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Nutrition Analysis Using $\mathfrak{D}_{\mathfrak{L}}$-Nano Topological Spaces and $\mathfrak{D}_{\mathfrak{R}}$-Nano Topological Spaces through Attribute Reduction Method

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Abstract: ***This study offers a thoughtful and methodical application of Nano-topological concepts, grounded in rough set approximations and the theory of ideals, to address real-world decision-making challenges-in this case, nutritional health assessment. Also, this research wok formalized the structures of left and right dynamic Nano-topologies and applied them in a layered, stepwise methodology to distill the most influential nutritional factors from a set of conditional attributes. The use of equivalence relations, neighborhood analysis, and core factor extraction demonstrates a disciplined analytical framework that bridges abstract mathematical theory with tangible health-related outcomes. What stands out is the dual-stage computational process, which ensured that the core attributes-carbohydrates, calcium, proteins, and fat-emerged consistently across both topological perspectives, reinforcing their importance in sustaining well-being. This suggests that the approach has the flexibility to move beyond nutrition into other domains such as market analysis, clinical diagnostics, and academic evaluation, where complex attribute interdependencies influence decisions. By translating a mathematically sophisticated model into a tool for practical, evidence-based judgments, the work strikes a meaningful balance between theoretical elegance and applied relevance.***

Keywords: Nano Topology, Attribute reduction, Ideal, $\mathfrak{D}_{\mathfrak{L}}$ - Nano topological spaces, $\mathfrak{D}_{\mathfrak{R}}$-Nano topological space

1. Introduction

A Nano-Topology is a topology generated by rough set approximations. The ideal is an important concept in analysing the topological problems [1,2] .The idea of Nano – topology was applied to make accurate decisions in real life problems [3-8].The notion of ideals has applied to present the concept of $\mathfrak{D}_{\mathfrak{L}}$ - Nano topological spaces induced by the initial left neighborhood[9]. Maheswari and Rekha have introduced a new kind of ideal Nano-topological space called $\mathfrak{D}_{\mathfrak{R}}$-Nano topological space and explored different kinds topological properties in $\mathfrak{D}_{\mathfrak{R}}$-Nano topological space [10,11].In this research the applications of $\mathfrak{D}_{\mathfrak{L}}$ - Nano topological spaces and $\mathfrak{D}_{\mathfrak{R}}$ - Nano topological spaces are analysed.

2. Preliminaries

Definition: 2.1[1] A nonempty collection $\mathfrak{K}$ of subsets of a set $\mathfrak{U}$ is said to be an ideal on $\mathfrak{U}$, if it satisfies the following two conditions: (i) if $\mathcal{G} \in \mathfrak{K}$ and $\mathcal{T} \subseteq \mathcal{G}$, then $\mathcal{T} \in \mathfrak{K}$ (ii) if $\mathcal{G} \in \mathfrak{K}$ and $\mathcal{T} \in \mathfrak{K}$ then $\mathcal{G} \cup \mathcal{T} \in \mathfrak{K}$.

Definition: 2.2[8] Let $\mathfrak{K}$ be an ideal in the universal set $\mathfrak{U}$ and $\mathcal{M} \subseteq \mathfrak{U}$. Then the Nano left dynamic lower, upper and boundary regions are defined as,

$\underline{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M}) = \{\eta \in \mathfrak{U}: \mathfrak{R}_i{}'(\eta) \cap \mathcal{M}^c \in \mathfrak{K} \; and \; \mathfrak{R}_i{}'(\eta) \subseteq \mathcal{M}\}$,

$\overline{\overline{\mathfrak{D}_{\mathfrak{L}}}}(\mathcal{M}) = \{\eta \in \mathfrak{U}: \mathfrak{R}_i{}'(\eta) \cap \mathcal{M} \notin \mathfrak{K} \; and \; \mathfrak{R}_i{}'(\eta) \cap \mathcal{M} \neq \emptyset\}$

where $\mathfrak{R}_i{}'(\eta) = \{\beta \in \mathfrak{U}: \mathfrak{R}_\ell(\eta) \subseteq \mathfrak{R}_\ell(\beta)\}$ and

$\mathfrak{B}_{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M}) = \overline{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M}) - \underline{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M})$.

