Routh Approximation Based Evolutionary Techniques for Reduced Order Modeling

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Abstract: ***The aim of the authors is to develop the two mixed Model Order Reduction (MOR) techniques in connection with the reduction of Linear Time-Invariant (LTI) higher order systems. The first technique is particularly based on Routh approximation and Particle Swarm Optimization (PSO) and the second technique is based on Routh approximation and Cuckoo Search Algorithm (CSA). The time and frequency responses of the original and reduced order models were analyzed and plotted for comparison. From the analysis and plots for different examples considered, it is observed that the Reduced Order Model (ROM) obtained by the two mixed proposed techniques provided quite close approximation with the original higher order systems. The newly suggested methods are also justified in that the stability of the ROM provided the original higher order system is stable. Further the proposed techniques are also extended to MIMO systems. Furthermore, to show the effectiveness and superiority of the proposed methods, the obtained results are compared with different reduction methods existing in the literature with reference to IEA, RISE, IAE and ITAE.***

Keywords: L Linear Time-Invariant higher order systems, Particle Swarm Optimization, Model Order Reduction, Routh approximation based evolutionary techniques, Cuckoo Search Algorithm, Reduced Order Model

1. Introduction

In the design and analysis of higher order systems, model reduction has become an important tool [1 -5]. The MOR technique is defined as to yield the reduced order model (ROM) without loss of important control properties of the higher order system such as stability and steady state value. The MOR of higher order models is one of the most significant fields of research in engineering and science. Thus, MOR can be used for better understanding of the higher order systems. Some other reasons for computing the reduced order systems are as follows:

In simulation problems it can be used for reducing the computational efforts-

i) for the design of controller efficiently
ii) for the reduction of hardware complexity and obtaining simpler control laws.

In modern control systems, the MOR becomes an important feature for analysis and design [6]. Currently, the MOR has become a prominent tool in the prosperous field of engineering design such as fluid dynamics [17], power system [8-10], control theory [11-13] etc. Among these applications, reduced order models are mostly used for the synthesis and analysis of large-scale models [14].

In frequency domain, frequently used model reduction methods are Padé approximation [15], Routh stability [16], Routh approximation [17], stability equation [18] and pole clustering [19] etc. Routh stability method is suitable for matching Markov parameters of ROM to that of original complex system and for that it is relevant method for the matching of transient responses of ROM and original complex system [20]. Sometimes Routh stability technique gives same ROM of different large-scale systems, and this non-uniqueness was first discussed by Singh in 1979 [21]. The Routh stability method fails for the systems in which pole zero cancellation occur. Hutton and Friedland [17] proposed a method for computing the stable reduced model for the stable large scale single input single output (SISO) LTI systems, which is known as Routh approximation method. Papadopoulo and Bandekas proposed Routh approximation method for multiple-input multiple-output (MIMO) LTI systems [22]. Routh approximation method is applicable only for those higher order systems in which order of the numerator polynomial is one less than the order of the denominator polynomial [23]. To circumvent this problem, Langholz and Feinmesser proposed a modified Routh approximation method [23]. For obtaining the reduced models by Routh approximation method two tables are required to formulate which are known as alpha table (α-table) and beta table (β-table). In these tables, alpha tables are computed as simple as Routh Hurwitz table [16] but formulation of beta table is complicated as compared to alpha table. To avoid the formulation of beta table, several Routh approximations based mixed model reduction techniques have been proposed [24].

In time domain, balanced truncation [25], singular perturbation [26], aggregation method [27] and Krylov subspace [28] methods are frequently used model reduction techniques. Among these methods, balanced truncation for model reduction is a more popular method because it guarantees the error bound and stability of a reduced model. The balanced truncation technique was further simplified [29-30]. Abbas and Werner extended the balanced truncation as well as frequency-weighted balanced truncation [6] techniques of LTI systems to the discrete-time linear parameter-varying systems [31]. Besselink *et al.* [32] proposed the balanced truncation technique for the nonlinear systems by computing incremental controllability and observability function without linearization. Perev and Shafai

[33] discussed the limitations of balanced realization and MOR of a singular system.

To overcome the problem of Routh approximation technique [23] and complexity in formulation of beta table, a new mixed model order reduction technique has been proposed in this paper. The proposed method guarantees the preservation of stability of the original system in the reduced model and preserves the features of Routh approximation technique. In this method, the denominator polynomial of the lower order system is obtained by using Routh approximation method. The numerator polynomial is computed by comparing the transfer function of original higher order system with its reduced model whose denominator polynomial is already obtained by Routh approximation method.

