Topological Indices in Fuzzy Graphs

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Abstract: ***The fuzzy first Zagreb index $M_1(G)$ is defined as the sum of the product of the membership value of the vertex u and the square of the degree of the vertex. The fuzzy second Zagreb index $M_2(G)$ is defined as half of the sum of the products of the degrees of adjacent vertices and the membership value of fuzzy graph. In this paper, first, second Zagreb indices, forgotten-index, Randić index, harmonic index, Sombor index, Nirmala index, first entire Zagreb index, hyper Zagreb index and second reduced Zagreb index of fuzzy graph, complement fuzzy graph, complete fuzzy graph, complete bipartite fuzzy graph, wheel fuzzy graph and anti-fuzzy graph are studied.***

Keywords: Anti-fuzzy, complement, complete, complete bipartite, fuzzy graph, fuzzy topological indices, Randić index, Sombor index, wheel graph

1. Introduction

The fuzzy graph G is an extension of a classical (crisp) graph, wherein vertices and edges are identified with degrees of membership rather than binary values. Graph wherein the edges are unordered vertex pair is called an undirected graph. Fuzzy graphs are extension of classical graphs in which each edge and or vertex is assigned a membership value in [0,1] indicating the level of association or certainty. A graph is a symmetric binary relation on a nonempty set V. Similarly, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. In graph theory, crisp topological indices are computed based on binary vertex and edge relationship, assuming precise structural information. In contrast fuzzy topological indices incorporate the fuzzy membership values of the vertices and fuzzy degrees of vertices through fuzzy set, allowing the modelling of uncertainty. Fuzzy graph is an expression of fuzzy relation and thus the fuzzy graph is frequently expressed as fuzzy matrix. Fuzzy graph is combination of graph theory and fuzzy set theory. This is known as fuzzy graph theory. Fuzzy graph theory has wider range of applications in logic, algebra, topology, operations research, pattern recognition, artificial intelligence, neural networks and several other fields [1-2]. Fuzzy graph theory marks a notable departure from traditional graph theory by incorporating the notion of fuzziness into graph structures, thus allowing for the representation of imprecise and uncertain information [3-4].

Some results on energy of a fuzzy graph are studied in [5]. Mordeson and Peng introduced the concept of operations on fuzzy graphs [6]. Significance of the Sombor index in fuzzy graph were studied in [7]. The topological indices of fuzzy random graphs were determined by merging the fuzzy and random graph indices and applying them to the corresponding edges in [8]. Some results of union of two fuzzy graphs were studied in [9]. A fuzzy set A on a set X is characterized by a mapping m:X→ [0,1], which is called the membership function. A fuzzy set is denoted by A= (X, m). If there is any uncertainty in the set boundaries of objects, the concept of fuzzy set is used [10].

A fuzzy graph with V as the underlying set is a pair of functions G:(σ, μ), where σ: V → [0,1] is a fuzzy subset, μ: V × V → [0,1] is a fuzzy relation on the fuzzy subset such that for all u, $v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, where $\wedge$ stands for minimum [11-18]. Here σ(v) and μ(u, v) represent the membership values of the vertex u and of the edge (u, v) respectively. We assume that V is finite and non-empty, μ is reflexive and symmetric. Two nodes u and v are said to be neighbours if $\mu(u, v)>0$. The underlying crisp graph of a fuzzy graph G:(σ, μ) is denoted by G*:$(\sigma*, \mu*)$, where $\sigma* = \{u \in V / \sigma(u) > 0\}$ and $\mu* = \{(u,v) \in V \times V / \mu(u, v) > 0\}$. The degree of a vertex u is $d_G(u)=\sum_{u \neq v} \mu(uv)$. Since μ(uv)> 0 for uv∈E and μ(uv) = 0 for uv ∉ E, this is equivalent to $d_G(u) = \sum_{uv \in E} \mu(uv)$. The degree of an edge is $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$. The minimum degree of G is $\delta(G) = \wedge \{d(v) / v \in V\}$ and the maximum degree of G is $\Delta(G) = \vee \{d(v) / v \in V\}$. The order of a fuzzy graph G is $O(G) = \sum_{u \in V} \sigma(u)$.The size of a fuzzy graph G is $S(G) = \sum_{uv \in E} \mu(uv)$ [19-23]. If $d_G(u)$ = k ∀ u ∈ G, G is said to be a regular fuzzy graph of degree 'k' or k-regular [24].A fuzzy graph G:(σ, μ) is a strong fuzzy graph if $\mu(u, v)=\sigma(u)\wedge\sigma(v) \forall (u, v) \in \mu*$).A fuzzy graph G:$(\sigma, \mu)$ is a weak fuzzy graph if $\mu(u, v) < \sigma(u)\wedge\sigma(v) \forall (u, v) \in \mu*$, $\mu*=\{(u,v):\mu(u,v) > 0\}$. The complement of a fuzzy graph G:(σ, μ) is a fuzzy graph $\bar{G}$:$(\bar{\sigma}, \bar{\mu})$, where $\bar{\sigma} = \sigma$ and $\bar{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v) \forall u,v \in V$ [25-26].