Then, the Nano left dynamic topology (called $\mathfrak{D}_{\mathfrak{L}}$ - Nano topology) on $\mathfrak{U}$ with respect $\mathcal{M} \subseteq \mathfrak{U}$ is defined as, $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(\mathcal{M}) = \{\mathfrak{U}, \emptyset, \overline{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M}), \underline{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M}), Bnd_{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M})\}$.

The collection $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M}) = \{\mathfrak{U}, \underline{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M}), \mathfrak{B}_{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M})\}$ forms a basis for $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(\mathcal{M})$ and the $\mathfrak{D}_{\mathfrak{L}}$ – Nano topological space is denoted by $(\mathfrak{U}, \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(\mathcal{M}), \mathfrak{K})$.

The elements of $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(\mathcal{M})$ are called $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}}$- open sets and its complements are called $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}}$- closed sets.

Definition: 2.3[9] Let $\mathscr{k}$ be an ideal in the universal set $\mathfrak{U}$. Let $\mathcal{G} \subseteq \mathfrak{U}$.
Then the Nano right dynamic lower, upper and boundary regions are given by

$\underline{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G}) = \{\varrho \in \mathfrak{U}: \mathfrak{R}_i(\varrho) \cap \mathcal{G}^c \in \mathscr{k} \; and \; \mathfrak{R}_i(\varrho) \subseteq \mathcal{G}\}$,

where $\mathfrak{R}_i(\varrho) = \{\beta \in U: \mathfrak{R}_r(\eta) \subseteq \mathfrak{R}_r(\beta)\}$and

$\overline{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G}) = \{\varrho \in \mathfrak{U}: \mathfrak{R}_i(\varrho) \cap \mathcal{G} \notin \mathscr{k} \; and \; \mathfrak{R}_i \cap \mathcal{G} \neq \emptyset\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G}) = \overline{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G}) - \underline{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G})$. Also, the generated Nano right dynamic topology (called $\mathfrak{D}_{\mathfrak{R}}$ - Nano topology) with respect $\mathcal{G} \subseteq \mathfrak{U}$ is given by $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(\mathcal{G}) = \{\mathfrak{U}, \emptyset, \overline{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G}), \underline{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G}), \mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G})\}$.

The collection $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G}) = \{\mathfrak{U}, \underline{\mathfrak{D}_{\mathfrak{R}}}(\mathcal{G}), \mathfrak{B}_{\mathfrak{C}_{\mathfrak{R}}}(\mathcal{G})\}$ forms a basis for $\mathfrak{Y}\mathfrak{C}_{\mathfrak{R}}$ and the Nano right dynamic topological space is denoted by $(\mathfrak{U}, \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(\mathcal{G}), \mathscr{k})$.The elements of $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(\mathcal{G})$ are called $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}}$- open sets and its complements are called $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}}$- closed sets.

3. Application of $\mathfrak{D}_{\mathfrak{L}}$ and $\mathfrak{D}_{\mathfrak{R}}$ - Nano topological space

Proposed Methodology

Strategy of the present paper employs the following approaches:

Step 1: Determine particular indicators of the disease and make them as conditional attributes.

Step 2: Create a prospective information resource table that indicates the sickness correlates under relevant situational indicators characterized properly.

Step 3: Establish the equivalence relation, left neighbourhoods and initial left neighbourhoods of each elements in the universal set.

Step 4: Compute $\underline{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M})$, $\overline{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M})$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{L}}}(\mathcal{M})$ and generate $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(\mathcal{M})$ with respect to the ideal $\mathscr{k}$(recommendations from experts).

Step 5: Determine $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{c_i\}}(\mathcal{M})$.If $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{c_i\}}(\mathcal{M}) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(\mathcal{M})$ then the factor c_i is described as core factor.

Step 6: complete step 4 and step 5 for each conditional attributes.

Step 7: Ultimately a substantial core set with vital components is generated.

