

# Integrating Physics and Biomechanics in Sprinting: Modeling Forces and Validating Motion through Kinematic Data

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**Abstract:** *This article presents an analytical study of sprinting mechanics by modeling the forces acting on a sprinter and validating these through experimental data. The study considers resistive forces such as air drag, ground friction, internal resistance, and gravity acting on sloped surfaces. A reduced-order model is developed to predict performance, with special attention to sprinting on curved tracks and the role of centripetal force. Empirical validation is achieved through time-split data from national-level sprinters, allowing segmentation into acceleration, constant speed, and deceleration phases. The study also investigates sprint start biomechanics, analyzing block position, acceleration patterns, and reaction times. The results support improved training techniques, optimized track design, and enhanced sprint efficiency through physics-informed coaching strategies.*

**Keywords:** Sprint modeling, Kinematics, Resistive forces, Biomechanics, Acceleration phases

## 1. Introduction

### 1.1. Context and Significance

Over the past few years, application of mathematical models to human and animal locomotion has been rising, especially in athletic contexts. A key research challenge is to develop a model that accurately represents the propulsive and resistive forces acting on a human runner. Such a model can then be used to study the mechanics of sprinting and to optimize training techniques for different phases of the sprint.

Several studies have been conducted to develop comprehensive models of sprinting mechanics.

### 1.2. Literature Review

Past research has explored various aspects of the physics behind sprinting. A model has been described that considers both propulsive and resistive forces acting on a sprinter, leading to analytical solutions for velocity and position as a function of time. The model assumes a constant propulsive force and an air resistance force proportional to velocity.

Another study analyzed the 10m split times of a 100m race for 5 males and 3 females. This paper reports on the kinematic characteristics, such as velocity and acceleration. The split times are used to calculate velocity and acceleration at every 10m interval using the kinematic models developed in this paper.

In 8 male sprinters, the activity of key lower limb muscles was analyzed during maximal 3-meter starts taken on a force platform. Total reaction time was measured, highlighting individual differences in neuromuscular response during sprint starts. This study takes a more biomechanical approach, highlighting the importance of ground reaction forces and the runner's ability to generate high forces for achieving high speeds.

More recent work has investigated the role of attentional focus in sprint start performance. This suggests that an external focus of attention, such as on the starting blocks rather than the body, can improve reaction times and movement coordination during the initial acceleration phase.

### 1.3. Research Question and Hypothesis:

“Is it possible to create a simplified model that incorporates all the forces acting on a body during sprinting, considering multiple factors such as air resistance, ground reaction forces, and the sprinter's biomechanics, to predict and optimize performance?”

### 1.4 Objectives:

1. To analyze and quantify all the forces acting on a body during the sprinting motion.
2. To develop a comprehensive model that will integrate all the forces acting on a body during the sprint motion.
3. To create a simplified model for the net resistive force acting on the runner.
4. Mathematical Modeling of a Sprinter's Motion in the 100m Race.
5. Analyzing the Biomechanics of the Track and Field Sprint Start.
6. This study is significant as it bridges theoretical physics with practical sprint training, offering predictive tools for athletic performance and contributing to sports science literature by validating models through real-world biomechanical data.

## 2. Methods

We started by quantifying all the resistive forces acting on a body during a sprint - both on straight and curved tracks. After this we created a comprehensive model of the net force the body produces in the forward direction. The net force is calculated by subtracting all resistive forces from the forward propulsive force. Using this comprehensive model for all the resistive forces acting on the body, a simplified model was

created for ease of calculation. In the simplified model a parameter/constant was used which encompassed the effects of air resistance, ground friction, and internal resistance.

To analyze the kinematics of a 100m race and improve the kinematic modelling of the 100m, we split the race into 3 parts: acceleration phase, constant velocity and deceleration phase. By doing so, we created mathematical models to determine kinematic characteristics such as velocity and acceleration at each point for each of the three phases.

To experimentally verify the validity of these models we used data from an experiment conducted in Rajshahi University, Bangladesh where a total of eight (five male and three female) sprinters were chosen as the subject. This experiment provided the 10m split times through the entire 100m race for all participants. We used this data to determine the displacement and velocity at each of the time intervals and verify them with the displacement-time and velocity time graph for the fastest male and fastest female sprinters.