A fuzzy graph G:(σ, μ) is a complete fuzzy graph if $\mu(u, v)=\sigma(u)\wedge\sigma(v) \forall (u, v) \in \sigma*$ [27-29]. In a complete fuzzy graph K_n all the edges are strong edges and hence by the definition the membership values of the edges will be equal to the minimum membership value of the corresponding vertices, the edge domination number of a fuzzy graph is defined as the cardinality of the minimum edge dominating set [30].A fuzzy graph G is said to be bipartite fuzzy graph if the vertex set V can be partitioned as $V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$,such that $\mu(u, v) = 0$ ∀ u,v∈ V_1 ,or u,v ∈V_2.If vertex in V_1 is connected to every vertex in V_2 , it becomes a fuzzy complete bipartite graph [31].

An anti-fuzzy graph G_A = (V,σ, μ) of a (crisp) graph G* is a triple with a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ where V is a non-empty set of vertices, σ is a fuzzy subset of V and μ is an anti-fuzzy relation on $V \times V$ such that $\mu(u, v) \geq \sigma(u) \vee \sigma(v)$, ∀ u,v∈V [32-34]. Degree of vertex u∈V is $d_G(u) = \sum_{u \neq v} \mu(uv)$. Degree of edge $uv \in E$ is $d_G(uv) = \sum_{u \neq w} \mu(uw) + \sum_{w \neq v} \mu(wv) - 2\mu(uv)$. A fuzzy graph G = (σ, μ) is a strong fuzzy graph if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$∀(u, v)∈μ*. An anti-fuzzy graph H_A= (□,□) is called an anti-fuzzy subgraph of G_A(□,□) if τ (u) ≤ σ(u) ∀ u∈V and ρ(u,v) ≤ μ(u,v)∀ u,v ∈V. If every edge on G has same degree of edge m, then G is called m-anti-fuzzy graph edge regular.

A general weak fuzzy graph with V as the underlying set is a pair of functions G:(σ,μ), where σ:V→[0,1] is a fuzzy subset of V and μ:V ×V→[0,1], is a fuzzy subset of V × V such that for all $(u, v) \in \mu*$, $\mu(u,v) \neq \sigma(u) \wedge \sigma(v)$.In irregular fuzzy graph there exists a vertex which is adjacent to vertices with distinct degrees. Some fuzzy topological are defined as follows [35].

The fuzzy first Zagreb index $M_1(G)$ is defined as

$$M_1(G)= \sum_{u \in V} \sigma(u)(d_G(u))^2, \quad (1)$$

where σ(v), μ(u,v) represent the membership values of the vertex u and of the edge (u, v) respectively and $d_G(u)$ is the degree of the vertex u in the fuzzy graph G.