2. Statement of Problem

Consider an nth order large scale SISO LTI dynamic system, described by the following transfer function

$$G(s) = \frac{N(s)}{D(s)} = \frac{d_0+d_1 s+\cdots+d_{n-1}s^{n-1}}{e_0+e_1 s+e_2 s^2+\cdots+e_n s^n} \quad (1)$$

where $d_0, d_1 \dots d_{n-1}$ and $e_0, e_1 \dots e_n$ are the known constants of original system. The objective of the paper is to obtain the unknown scalar constants of rth-order $(r < n)$ reduced order model having the following transfer function:

$$R_r(s) = \frac{Q_r(s)}{P_r(s)} = \frac{q_0+q_1 s+q_2 s^2+\cdots+q_{r-1}s^{r-1}}{p_0+p_1 s+p_2 s^2+\cdots+p_{r-1}s^{r-1}+p_r s^r} \quad (2)$$

where $q_0, q_1 \dots q_{r-1}$ and $p_0, p_1 \dots p_r$ are unknown constants of the lower order model.

Let us consider an nth-order large scale MIMO LTI system with u inputs and v outputs represented in the following transfer matrix form,

$$[G(s)] = \frac{1}{D(s)}\begin{bmatrix} A_{11}(s) & A_{12}(s) & \cdots & A_{1u}(s) \\ A_{21}(s) & A_{22}(s) & \cdots & A_{2u}(s) \\ \vdots & \vdots & \vdots & \vdots \\ A_{v1}(s) & A_{v2}(s) & \cdots & A_{vu}(s) \end{bmatrix} \quad (3)$$

$$= [g_{ij}(s)]_{v\times u} \quad (4)$$

where $i = 1, 2, 3, \dots, v$; $j = 1, 2, 3, \dots, u$. Hence $g_{ij}(s)$ can be written as

$$g_{ij}(s) = \frac{A_{ij}(s)}{D(s)} \quad (5)$$

The aim of the paper is to obtain the rth order reduced model $[R_r(s)]$ in the form of Equation (6) with u inputs and v outputs, such that it retains the important features of the higher order original system, and which is expressed in the following transfer matrix form,

$$[R_r(s)] = \frac{1}{P_r(s)}\begin{bmatrix} B_{11}(s) & B_{12}(s) & \cdots & B_{1u}(s) \\ B_{21}(s) & B_{22}(s) & \cdots & B_{2u}(s) \\ \vdots & \vdots & \vdots & \vdots \\ B_{v1}(s) & B_{v2}(s) & \cdots & B_{vu}(s) \end{bmatrix} \quad (6)$$

$$= [r_{ij}(s)]_{v\times u} \quad (7)$$

where $i = 1, 2, 3, \dots, v$; $j = 1, 2, 3, \dots, u$. Hence $r_{ij}(s)$ can be written as

$$r_{ij}(s) = \frac{B_{ij}(s)}{P_r(s)} \quad (8)$$

where $g_{ij}(s)$ and $r_{ij}(s)$ are the various elements of the original and the reduced transfer function matrices respectively.

3. Description of the Proposed Method

The proposed method involves two steps for the determination of lower order system of large-scale dynamic system.

3.1. Determination of denominator polynomial of rth order reduced model

The denominator coefficients of the desired reduced model are calculated by using Routh approximation method discussed by Hutton and Friedland [17] as

(i) Determine the reciprocal of the denominator polynomial of the original system,

$$\widetilde{D}(s) = s^n D\left(\frac{1}{s}\right) \quad (9)$$

(ii) From the alpha table [17] from the coefficients of $\widetilde{D}(s)$ and obtain the values of $\alpha_1, \alpha_2, \dots \alpha_r$, parameters from that alpha table (Table 1).