Data from previous research was used to examine how variations in set positions and block placements influence sprint performance, particularly in relation to leg positioning and overall start mechanics.

Using data from this entire paper, improved training techniques were developed to maximise sprint performance. The techniques individually address the resistive forces and how to face them to increase sprint efficiency and minimise unnecessary loss of energy.

### 3. Results

#### Modeling All Resistive Forces in Sprinting

There are multiple forces acting on the body during a sprint. A comprehensive model must be developed for the same.

##### I. Air Resistance

- **Drag Force:** the first resistive force which affects the parameter is drag force. it is characterised by:
  - Air density ( $\rho$ )
  - Sprinter's velocity ( $v$ )
  - Sprinter's cross-sectional area ( $A$ )
  - Drag coefficient ( $C_d$ )
- **Formula:**

$$F_D = \frac{1}{2} \rho v^2 C_d A$$

- **Simplified Model:** The Drag force is directly proportional to the drag coefficient. The drag coefficient essentially gives how aerodynamic a body is. A higher drag coefficient means the body is less aerodynamic. For example, the drag coefficient of a teardrop will be very low. drag force is also directly proportional to the density of the medium, for example, a denser fluid such as honey causes more drag than air.

##### II. Ground Contact Friction

- **Horizontal Component:** A small horizontal component of ground reaction force opposes motion.
- **Modeling:** Since the acceleration is in the forward direction, we can assume that the propulsive force acting forwards can be given as " $ma$ ", wherein " $m$ " is the mass of the body and " $a$ " is the instantaneous acceleration of the body. In the Acceleration phase from this propulsive force, we can subtract a small resistive force due to friction, which can be modeled as " $\mu mg$ ", where " $\mu$ " is the coefficient of friction for the given track, " $m$ " is mass of the body and " $g$ " is the acceleration due to gravity.

In the Constant Velocity Phase, this Friction force will be equal to the propulsive force.

##### III. Internal Resistance

- **Viscous Resistance:** Forces within muscles and joints that oppose movement.
- **Inertial Resistance:** The body's inertia resists changes in motion, particularly during the initial acceleration phase and at each stride.
- **Modeling:** These are often incorporated into more complex musculoskeletal models. A simple approximation sometimes used is  $F_{internal} \approx cv$ , where " $c$ " is a constant for every individual.

##### IV. Gravity (on inclines)

- **Component along the slope:** If sprinting uphill, a component of the gravitational force opposes motion. If sprinting downhill, it assists motion.
- **Formula:**  
 $F_g = mg \sin(\theta)$ , where  $\theta$  is the incline angle.

##### V. Comprehensive Model

A more complete model would combine these forces:

$$F_{net} = F_{propulsive} - F_{air} - F_{friction} - F_{internal} - F_g$$

##### Experimental Validation

The models discussed above have been validated through experiments that show good agreement between theory and observed sprinting data. This gives confidence in using the models to understand sprinting mechanics and make performance predictions.

For example, one study examined the mechanics of world-class sprinters and found that the model accurately captured the relationships between key variables like forward velocity, ground reaction forces, and stride characteristics.

$$F_{resistive} = F_D + F_{friction} + F_{internal} + F_g$$

##### Simplified Model

Since resistive forces are proportional to speed.

$$F_{\text{res}}(u) \propto u$$

In order to scale this relationship to the runner's mass,

$$F_{\text{res}}(u) = -\sigma Mu$$

the complexity of accurately modeling all these factors often necessitates simplifications, especially in less resource-intensive analyses. For instance, a simplified model used,  $F_r = -\sigma Mu$ , where “ $\sigma$ ” is a constant and “ $M$ ” is the runner's mass. This model combines several resistive forces into a single term proportional to velocity.

Although such a form is suggested by analogy with viscous forces, it is far from clear that it can adequately represent human runners where the forces are rapidly varying and, in general, complex.