Steps 1 to 8 will be applied for $\mathfrak{D}_{\mathfrak{R}}$ - Nano topological spaces. Then the core factors which are common to both topological spaces are considered as core factors.

4. Results and Discussions

Computational Assessment

Stage 1:

The following data in the Table 1 is inspected to analyze the efficiency of the current method. In this information set decision attribute is Healthy (D) and conditional attribute set C consists of Carbohydrates(c_1), Calcium(c_2), Proteins(c_3), Fat(c_4), Fiber(c_5), Vitamins (c_6), Minerals(c_7),Water(c_8) and Glucose(c_9).Three qualitative factors are used to categorize each of these characteristics: 1-Low level,2-Medium level and 3-High level.Also,15 entities are taken, U={ l_i, i=1 to 15 } and the ideal $\mathscr{k} = \{\varphi, \{l_{13}\}, \{l_{14}\}, \{l_{13}, l_{14}\}$ is considered.

Table 1: The data table of Nutrition Analysis

Adults	j_1	j_2	j_3	j_4	j_5	j_6	j_7	j_8	j_9	Decision (D)
l_1	3	1	1	2	1	2	2	2	2	Unhealthy
l_2	3	2	1	2	2	1	1	2	2	Healthy
l_3	3	2	2	1	2	2	2	3	3	Unhealthy
l_4	2	2	1	2	1	3	3	1	2	Healthy
l_5	3	1	1	3	2	1	1	3	3	Unhealthy
l_6	2	1	2	2	2	1	2	2	2	Unhealthy
l_7	2	2	1	2	1	1	2	2	1	Healthy
l_8	2	2	1	3	3	2	1	3	2	Unhealthy
l_9	2	2	2	2	2	2	2	1	1	Healthy
l_{10}	2	2	2	1	2	2	2	2	3	Healthy
l_{11}	2	1	3	1	3	2	2	3	1	Healthy
l_{12}	3	1	1	1	1	1	1	2	2	Unhealthy
l_{13}	3	1	2	1	1	1	1	2	2	Unhealthy
l_{14}	3	2	2	1	2	1	2	2	2	Unhealthy
l_{15}	3	2	1	1	2	2	1	2	2	Healthy

From the data table 1, the different equivalence classes are as follows:

G_1={ l_2, l_4, l_7, l_9,l_{10}, l_{11},l_{15} }G_2={ l_1,l_3,l_5,l_6, l_8, l_{12},l_{13}, l_{14}} and the equivalence relation is

R={ (l_1, l_1), (l_{12}, l_1), (l_2, l_2), (l_{12}, l_2), (l_3, l_3), (l_{10}, l_3), (l_{12}, l_3), (l_{13}, l_3), (l_{14}, l_3), (l_{15}, l_3), (l_4, l_4), (l_5, l_5), (l_{12}, l_5), (l_6, l_6), (l_7, l_7), (l_8, l_8), (l_9, l_9), (l_{10}, l_{10}), (l_{11}, l_1), (l_{12}, l_{12}), (l_{12}, l_{13}), (l_{12}, l_{14}), (l_{13}, l_{13}),(l_{13}, l_{14}), (l_{14}, l_{14}), (l_{15}, l_{15}) }.