(iii) Compute the rth order denominator polynomial by using the following expression.

$$\tilde{P}_r(s) = \alpha_r s\tilde{P}_{r-1}(s) + \tilde{P}_{r-2}(s), \quad r = 1, 2, \cdots \quad (10)$$

with

$$\tilde{P}_0(s) = \tilde{P}_{-1}(s) = 1$$

(iv) Apply the reciprocal transformation on rth order denominator polynomial $\tilde{P}_r(s)$ to obtain the denominator polynomial $P_r(s)$ of the desired reduced model,

$$P_r(s) = s^r \tilde{P}_r\left(\frac{1}{s}\right) \quad (11)$$

Table 1: Alpha Table

	$e_0^0 = e_0$ $e_0^1 = e_1$	$e_2^0 = e_2$ $e_2^1 = e_3$	$e_4^0 = e_4$ $e_4^1 = e_5$	$e_6^0 = e_6$ …	…
$\alpha_1 = \frac{e_0^0}{e_0^1}$	$e_0^2 = e_2^0 - \alpha_1 e_2^1$	$e_2^2 = e_4^0 - \alpha_1 e_4^1$	$e_4^2 = e_6^0 - \alpha_1 e_6^1$	…	
$\alpha_2 = \frac{e_0^1}{e_0^2}$	$e_0^3 = e_2^1 - \alpha_2 e_2^2$	$e_2^3 = e_4^1 - \alpha_2 e_4^2$	…		
$\alpha_3 = \frac{e_0^2}{e_0^3}$	$e_0^4 = e_2^3 - \alpha_3 e_2^3$	…			
…	…				

3.2 Determination of numerator polynomial of the reduced model

For obtaining the numerator polynomial of the low order model, the transfer function of large-scale original system of Equation (1) and the transfer function of rth order reduced system of Equation (2) is compared. This gives the unknown constants of the numerator polynomial as

$$\frac{d_0+d_1s+\cdots+d_{n-1}s^{n-1}}{e_0+e_1s+e_2s^2+\cdots+e_ns^n} = \frac{q_0+q_1s+q_2s^2+\cdots+q_{r-1}s^{r-1}}{p_0+p_1s+p_2s^2+\cdots+p_{r-1}s^{r-1}+p_rs^r} \quad (12)$$

By cross-multiplication of Equation (12) and comparing the same powers of 's' from s^0 to s^{r-1} on both sides, it gives "r" number of equations as below.

$$\begin{cases} d_0p_0 = e_0q_0 \\ d_0p_1 + d_1p_0 = e_0q_1 + e_1q_0 \\ d_0p_2 + d_{1,}p_1 + d_2p_0 = e_0q_2 + e_{1,}q_1 + e_{2,}q_0 \\ \vdots \end{cases} \quad (13)$$

By solving these "r" number equations, the "r" number of unknown parameters $(q_0, q_1, q_2, \dots, q_{r-1})$ of the reduced model are obtained.

4. Mathematical Modeling of DC-DC Converter

Cuk converter is a dc transformer analogous to ac transformer, which is used to convert the constant dc voltage to the variable dc voltage. It delivers output voltage which is either higher than or lower than the input voltage. Therefore, it is similar like buck-boost chopper.

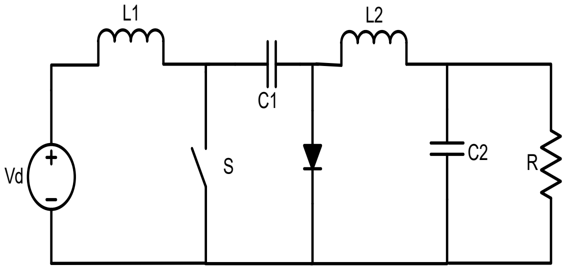

Figure 1: Cuk Converter [17].

In Figure 1, the circuit diagram of Cuk converter is shown, which has two inductors and two capacitors. This system also has one power switch and diode; hence it is a fourth order nonlinear model. For the design of feedback controllers of non-linear models, a linear model is required. The linear circuit model of the Cuk converter is obtained by small signal averaged switch model in place of diode and power switch by [34]. The small signal linear model is obtained by using state space averaging (SSA) method [35]. The Cuk converter taken in this paper is working in CCM and in this mode the inductor current never drops to zero throughout one switching period. In this mode, two states one when the switch is on and another when switch is off are exist.