The fuzzy second Zagreb index

$$M_2(G)= \frac{1}{2} \sum_{uv \in E} \sigma(u) d_G(u) \times \sigma(v) d_G(v) . \quad (2)$$

The fuzzy forgotten topological index is defined as

$$F(G)= \sum_{u \in V} (\sigma(u) d_G(u))^3. \quad (3)$$

The fuzzy Randić index is defined as

$$R(G) = \frac{1}{2} \sum_{uv \in E} \frac{1}{\sqrt{\sigma(u) d_G(u) \times \sigma(v) d_G(v)}} . \quad (4)$$

The fuzzy Harmonic index is defined as

$$H(G) = \frac{1}{2} \sum_{uv \in E} \frac{1}{\sigma(u) d_G(u) + \sigma(v) d_G(v)} . \quad (5)$$

The fuzzy Sombor index is defined as

$$SO(G) = \sum_{uv \in E} \sqrt{(\sigma(u) d_G(u))^2 + (\sigma(v) d_G(v))^2}. \quad (6)$$

The fuzzy Nirmala index is defined as

$$N(G) = \sum_{uv \in E} \sqrt{\sigma(u) d_G(u) + \sigma(v) d_G(v)}. \quad (7)$$

The fuzzy first entire Zagreb index is defined as [36]

$$M_1^Z(G) = \sum_{u \in V} (\sigma(u) d_G(u))^2 + \sum_{e \in E} (\sigma(e) d_G(e))^2, \quad (8)$$

where $d_G(e)$ edge degree of an edge e.

The fuzzy hyper Zagreb index is defined as

$$HZ(G) = \sum_{uv \in E} \left(\sigma(u) d_G(u) + \sigma(v) d_G(v)\right)^2. \quad (9)$$

The fuzzy second reduced Zagreb index can be defined as

$$RM_2(G)= \sum_{uv \in E} \sigma(u)(d_G(u) - 1) \times \sigma(v)(d_G(v) - 1) \quad (10)$$

In this paper first, second Zagreb indices, forgotten index, Randić index, harmonic index, Sombor index, Nirmala index, first entire Zagreb index, hyper Zagreb index and second reduced Zagreb index of fuzzy graph, complement fuzzy graph, complete fuzzy graph, complete bipartite fuzzy graph, wheel fuzzy graph and anti-fuzzy graph are computed. All the symbols and notations used in this paper are standard and taken mainly from books of fuzzy graph theory and fuzzy mathematics [37-40].

2. Materials and Methods

The weighted graph shows the strength of the relations between the vertices. A fuzzy graph can be drawn in many different ways by changing the positions of the vertices and drawing the edges by straight lines or curved lines. The fuzzy graphs considered here are finite, undirected, non-trivial without loops or multiple edges and μ is reflective (that is μ(u,v) = σ(v)∀ v ∈ V) and symmetric (that is $\mu(u, v) = \mu(v, u)$)∀(u,v)∈E.In a fuzzy graph G, the degree of a vertex u is calculated as the total of the membership values of the edges adjacent to u. Degree of membership values lie in the range [0….1]. The complement of a fuzzy graph is derived by reversing the membership degrees, highlighting what is absent in the original graph. The fuzzy graph, complement fuzzy graph, complete fuzzy graph, complete bipartite fuzzy graph, wheel fuzzy graph and anti-fuzzy graph are shown in figures (1-6) respectively.

3. Results and Discussion

The values of membership function of vertex and edge are computed in fuzzy graph, complement fuzzy graph, complete fuzzy graph, complete bipartite fuzzy graph, wheel fuzzy graph and anti-fuzzy graph from figures (1-6) and are used to compute the fuzzy topological indices. The values of these fuzzy topological indices thus computed are given in table (1).

Theorem 1. The fuzzy first Zagreb index of fuzzy graph is 5.474.

Proof. By using figure (1), the membership values of each vertex and membership values of each edge of a fuzzy graph, we have

$M_1(G)= \sum_{u \in V} \sigma(u)(d_G(u))^2$

$=\sigma(v_1)(d_G(v_1))^2+\sigma(v_2)\left(d_G(v_2)\right)^2 + \sigma(v_3)\left(d_G(v_3)\right)^2 + \sigma(v_4)(d_G(v_4))^2$

$=0.9 \times (1.5)^2+0.7 \times (0.7)^2 + 1 \times (1.7)^2 + 0.6 \times (0.6)^2$=5.474.

Theorem 2. The fuzzy second Zagreb index of complement fuzzy graph is 1.3612.