Table 2: Right Neighbourhoods and Initial Right Neighbourhoods of each $\{l_i\}$

$\{l_i\}$	Right Neighbourhoods $\mathfrak{R}_r\{l_i\}$	Initial Right Neighbourhoods $\mathfrak{R}_i\{l_i\}$	$\{l_i\}$	Right Neighbourhoods $\mathfrak{R}_r\{l_i\}$	Initial Right Neighbourhoods$\mathfrak{R}_i\{l_i\}$
$\{l_1\}$	$\{l_1\}$	$\{l_1, l_{12}\}$	$\{l_9\}$	$\{l_9\}$	$\{l_9\}$
$\{l_2\}$	$\{l_2\}$	$\{l_2, l_{12}\}$	$\{l_{10}\}$	$\{l_3, l_{10}\}$	$\{l_{10}\}$
$\{l_3\}$	$\{l_3\}$	$\{l_i, i = 3,10,12,13,14,15\}$	$\{l_{11}\}$	$\{l_{11}\}$	$\{l_{11}\}$
$\{l_4\}$	$\{l_4\}$	$\{l_4\}$	$\{l_{12}\}$	$\{l_i, i = 1,2,3,5,12-15\}$	$\{l_{12}\}$
$\{l_5\}$	$\{l_5\}$	$\{l_5, l_{12}\}$	$\{l_{13}\}$	$\{l_3, l_{13}, l_{14}\}$	$\{l_{12}, l_{13}\}$
$\{l_6\}$	$\{l_6\}$	$\{l_6\}$	$\{l_{14}\}$	$\{l_3, l_{14}\}$	$\{l_{12}, l_{13}, l_{14}\}$
$\{l_7\}$	$\{l_7\}$	$\{l_7\}$	$\{l_{15}\}$	$\{l_3, l_{15}\}$	$\{l_{12}, l_{15}\}$
$\{l_8\}$	$\{l_8\}$	$\{l_8\}$			

From the table 2,the following different approximations corresponding to G_1 are obtained $\underline{\mathfrak{D}_{\mathfrak{R}}}(G_1) = \{l_4, l_7, l_9, l_{10}, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{R}}}(G_1) = \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}}}(G_1) = \{l_2, l_3, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_7, l_9, l_{10}, l_{11}\}, \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$.