It encompasses the effects of air resistance, ground friction, and internal resistance. A basic question is: What are typical values of the parameter  $\sigma$ ? It is to be expected that  $\sigma$  will depend on the runner, his speed, the type of race, and possibly on external conditions such as the track surface, altitude, etc. Keller uses a value of  $\sigma = 0.44 \text{ sec}^{-1}$  while Whitt and Wilsons quote similar values. The latter, however, were measured at walking speeds. To our knowledge there exists no measurement of  $\sigma$  at sprinting speeds.

**Limitations of Simplified Model:** However, the simplified model is not fully accurate- for example, the air resistance is directly proportional to  $v$  square, not just  $v$ . however this simplified model can be used for estimation of values, especially at lower speeds.

#### 4. Application to Curved Track Running

One application of this simplified model is, displaying the differences between straight track and curved track running on the curve, there is an additional centripetal acceleration that a runner has to overcome which must be taken into consideration.

The additional horizontal force, opposing the runners motion is given by;

$$F_c = Mu^2/r, \text{ where } r \text{ is the radius of the curve.}$$

The total resistive force on a curved track is then:

$$F_r = F_D + F_{\text{friction}} + F_{\text{internal}} + F_g + F_c$$

This clearly shows that running on a curve has an additional force component caused by centripetal acceleration.

This additional force can lead to differences in timings of running on a straight track vs running on a curve of the same distance.

The timings can vary from 5-10% for elite sprinters, the curved track being slower due to an additional force being overcome.

#### Sprint Kinematics of a 100m Race

##### Mathematical Modeling of a Sprinter's Motion

**1. To improve the accuracy of the kinematic model for a 100m sprint, we need to consider more realistic dynamics. Hence, we split the race into the following parts:**

- Phase 1: Reaction Time**  
Reaction time is normally 0.15–0.2s in world class sprinters, resulting in negligible displacement.
- Acceleration Phase:**
  - Acceleration is not a constant. It decreases gradually as the sprinter approaches the maximum velocity: ( $v_{\text{max}}$ )
  - A realistic model for acceleration can be expressed as a decaying function, such as  $a(t) = a_0 e^{-kt}$  where  $a_0$  is the initial acceleration and  $k$  controls the rate of decay.
  - Initial Acceleration:** At the very start ( $t = 0$ ), the exponential term becomes  $e^0 = 1$  so the acceleration is simply  $a_0$ .
  - The constant  $k$  controls how quickly the acceleration decays. A larger  $k$  means a faster decay as it increases the value of the exponent.
  - $$v(t) = \int_0^t a(t) dt$$

$$a(t) = a_0 e^{-kt}, \text{ we get } v(t) = \frac{a_0(1 - e^{-kt})}{k}$$

#### 3. Constant Velocity Phase:

acceleration here is negligible

$$v(t) \approx v_{\text{max}}$$

$$s(t) = v_{\text{max}} \cdot t$$

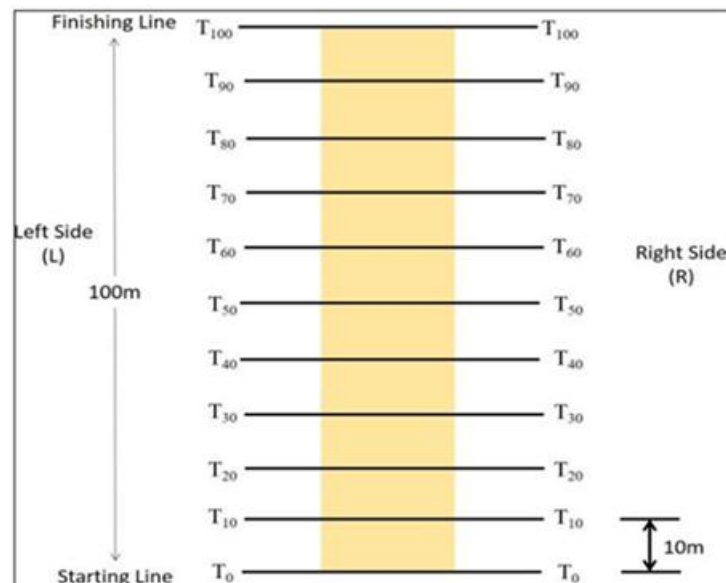
#### 4. Deceleration phase:

closer to the finish line, air resistance and muscle fatigue cause the sprinter to decelerate.

$$v(t) = v_{\text{max}} - b(t - t_0)^2$$

##### Experimental Validation:

A total of eight (five male and three female) sprinters from Rajshahi University, Bangladesh were chosen as the subject. They were selected through the inter-university athletics championship-2022. These sprinters can be classified as national or average-level performers based on their performance level. Their ages ranged from 21 to 24 years.