Proof. By using figure (2), we have the membership values of each vertex $\sigma_a(0.4), \sigma_b(0.5), \sigma_c(0.6), \sigma_d(0.7)$ and $\mu(a, b) = 0.5, \mu(a, c) = 1, \mu(b, d) = 1$ and $\mu(c, d) = 0.5$ values in complement fuzzy graph. We compute fuzzy second Zagreb index

$M_2(G)= \frac{1}{2} \sum_{uv \in E} \sigma(u) d_G(u) \times \sigma(v) d_G(v)$

$=\frac{1}{2}[(0.4 \times 1.5) \times (0.5 \times 1.5) + (0.5 \times 1.5) \times (0.7 \times 1.5)+(0.6 \times 1.5) \times (0.4 \times 1.5) + (0.6 \times 1.5) \times (0.7 \times 1.5)]$=1.3612.

Theorem 3. The fuzzy forgotten index of complete fuzzy graph is 0.5195.

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Proof. By using figure (3), the membership values of vertices and membership values of edges. We get fuzzy forgotten index

$F(G)=\sum_{u\in V}(\sigma(u)d_G(u))^3$
$=(\sigma(u_1)d_G(u_1))^3+(\sigma(u_2)d_G(u_2))^3+(\sigma(u_3)d_G(u_3))^3+(\sigma(u_4)d_G(u_4))^3$
$=(0.2\times 0.6)^3+(0.4\times 1.0)^3+(0.5\times 1.3)^3+(0.6\times 1.1)^3$
$=0.5195.$

Theorem 4. The fuzzy Randić index of complete bipartite fuzzy graph is 13.568.

Proof. By using figure (4) and the degree of each vertex of a complete bipartite fuzzy graph, we have

$R(G)=\frac{1}{2}\sum_{uv\in E}\frac{1}{\sqrt{\sigma(u)d_G(u)\times\sigma(v)d_G(v)}}$
$=\frac{1}{2}[\frac{1}{\sqrt{(0.2\times0.6)\times(0.3\times0.5)}}+\frac{1}{\sqrt{(0.2\times0.6)\times(0.5\times0.6)}}+\frac{1}{\sqrt{(0.2\times0.6)\times(0.4\times0.8)}}+\frac{1}{\sqrt{(0.4\times1.1)\times(0.3\times0.5)}}+\frac{1}{\sqrt{(0.4\times1.1)\times(0.5\times0.6)}}+\frac{1}{\sqrt{(0.4\times(1.1)\times(0.4\times0.6)}}]=13.568.$

Theorem 5. The fuzzy harmonic index of wheel fuzzy graph is 8.9595.

Proof. Consider W_8 wheel fuzzy graph (figure 5) with σ(v1) = 0.1, σ(v2) = 0.2, σ(v3) = 0.3, σ(v4)= 0.4, σ(v5)=0.5, σ(v6) =0.6, σ(v7) =0.7 and σ(v8)=0.8 vertices and degrees as $d_G(v1)=d_G(v2)=d_G(v3)=d_G(v4)=d_G(v5)=d_G(v6)=d_G(v7)=0.8$ and $d_G(v8)=2.8$. Then fuzzy harmonic index of wheel fuzzy graph

$H(G)=\frac{1}{2}\sum_{uv\in E}\frac{1}{\sigma(u)d_G(u)+\sigma(v)d_G(v)}$
$=\frac{1}{2}[\frac{1}{(0.1\times0.8+0.2\times0.8)}+\frac{1}{(0.2\times0.8+0.3\times0.8)}+\frac{1}{(0.3\times0.8+0.4\times0.8)}+\frac{1}{(0.4\times0.8+0.5\times0.8)}+\frac{1}{(0.5\times0.8+0.6\times0.8)}+\frac{1}{(0.6\times0.8+0.7\times0.8)}+\frac{1}{(0.7\times0.8+0.1\times0.8)}+\frac{1}{(0.1\times0.8+7\times2.8)}+\frac{1}{(0.2\times0.8+0.8\times2.8)}+\frac{1}{(0.3\times0.8+0.8\times2.8)}+\frac{1}{(0.4\times0.8+0.8\times2.8)}+\frac{1}{(0.5\times0.8)+0.8\times2.8)}+\frac{1}{(0.6\times0.8+0.8\times2.8)}+\frac{1}{(0.7\times0.8+0.8\times2.8)}]=8.9595$

Theorem 6. The fuzzy Sombor index of anti-fuzzy graph is 6.33.

