table 4 = $j0.1498$. Sum of loss in each line= $0.0312-j0.0982$. Network loss=Sum of loss in each line + effect of capacitance $0.0312-j0.0982+j0.1498=0.0312+j0.0516$. Note that V_1 and V_2 are equal to $1.0331+j0.0$ and λ is equal to the total generation $1.4812-j0.2484$

Table 5 Sharing of Loads calculated by PDT $V_1=1.0331+j0$, $V_2=1.033+j0$, $V_3=0.9994-j0.0575$, $V_4=0.9993-j0.0596$, $V_5=0.9965-j0.0643$, $k_1=0.1778+j0.0117$, $k_2=0.8222-j0.0117$, $IT=1.4338-j0.2405$ and $\lambda=1.4812-j0.2484$

Loads/ (Columns) Generators (Rows)	Bus 3 Load 1 S_{01}	Bus 4 Load 2 S_{02}	Bus 5 Load 3 S_{03}
Bus1 Generator 1	0.0818- j0.0214	0.0717- j0.0042	0.1079- j0.0108
Bus2 Generator 2	0.3682- j0.1286	0.3283- j0.0458	0.4921- j0.0892

The same results in Table 5 are obtained by the Novel Direct Load Flow Method just by assigning $1.0331+j0.0$ to the Common Source Node Voltage V . The equations for the

method are given below. In the equations 1 and 2 the term V_1-V_2 cancelled since they are equal. The solution validates the method. The variables are the source currents I_1 , I_2 and voltages of other nodes V_3 , V_4 , V_5 . Five variables only present. No slack bus.

Novel Direct Load Flow Method for Minimization of Network Loss

V =Common Source Node Voltage

$$\frac{V - V_3}{Z_{13}} + B_{11} V = I_1 \quad (1)$$

Node 2

$$\frac{V - V_3}{Z_{23}} + \frac{V - V_4}{Z_{24}} + \frac{V - V_5}{Z_{25}} + B_{22} V = I_2 \quad (2)$$

Node 3

$$\frac{V_3 - V}{Z_{13}} + \frac{V_3 - V}{Z_{23}} + \frac{V_3 - V_4}{Z_{34}} + B_{33} V_3 + \frac{S_{01}}{V_3} = 0 \quad (3)$$

Node 4

$$\frac{V_4 - V}{Z_{24}} + \frac{V_4 - V_3}{Z_{34}} + \frac{V_4 - V_5}{Z_{45}} + B_{44} V_4 + \frac{S_{02}}{V_4} = 0 \quad (4)$$

Node 5

$$\frac{V_5 - V}{Z_{25}} + \frac{V_5 - V_4}{Z_{45}} + B_{55} V_5 + \frac{S_{03}}{V_5} = 0 \quad (5)$$

The minimum loss and source sharing on loads are directly determined by any value of CSNV.

3. Conclusion

The work presented brought the very important novel concepts. Any Power System Networks (Grids) operating with equal source node voltage results minimum loss, proper sharing of sources on each load and loss. This concept is the outcome of the approach by applying PDT along with Lagrangian constraints in load flow (LMLPDT). Also, The Lagrangian multiplier is equal to the total of source powers in the solution. The concept of Common Source Node Voltage (CSNV) is used to frame the equations in the innovated Novel Direct Load Flow (NDLF) method which gives the same results as in LMLPDT approach. The ratios of source currents to its total value k_i decides the sharing of each load and loss (PDT). The k_i is the same value in the solution of NDLF for any value of CSNV, but the minimum loss value and other node voltages are different. If the CSNV increased from 101.4538 V to 105.87 in the DC network, the total source current reduces from 5.5 A to 5.25 A and the minimum loss reduced from 24 W to 21.84W. The same changes are present in the IEEE-5 Bus system, when the CSNV increased from 1.0331+j0.0 V to a higher value. This can be verified by any engineer, staff and students with the NDLF method presented in this paper. The NDLF method will reduce/eliminate effectively the laborious work needed in the conventional methods applied for Power System operation and control. Just deciding the CSNV in NDLF gives all values at minimum network Loss.

At present, the grids are not being operated with equal source node voltage. The Electrical Engineers must carry out practically the novel concept of equal source node voltage operation reported in this work in order to reduce the losses automatically. The loss is minimum and saves a lot of energy consumption. This is a boom for design Engineers. The students and staff can verify/realize the concept practically with DC circuit setup in the laboratory. All Universities/Engineering Colleges must include the Fundamental Theorem (PDT) in the circuit theory subject for recognition of such very important theorem world over.

References

- [1] An Equivalent Circuit-Based Approach for Power and Emission Tracing in Power Networks by Caisheng Wang, *Senior Member, IEEE*, Yang Wang, *Senior Member, IEEE*, Mahdi Rouholamini, *Senior Member, IEEE*, and Carol Miller. 2021; accepted March 11, 2021. This work was supported by the National Science Foundation of the USA under Grant ECCS-1508910. (Corresponding author: Caisheng Wang.)
- [2] A. Enshaee, G. R. Yousefi, and A. Ebrahimi, "Allocation of transmission active losses through a novel power tracing-based technique," *IET Gener., Transmiss. Distrib.*, vol. 12, no. 13, pp. 3201–3211, 2018
- [3] Adhip, D. Thukaram, and G. Gurralla, "Loss allocation based on active power flow tracing," in *Proc. Nat. Power Syst. Conf.*, Bhubaneswar, India, 2016, pp. 1–5
- [4] E. Vega-Fuentes, J. Yang, C. Lou, and N. K. Meena, "Transaction-oriented dynamic power flow tracing for distribution networks—Definition and implementation in GIS environment," *IEEE Trans. Smart Grid*, vol. 12, no. 2, pp. 1303–1313, Mar. 2021.
- [5] K. Dhayalini and R. Mukesh, "Optimal tracing based real power loss allocation in transmission lines," in *Proc. Int. Conf. Power Embedded Drive Control*, Chennai, India, 2017, pp. 20–25.
- [6] K. Berg, "Power flow tracing: Methods and algorithms," Master's thesis, Dept. Elect. Power Eng. Norwegian Univ. Sci. Technol., Trondheim, Norway, 2017.
- [7] K. Dhayalini and R. Mukesh, "Optimal tracing based real power loss allocation in transmission lines," in *Proc. Int. Conf. Power Embedded Drive Control*, Chennai, India, 2017, pp. 20–25.