**Figure 1:** A 100m linear runway divided into ten equal distance intervals of 10m each.

**Experimental Setup:** The experiment was conducted at the Rajshahi University central playground. The 100-meter track was properly marked, and cones were used to indicate specific distances.

#### Data Collection:

The estimated zonal times for all participants are presented in Table 1. First five rows (Registered as RUM1 to RUM5) are for the male sprinters and last three (registered as RUF1 to RUF3) are for the female sprinters. These times were obtained from the distance-time data measured in this study. Table 1 demonstrates that all the male sprinters spent the shortest time in the zone 50-60m. Whereas, for the female sprinters the shortest times were seen at 60-70m zone. A

deceleration phase for all male sprinters has been observed after 80m, which is absent in the case of female sprinters. Overall, the female sprinters took much time as compared to the male sprinters to cover full length (100m). It is also evident from Table 1 that the sprinters RUM5 and RUF1 took the shortest time among the male and female sprinters respectively. The displacement versus time curves of these two fastest sprinters are depicted in Fig. 3. These two curves exhibit a similar pattern but with differences in times to cover the same displacement. The male sprinter (RUM5) is faster than the female one (RUF1). A similar picture has been observed for the case of other male and female sprinters, but not included in this paper because of space constraints. To finish the full length of 100m the sprinter RUM5 took 12.11 seconds whereas it was 15.37 seconds for RUF1.

Subject	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>	T <sub>6</sub>	T <sub>7</sub>	T <sub>8</sub>	T <sub>9</sub>	T <sub>10</sub>	Total
RUM1	1.97	1.37	0.91	1.46	0.91	0.75	1.24	1.38	1.23	1.36	12.39
RUM2	1.95	1.32	0.99	1.33	0.96	0.88	0.94	1.52	1.24	1.29	12.26
RUM3	2.05	1.31	1.05	1.31	1.02	0.90	0.95	1.54	1.06	1.38	12.41
RUM4	1.97	1.23	1.06	1.31	1.02	0.91	0.92	1.55	1.04	1.28	12.35
RUM5	1.92	1.31	1.05	1.30	1.02	0.94	0.95	1.50	1.04	1.08	12.11
RUF1	2.28	1.52	1.41	1.61	1.33	1.37	1.03	2.14	1.56	1.12	15.37
RUF2	2.36	1.60	1.56	1.59	1.43	1.60	1.14	2.22	1.75	1.30	16.52
RUF3	2.26	1.58	1.49	1.55	1.42	1.53	1.15	2.21	1.69	1.24	16.02

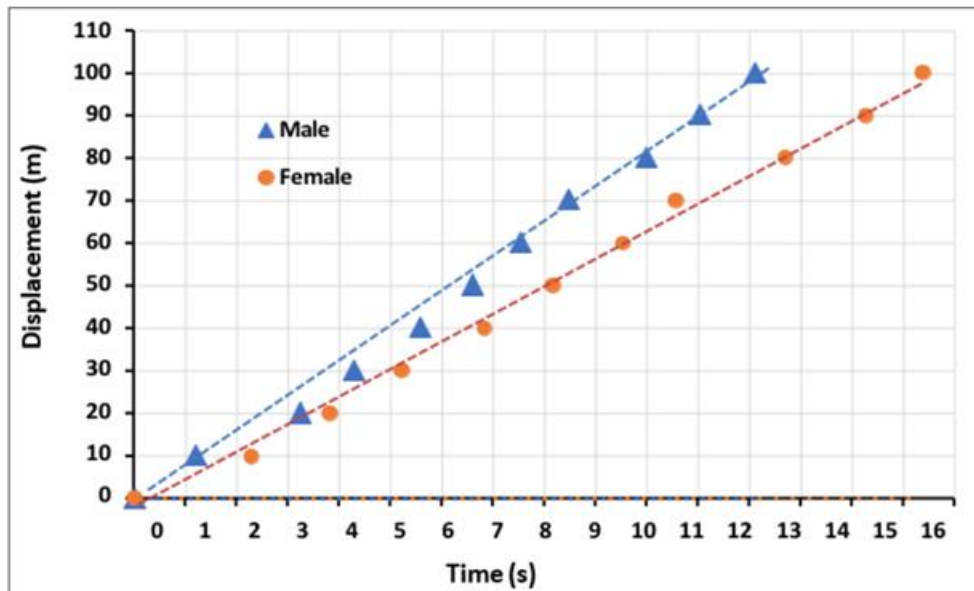
**Figure 2:** Regional time in second while passing the respective area in the subscripts