Proof. By using figure (6), we have

$SO(G)=\sum_{uv\in E}\sqrt{(\sigma(u)d_G(u))^2+(\sigma(v)d_G(v))^2}$
$=\sqrt{(04\times0.9)^2+(0.5\times1.5)^2}+\sqrt{(0.5\times1.5)^2+(0.7\times2.3)^2}+\sqrt{(0.7\times2.3)^2+(1\times0.3)^2}+\sqrt{(1\times0.3)^2+(0.4\times0.9)^2}+\sqrt{(0.7\times2.3)^2+(0.1\times0.7)^2}$
$=6.33.$

Theorem 7. The fuzzy Nirmala index of fuzzy graph is 7.2309.

Proof. By using figure (1), membership values of each vertex and of each edge, we have fuzzy Nirmala index

$N(G)=\sum_{uv\in E}\sqrt{\sigma(u)d_G(u)+\sigma(v)d_G(v)}$
$=\sqrt{\sigma(v_1)d_G(v_1)+\sigma(v_2)d_G(v_2)}+\sqrt{\sigma(v_2)d_G(v_2)+\sigma(v_3)d_G(v_3)}+\sqrt{\sigma(v_3)d_G(v_3)+\sigma(v_4)d_G(v_4)}+\sqrt{\sigma(v_4)d_G(v_4)+\sigma(v_1)d_G(v_1)}+\sqrt{\sigma(v_3)d_G(v_3)+\sigma(v_1)d_G(v_1)}$
$=\sqrt{(0.9\times1.4+0.7\times0.7)}+\sqrt{(0.7\times0.7+1\times1.7)}+\sqrt{(1\times1.7+0.6\times0.6)}+\sqrt{(0.6\times0.6+0.9\times1.4)}+\sqrt{(1\times1.7+0.9\times1.4)}=7.2309.$

Theorem 8. The fuzzy first entire Zagreb index of complement fuzzy graph is 6.385.

Proof. The membership values of each vertex and of each edge are obtained from figure (2) as $\sigma_a(0.4)$, $\sigma_b(0.5)$, $\sigma_c(0.6)$ and $\sigma_d(0.7)$, $\mu(a,b)=0.5$, $\mu(a,c)=1$, $\mu(b,d)=1$ and $\mu(c,d)=0.5$. Then fuzzy first entire Zagreb index is

$M_1^Z(G)=\sum_{u\in V}(\sigma(u)d_G(u))^2+\sum_{e\in E}(\sigma(e)d_G(e))^2$
$=[(0.4\times1.5)^2+(0.5\times2)^2]+[(0.5\times1.5)^2+(1\times1)^2]+[(0.4\times1.5)^2+(1\times1)^2]+[(0.7\times1.5)^2+(0.5\times2)^2]=6.385.$

Theorem 9. The fuzzy hyper Zagreb index of complete fuzzy graph is 4.8179.

Proof. By using figure (3), we have

$HZ(G)=\sum_{uv\in E}\left(\sigma(u)d_G(u)+\sigma(v)d_G(v)\right)^2=$
$(0.2\times0.6+0.4\times1.0)^2+(0.4\times1.0+0.5\times1.1)^2+(0.5\times1.1+0.6\times1.1)^2+(0.6\times1.1+0.2\times0.6)^2+(0.2\times0.6+0.5\times1.1)^2+(0.4\times1.1+0.6\times1.1)^2=4.8179.$

Theorem 10. The fuzzy reduced second Zagreb index of complete bipartite fuzzy graph is 0.0204.

Proof. By using figure (4) and computing $d_G(u)$ is the degree of the vertex u in complete bipartite fuzzy graph G, we have

$RM_2(G)=\sum_{uv\in E}\sigma(u)(d_G(u)-1)\times\sigma(v)(d_G(v)-1)$
$=(0.2\times-0.4)\times(0.3\times-0.5)+(0.2\times-0.4)\times(0.5\times-0.2)+(0.2\times-0.4)\times(0.4\times-0.2)+(0.4\times0.1)\times(0.3\times-0.5)+(0.4\times0.1)\times(0.5\times-0.4)+(0.4\times0.1)\times(0.4\times-0.2)=0.0204.$