For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}}}(G_2) = \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}}}(G_2) = \{l_2, l_3, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}, \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$
For $C - \{l_1\}$, the equivalence relation is given below
$\underline{\mathfrak{D}_{\mathfrak{R}-\{l_1\}}}(G_1) = \{l_4, l_9\}$, $\overline{\mathfrak{D}_{\mathfrak{R}-\{l_1\}}}(G_1) = \{l_2, l_3, l_4, l_7, l_8, l_9, l_{10}, l_{11}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_1\}}}(G) = \{l_2, l_3, l_7, l_8, l_{10}, l_{11}, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_1\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_9\}, \{l_2, l_3, l_4, l_7, l_8, l_9, l_{10}, l_{11}, l_{15}\}, \{l_2, l_3, l_7, l_8, l_{10}, l_{11}, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}-\{l_1\}}}(G_2) = \{l_1, l_5, l_6, l_{12}, l_{13}, l_{14}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}-\{l_1\}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_7, l_8, l_{10}, l_{11}, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_1\}}}(G_2) = \{l_2, l_3, l_7, l_8, l_{10}, l_{11}, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_1\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_5, l_6, l_{12}, l_{13}, l_{14}\}, \{l_1, l_2, l_3, l_5, l_6, l_7, l_8, l_{10}, l_{11}, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_7, l_8, l_{10}, l_{11}, l_{15}\}\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_1\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_1\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2)$.
For $-\{l_2\}$,
$\underline{\mathfrak{D}_{\mathfrak{R}-\{l_2\}}}(G_1) = \{l_4, l_7, l_9, l_{10}, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{R}-\{l_2\}}}(G_1) = \{l_2, l_3, l_4, l_5, l_7, l_9, l_{10}, l_{11}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_2\}}}(G) = \{l_2, l_3, l_5, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_2\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_7, l_9, l_{10}, l_{11}\}, \{l_2, l_3, l_4, l_5, l_7, l_9, l_{10}, l_{11}, l_{15}\}, \{l_2, l_3, l_5, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}-\{l_2\}}}(G_2) = \{l_1, l_6, l_8, l_{12}, l_{13}, l_{14}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}-\{l_2\}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_2\}}}(G_2) = \{l_2, l_3, l_5, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_2\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_6, l_8, l_{12}, l_{13}, l_{14}\}, \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_5, l_{15}\}\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_2\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_2\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2)$.
For $C - \{l_3\}$,
$\underline{\mathfrak{D}_{\mathfrak{R}-\{l_3\}}}(G_1) = \{l_4, l_7, l_9, l_{10}, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{R}-\{l_3\}}}(G_1) = \{l_2, l_3, l_4, l_7, l_8, l_9, l_{10}, l_{11}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_3\}}}(G) = \{l_2, l_3, l_5, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_3\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_7, l_9, l_{10}, l_{11}\}, \{l_2, l_3, l_4, l_5, l_7, l_8, l_{10}, l_{11}, l_{15}\}, \{l_2, l_3, l_5, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}-\{l_3\}}}(G_2) = \{l_1, l_5, l_6, l_{12}, l_{13}, l_{14}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}-\{l_3\}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_3\}}}(G_2) = \{l_2, l_3, l_8, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_3\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_5, l_6, l_{12}, l_{13}, l_{14}\}, \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_8, l_{15}\}\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_3\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_3\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2)$.
For $C - \{l_4\}$,
$\underline{\mathfrak{D}_{\mathfrak{R}-\{l_4\}}}(G_1) = \{l_4, l_7, l_9\}$, $\overline{\mathfrak{D}_{\mathfrak{R}-\{l_4\}}}(G_1) = \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_4\}}}(G) = \{l_2, l_3, l_{10}, l_{11}, l_{14}, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_4\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_7, l_9\}, \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{14}, l_{15}\}, \{l_2, l_3, l_{10}, l_{11}, l_{14}, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}-\{l_4\}}}(G_2) = \{l_1, l_5, l_6, l_8, l_{12}, l_{13}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}-\{l_4\}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_8, l_{10}, l_{11}, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_4\}}}(G_2) = \{l_2, l_3, l_{10}, l_{11}, l_{14}, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_4\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_5, l_6, l_8, l_{12}, l_{13}\}, \{l_1, l_2, l_3, l_5, l_6, l_8, l_{10}, l_{11}, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_{10}, l_{11}, l_{14}, l_{15}\}\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_4\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_4\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2)$.
For $C - \{l_5\}$,
$\underline{\mathfrak{D}_{\mathfrak{R}-\{l_5\}}}(G_1) = \{l_4, l_7, l_9, l_{10}, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{R}-\{l_5\}}}(G_1) = \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_5\}}}(G) = \{l_2, l_3, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_5\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_7, l_9, l_{10}, l_{11}\}, \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}-\{l_5\}}}(G_2) = \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}-\{l_5\}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_5\}}}(G_2) = \{l_2, l_3, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_5\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}, \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_5\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_5\}}(G_2) = \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2)$.
For $C - \{l_6\}$
$\underline{\mathfrak{D}_{\mathfrak{R}-\{l_6\}}}(G_1) = \{l_4, l_7, l_9, l_{10}, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{R}-\{l_6\}}}(G_1) = \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_6\}}}(G) = \{l_2, l_3, l_{14}, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_6\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_7, l_9, l_{10}, l_{11}\}, \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{14}, l_{15}\}, \{l_2, l_3, l_{14}, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}-\{l_6\}}}(G_2) = \{l_1, l_5, l_6, l_8, l_{12}, l_{13}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}-\{l_6\}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_6\}}}(G_2) = \{l_2, l_3, l_{14}, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_6\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_5, l_6, l_8, l_{12}, l_{13}\}, \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_{14}, l_{15}\}\}$