Using this data, we get the distance covered every 10m by the athlete. We use the third kinematic equation which is

$$s = ut + \frac{1}{2}at^2$$

$$s_i = v_0(t_i - t_{i-1}) + 0.5 * a * (t_i - t_{i-1})^2,$$

where  $t_i$  is the time at the  $i$ th segment and  $v_0$  is the initial velocity. Using this equation, we already have the values of displacement (10m) and the time in each segment for the athlete. We use these values to create a displacement time graph for a 100m sprint.



**Figure 3:** Displacement covered with time by the quickest male (RUM5) and female (RUF1) sprinters.

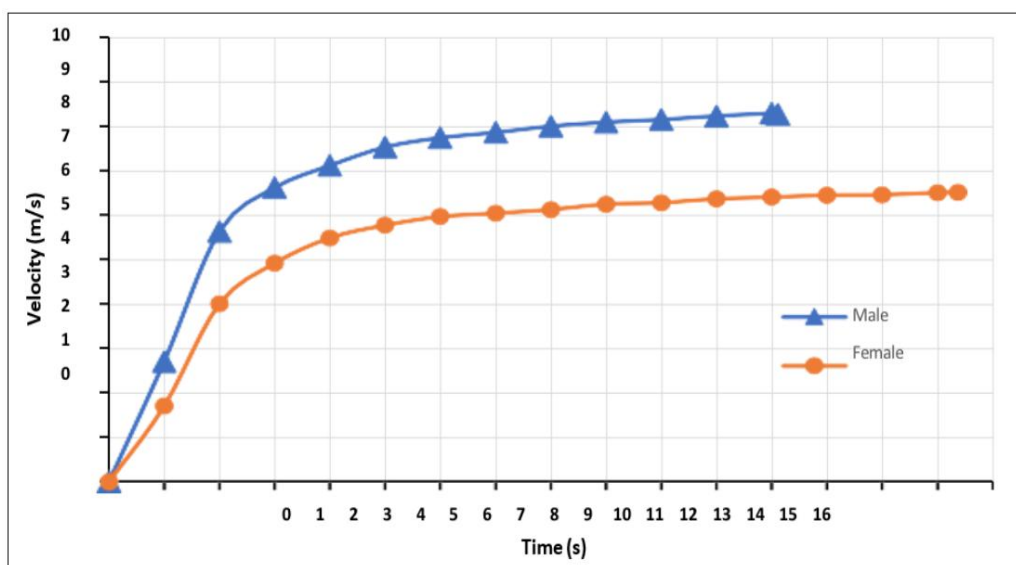
The slope of the best fit line of this graph gives us the average velocity of the runner throughout the race. The slope of the graph in each segment which can be taken as the slope of the line of the graph between the two points such as (0-10m) (10,  $T_1$ ), (20,  $T_2$ ) and so on gives us the velocity of the runner in each of the segments. This velocity can also be calculated by taking the distance for each segment as 10m and dividing it

by the time taken for each segment. Let's take for example RUM1:  $T_1 = 1.97s$ . Therefore, the velocity =  $10/1.97 = 5.07$  m/s

Similarly, we calculate the velocity for each segment for every runner to obtain the velocity at each segment.