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able 1: Fuzzy topological indices of fuzzy graph, complement fuzzy graph, complete fuzzy graph, complete bipartite fuzzy graph, wheel fuzzy graph and anti-fuzzy graph

Fuzzy topological index→	$M_1(G)$	$M_2(G)$	F(G)	R(G)	H(G)	SO(G)	N(G)	$M_1^Z(G)$	HZ(G)	$RM_2(G)$
Fuzzy graph	5.474	2.329	7.5376	2.9071	1.2342	8.2852	7.2309	8.7331	23.488	-0.2257
Complement fuzzy graph	4.95	1.3615	2.5245	2.5035	1.2378	4.7154	5.1247	6.385	10.775	0.3205
Complete fuzzy graph	1.803	0.5201	0.5195	8.876	3.76	3.9623	5.981	2.9021	4.8179	-0.0058
Complete bipartite fuzzy graph	0.955	0.1932	0.1311	13.568	6.3596	3.5542	4.133	1.044	1.737	0.0204
Wheel fuzzy graph	1.8547	2.8892	11.64	17.073	8.9595	19.186	15.667	10.918	49.369	-0.7588
Anti-fuzzy graph	5.501	1.1711	4.6691	3.3515	1.9792	6.33	6.0803	10.312	13.707	0.1902

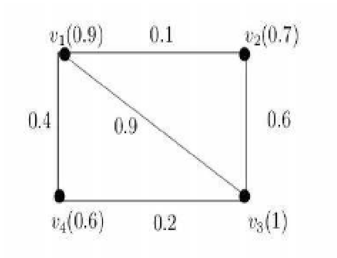

Figure 1

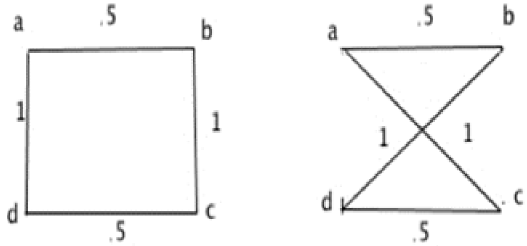

Figure 2

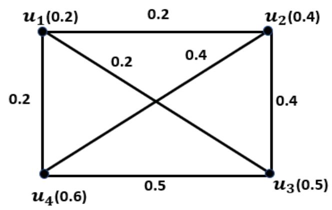

Figure 3

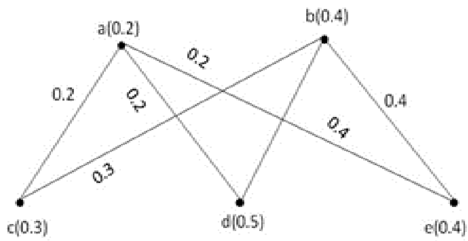

Figure 4

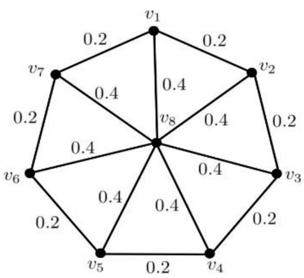

Figure 5

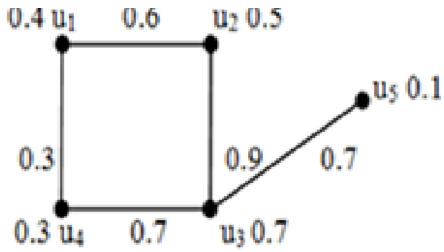

Figure 6

Figure 1. fuzzy graph, fig. 2. complement fuzzy graph (C_4 -cycle,$C_4^c -$ cycle),fig.3.complete fuzzy graph, fig.4.complete bipartite fuzzy graph, fig.5.wheel fuzzy graph W_8 and fig.6.anti-fuzzy graph.

4. Conclusion

Computation of topological indices of fuzzy graph depends on membership values of vertices and edges. The first, second Zagreb indices, forgotten index, Randić index, harmonic index, Sombor index, Nirmala index, first entire Zagreb index, hyper Zagreb index and second reduced Zagreb index of fuzzy graph, complement fuzzy graph, complete fuzzy graph, complete bipartite fuzzy graph, wheel fuzzy graph and anti-fuzzy graph are obtained. The fuzzy reduced second Zagreb index has least values among the fuzzy graphs studied.

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