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$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_6\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_6\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2)$.
For $C - \{l_7\}$
$\underline{\mathfrak{D}_{\mathfrak{R}-\{l_7\}}}(G_1) = \{l_4, l_7, l_9, l_{10}, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{R}-\{l_7\}}}(G_1) = \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_7\}}}(G) = \{l_2, l_3, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_7\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_7, l_9, l_{10}, l_{11}\}, \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}-\{l_7\}}}(G_2) = \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}-\{l_7\}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_7\}}}(G_2) = \{l_2, l_3, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_7\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}, \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_7\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_7\}}(G_2) = \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2)$.
For $C - \{l_8\}$,
$\underline{\mathfrak{D}_{\mathfrak{R}-\{l_8\}}}(G_1) = \{l_4, l_7, l_9, l_{10}, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{R}-\{l_8\}}}(G_1) = \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_8\}}}(G) = \{l_2, l_3, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_8\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_7, l_9, l_{10}, l_{11}\}, \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}-\{l_8\}}}(G_2) = \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}-\{l_8\}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_8\}}}(G_2) = \{l_2, l_3, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_8\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}, \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_8\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_8\}}(G_2) = \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2)$.
For $C - \{l_9\}$,
$\underline{\mathfrak{D}_{\mathfrak{R}-\{l_9\}}}(G_1) = \{l_4, l_7, l_9, l_{10}, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{R}-\{l_9\}}}(G_1) = \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_9\}}}(G) = \{l_2, l_3, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_9\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_4, l_7, l_9, l_{10}, l_{11}\}, \{l_2, l_3, l_4, l_7, l_9, l_{10}, l_{11}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{R}-\{l_9\}}}(G_2) = \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}$,
$\overline{\mathfrak{D}_{\mathfrak{R}-\{l_9\}}}(G_2) = \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{R}-\{l_9\}}}(G_2) = \{l_2, l_3, l_{15}\}$ .Also, the $\mathfrak{D}_{\mathfrak{R}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_9\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}\}, \{l_1, l_2, l_3, l_5, l_6, l_8, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_2, l_3, l_{15}\}\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_9\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{R}-\{l_9\}}(G_2) = \mathfrak{N}\mathfrak{D}_{\mathfrak{R}}(G_2)$.
From the above calculation, we conclude that l_1, l_2, l_3, l_4, l_6 are the major impact factors.

Stage 2:

Table 3: Left Neighbourhoods and Initial Left Neighbourhoods of each $\{l_i\}$

$\{l_i\}$	Left Neighbourhoods $\mathfrak{R}_\ell\{l_i\}$	Initial Left Neighbourhoods$\mathfrak{R}_i{}'\{l_i\}$	$\{l_i\}$	Left Neighbourhoods $\mathfrak{R}_\ell\{l_i\}$	Initial Left Neighbourhoods$\mathfrak{R}_i{}'\{l_i\}$
$\{l_1\}$	$\{l_1, l_{12}\}$	$\{l_1\}$	$\{l_9\}$	$\{l_9\}$	$\{l_9\}$
$\{l_2\}$	$\{l_2, l_{12}\}$	$\{l_2\}$	$\{l_{10}\}$	$\{l_{10}\}$	$\{l_3, l_{10}\}$
$\{l_3\}$	$\{l_i, i = 3,10,12,13,14,15\}$	$\{l_3\}$	$\{l_{11}\}$	$\{l_{11}\}$	$\{l_{11}\}$
$\{l_4\}$	$\{l_4\}$	$\{l_4\}$	$\{l_{12}\}$	$\{l_{12}\}$	$\{l_i, i = 1,2,3,5,12 - 15\}$
$\{l_5\}$	$\{l_5, l_{12}\}$	$\{l_5\}$	$\{l_{13}\}$	$\{l_{12}, l_{13}\}$	$\{l_3, l_{13}, l_{14}\}$
$\{l_6\}$	$\{l_6\}$	$\{l_6\}$	$\{l_{14}\}$	$\{l_{12}, l_{13}, l_{14}\}$	$\{l_3, l_{14}\}$
$\{l_7\}$	$\{l_7\}$	$\{l_7\}$	$\{l_{15}\}$	$\{l_{12}, l_{15}\}$	$\{l_3, l_{15}\}$
$\{l_8\}$	$\{l_8\}$	$\{l_8\}$			

From the table 3,the following different approximations corresponding to G_1 are obtained
$\underline{\mathfrak{D}_{\mathfrak{L}}}(G_1) = \{l_2, l_4, l_7, l_9, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{L}}}(G_1) = \{l_2, l_4, l_7, l_9, l_{10}, l_{11}, l_{12}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{L}}}(G) = \{l_{10}, l_{12}, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{L}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1) = \{\mathfrak{U}, \varphi, \{l_2, l_4, l_7, l_9, l_{11}\}, \{l_2, l_4, l_7, l_9, l_{10}, l_{11}, l_{12}, l_{15}\}, \{l_{10}, l_{12}, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{L}}}(G_2) = \{l_1, l_3, l_5, l_6, l_8, l_{13}, l_{14}, l_{15}\}$,
$\overline{\mathfrak{D}_{\mathfrak{L}}}(G_2) = \{l_j, j = 1,3,5,6,8,10,12 - 15\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{L}}}(G_2) = \{l_{10}, l_{12}\}$ .Also, the $\mathfrak{D}_{\mathfrak{L}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_3, l_5, l_6, l_8, l_{13}, l_{14}, l_{15}\}, \{l_j, j = 1,3,5,6,8,10,12 - 15\}, \{l_{10}, l_{12}\}\}$