Subject	V <sub>1</sub>	V <sub>2</sub>	V <sub>3</sub>	V <sub>4</sub>	V <sub>5</sub>	V <sub>6</sub>	V <sub>7</sub>	V <sub>8</sub>	V <sub>9</sub>	V <sub>10</sub>
RUM1	5.07	7.28	10.99	6.85	10.99	13.33	8.06	7.25	8.13	7.35
RUM2	5.12	7.58	10.10	7.52	10.42	11.36	10.64	6.58	8.06	7.75
RUM3	4.88	7.63	9.52	7.63	9.80	11.11	10.53	6.49	9.43	7.25
RUM4	5.09	7.63	9.43	7.63	9.80	10.99	10.87	6.45	9.63	7.81
RUM5	5.22	7.63	9.52	7.69	9.80	10.64	10.53	6.67	9.60	9.26
RUF1	4.39	6.58	7.09	6.21	7.52	7.30	9.71	4.67	6.41	8.93
RUF2	4.23	6.25	6.41	6.29	6.99	6.25	8.77	4.50	5.71	7.69
RUF3	4.43	6.33	6.71	6.45	7.04	6.54	8.70	4.52	5.92	8.06

**Figure 4:** Zonal velocity in meter per second while passing the respective area in the subscripts. Using these Values, we plot Zonal Velocity of the athlete against the split times of 100m distance to get a velocity-time graph



**Figure 5:** Variation of velocity with time for the fastest male (RUM5) and female (RUF1) athletes.



## Biomechanics and Physics of Sprinting

Sprinting is a complex movement requiring a lot of different explosive movements from one's body. Understanding the underlying physics and biomechanics is crucial for avoiding injuries and enhancing training.

### I. Biomechanics

#### A. Start and Acceleration Phase

The sprint start is crucial to set the stage for a successful race.

##### 1. Blocks starts:

At the start of any sprint event, sprinters commence from starting blocks, against which they must produce considerable acceleration. World-class 100 m sprinters can achieve around one-third of their maximum velocity in around only 5% of total race time by the instant they leave the blocks, and sprint start performance is strongly correlated with overall 100 m time, i.e. coefficient of relation between the two is near 1.

The most common measure of sprint start performance has been center of mass velocity at block exit. Block velocity is determined by push phase impulse and can therefore be increased by either greater force or greater time spent producing force. This is because  $\text{Impulse} = Ft$ . However, the ability to produce force is not consistent throughout the duration of the push against the blocks. Therefore, there comes a point when attempting to achieve further increases in block velocity by simply pushing for longer against the blocks may not be beneficial for overall sprint performance.

##### 2. Set position:

Sprinters can choose the location of the two-foot plate blocks during their sprint start.

Increasing the antero-posterior distance between the foot plates, leads to increased push phase duration along with total impulse and therefore it leads to greater block exit velocities. This is most likely due to the greater rear leg forces, which lead to a greater rear leg segmental kinetic energy.

Different footplate setups influence acceleration. Although wider spacing may increase exit velocity, it can slow the sprinter's time to reach 5m and 10m marks. A medium spacing seems to offer an optimal balance between push duration and force generation.

Wider medio-lateral foot placings do not seem to have any effect on block power. World class sprinters are supposed to use the blocks provided to them, however changing the width of the block seems to have no effect.

There is no effect of the habitual foot plate inclination on block power when analysed cross-sectionally across a wide range of sprinters. Front block inclination is also not related to any external force parameters, but a steeper rear foot plate is associated with a greater mean rear block horizontal force between sprinters.

### 3. Reaction Time

Reaction time has been defined as the time that elapses between the sound of the starter's gun and the moment the athlete is able to exert a certain pressure against the starting blocks. Reaction time measurement currently includes the time that it takes for the sound of the gun to reach the athlete, the time it takes for the athlete to react to the sound and the mechanical delay of measurements inherent in the starting blocks. An attempt has been made to separate premotor time and motor time components in the sprint start.

The reaction time during a sprint start includes both mental processing and muscle activation phases. After the gun signal, extensor muscles in the legs quickly activate to produce force. Faster activation leads to more effective block exit.

### 5. Discussion

This study provides insights into factors that influence sprint performance such as the physics and biomechanics, which are represented by the complex modeling of resistive forces.

The complexity of sprint dynamics is captured by models that integrate air resistance, ground friction, internal resistance, and gravitational components. In this report, these models are supported by empirical data collected from national-level sprinters, especially the system of sprinting with different phases (reaction, acceleration, constant velocity, and deceleration).

The mathematical model when applied to curve track running shows the difference due to the additional centripetal force acting on a body while moving in a curved path. There are empirically observed time losses of 5-10% when sprinting on curves.

Furthermore, the biomechanical analysis of the sprint start shows the critical role of block placement, reaction time, and the instantaneous neuromuscular activation in the body, determining the athlete's early race performance. While a steeper rear foot plate was associated with greater rear leg forces, the optimal inter-block distance remains individualized, balancing force production with duration of the push phase. The analysis also highlights that although reduced models offer useful approximations, they can be neglecting the subtle variation in resistance that can be witnessed during various phases of sprints and speeds. These models could be made more predictive by including more personalized parameters such as the strength of the muscles, fatigue levels, and aerodynamic characteristics, especially at elite sprinting speeds where marginal improvements are essential.

#### Practical Implications and Recommendations:

##### Improved Training Techniques to Minimize Resistive Forces in Sprinting: A Practical Approach

There are numerous resistive forces that limit sprint performance such as Based on the model developed, air drag, internal resistance, ground friction, gravitational components

(on slopes), and, during curved runs, centripetal force. This section outlines specific, targeted training techniques designed to counter these resistive forces and improve efficiency, speed, and race performance.

### 1. Reducing Air Drag: Posture and Apparel Training

**Technique:** Postural Optimization and Aerodynamic Gear Simulation

- Use video feedback and posture drills to refine the sprinter's form, focusing on maintaining a forward lean during acceleration and an upright but compact posture during the maximum velocity phase.
- A lower drag coefficient reduces air resistance, as per the formula  $F_D = \frac{1}{2} \rho v^2 C_d A$ . Training athletes to reduce their frontal area and optimize form reduces drag force considerably.

### 2. Enhancing Ground Contact Efficiency

**Technique:** Resisted Sled Sprints and High-Speed Plyometrics

- Use sled pulls (10–20% of bodyweight added to the sled) to train the sprinters in generating horizontal ground reaction forces. Pair these with other plyometric drills such as bounding and depth jumps to improve explosive ground contact.
- Stronger, more forceful ground contacts increase propulsive force and reduce the relative effect of resistive ground friction ( $\mu mg$ ). Plyometrics improve the stretch-shortening cycle of muscle contractions, enhancing stride efficiency.
- Improve time-to-20m and reduce time spent on the ground during each stride, optimizing frictional losses.

### 3. Minimizing Internal Resistance through Muscle Efficiency

**Technique:** Eccentric-Strength Training and Flexibility Regimens

- Introduce Nordic hamstring curls, eccentric leg press, and mobility training to improve force absorption and reduce energy lost to internal resistances like joint friction and muscle stiffness.
- Internal resistance is reduced when muscles and joints operate more fluidly. Eccentric training increases tendon resilience and neuromuscular control, leading to lower resistance during movement transitions.

### 4. Countering Gravitational Effects (Hill Sprints)

**Technique:** Incline and Decline Sprint Training

- Sprint training on a 5–10° incline or slope for strength development and downhill sprints (2–3°) for over-speed training. Note: Excessive downhill training can lead to knee injuries.
- Uphill running increases resistance, training the body to exert greater propulsive force. Downhill sprints allow neural adaptation to faster speeds, improving stride turnover and control.

### 5. Addressing Curved Track Forces

**Technique:** Centripetal Force Management via Curve Running Drills

- Include sprint intervals around curved sections of the track, focusing on inward lean, foot placement, and counter-rotation of arms.
- Curved running introduces  $F_c = \frac{Mu^2}{r}$ . Training under these conditions develops specific neuromuscular coordination to resist lateral drift and maintain momentum.
- Essential for 200m and 400m sprinters; drills improve symmetry and pacing when running on bends.

These findings can guide training programs, track design, and biomechanical adjustments, thereby enhancing sprint performance through scientifically grounded techniques

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