For $C - \{l_1\}$,$\underline{\mathfrak{D}_{\mathfrak{L}-\{l_1\}}}(G_1) = \{l_2, l_4, l_7, l_9, l_{11}\}$, $\overline{\mathfrak{D}_{\mathfrak{L}-\{l_1\}}}(G_1) = \{l_2, l_4, l_7, l_9, l_{10}, l_{11}, l_{12}, l_{13}, l_{14}, l_{15}\}$ and $\mathfrak{B}_{\mathfrak{D}_{\mathfrak{L}-\{l_1\}}}(G) = \{l_{10}, l_{12}, l_{13}, l_{14}, l_{15}\}$ .Here the $\mathfrak{D}_{\mathfrak{L}}$ - topology related to G_1 is $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_1\}}(G_1) = \{\mathfrak{U}, \varphi, \{l_2, l_4, l_7, l_9, l_{11}\}, \{l_2, l_4, l_7, l_9, l_{10}, l_{11}, l_{12}, l_{13}, l_{14}, l_{15}\}, \{l_{10}, l_{12}, l_{13}, l_{14}, l_{15}\}\}$.
For G_2, $\underline{\mathfrak{D}_{\mathfrak{L}-\{l_1\}}}(G_2) = \{l_1, l_3, l_5, l_6, l_8\}$,
$\overline{\mathfrak{D}_{\mathfrak{L}-\{l_1\}}}(G_2) = \{l_j, j = 1,3,5,6,8,10,12 - 15\}$ and
$\mathfrak{B}_{\mathfrak{D}_{\mathfrak{L}-\{l_1\}}}(G_2) = \{l_{10}, l_{12}, l_{13}, l_{14}, l_{15}\}$.Also, the $\mathfrak{D}_{\mathfrak{L}}$ - topology related to G_2 is
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_1\}}(G_2) = \{\mathfrak{U}, \varphi, \{l_1, l_3, l_5, l_6, l_8\}, \{l_j, j = 1,3,5,6,8,10,12 - 15\}, \{l_{10}, l_{12}, l_{13}, l_{14}, l_{15}\}\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_1\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_1\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2)$.

For $-\{l_2\}$, the equivalence relation is given below.
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_2\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_2\}}(G_2) = \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2)$.
For $C-\{l_3\}$,
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_3\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_3\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2)$.
For $C-\{l_4\}$,
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_4\}}(G_1) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1)$and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_4\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2)$.
For $C-\{l_5\}$,
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_5\}}(G_1) = \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_5\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2)$.
For $C-\{l_6\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_6\}}(G_1) = \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_6\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2)$.
For $C-\{l_7\}$
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_7\}}(G_1) = \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_7\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2)$.
For $C-\{l_8\}$,
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_8\}}(G_1) = \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_8\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2)$.
For $C-\{l_9\}$, $\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_9\}}(G_1) = \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_1)$ and
$\mathfrak{N}\mathfrak{D}_{\mathfrak{L}-\{l_9\}}(G_2) \neq \mathfrak{N}\mathfrak{D}_{\mathfrak{L}}(G_2)$.
From the above calculation, we conclude that l_1, l_2, l_3, l_4 are the major impact factors.
Also, from stage 1 and stage 2, $\{l_1, l_2, l_3, l_4, l_6\} \cap \{l_1, l_2, l_3, l_4\} = \{l_1, l_2, l_3, l_4\}$ are considered as appropriate core set.

5. Conclusion

We conclude that Carbohydrates, calcium, proteins and fat are important nutrients which play vital role in healthy life. They provide support for overall health and wellbeing. Also, the suggested technique can be applied in various fields, including business, marketing, academia, medicine and more for decision - making.

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