4.1 When Switch On

From Figure 2, the state space equations for Cuk converter during the switch on are obtained by applying Kirchhoff's voltage law as

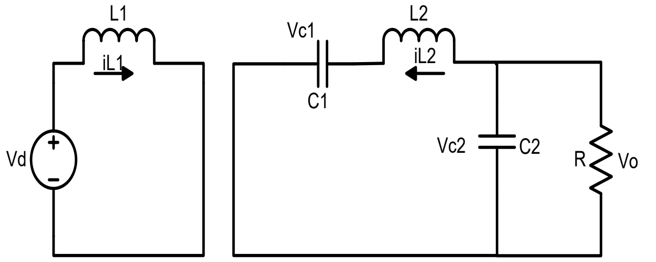

Figure 2: Cuk converter when switch is on

$$\frac{di_{L1}}{dt} = -\frac{r_{L1}i_{L1}}{L_1} + \frac{V_d}{L_1} \quad (14)$$

$$\frac{dI_{L2}}{dt} = \frac{V_{C1}}{L_2} - \frac{(r_{L2}+r_{C1}+r_{C2}\|R)i_{L2}}{L_2} - \left(\frac{r_{C2}}{R+r_{C2}} - 1\right)\frac{V_{C2}}{C_2} \quad (15)$$

$$\frac{dV_{C1}}{dt} = -\frac{i_{L2}}{C_1} \quad (16)$$

$$\frac{dV_{C2}}{dt} = -\frac{Ri_{L2}}{(R+r_{C2})C_2} - \frac{V_{C2}}{(R+r_{C2})C_2} \quad (17)$$

4.2 During Switch Off

From Figure 3, the state space equations of the converter during switched off condition are

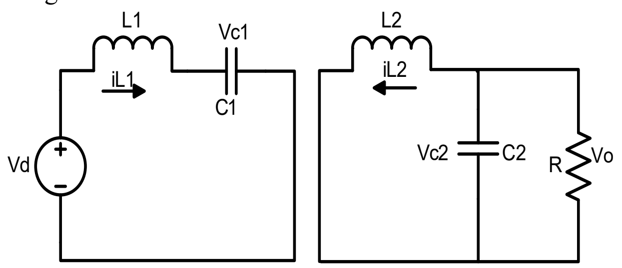

Figure 3: Cuk Converter when switch is off.

$$\frac{di_{L1}}{dt} = \frac{(r_{L1}+r_{C1})i_{L1}}{L_1} + \frac{V_d}{L_1} - \frac{V_{C1}}{L_1} \quad (18)$$

$$\frac{dI_{L2}}{dt} = \frac{(r_{L2}+r_{C2}\|R)i_{C2}}{L_2} + \frac{(\frac{r_{C2}}{R+r_{C2}}-1)V_{C2}}{R+r_{C2}} \quad (19)$$

$$\frac{dV_{C1}}{dt} = \frac{i_{L1}}{C_1} \quad (20)$$

$$\frac{dV_{C2}}{dt} = \frac{Ri_{L2}}{(R+r_{C2})C_2} - \frac{V_{C2}}{(R+r_{C2})C_2} \quad (21)$$

From the above equations, the matrices of state-space model of dc-dc converter during switch is on and off are written as

$$A_1 = \begin{bmatrix} -\frac{r_{L1}}{L_1} & 0 & 0 & 0 \\ 0 & \frac{(r_{L2}+r_{C1}+r_{C2}\parallel R)}{L_2} & \frac{1}{L_2} & \frac{-(\frac{r_{C2}}{R+r_{C2}}-1)}{C_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{R}{(R+r_{C2})C_2} & 0 & \frac{-1}{(R+r_{C2})C_2} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \frac{-r_{L1}+r_{C1}}{L_1} & 0 & \frac{1}{L_2} & 0 \\ 0 & \frac{(r_{L2}+r_{C2}\parallel R)}{L_2} & 0 & \frac{-(\frac{r_{C2}}{R+r_{C2}}-1)}{C_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{R}{(R+r_{C2})C_2} & 0 & \frac{-1}{(R+r_{C2})C_2} \end{bmatrix}$$

Paper ID: SR25720160034 DOI: https://dx.doi.org/10.21275/SR25720160034

$$B_1 = B_2 = B = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C_1 = C_2 = C = [0\ 0\ 0\ 1]$$

$D_1 = D_2 = D = [0]$.

where A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 are the matrices when switch is on and off respectively.

4.3 State Space Averaging Method

It is hard to model and predict the performance of nonlinear systems; hence approximation of non-linear model to a linear model is a general practice. For this approximation SSA technique is used [35]. The state space model when switch is on

$$\dot{x} = A_1 x + B_1 V_d \qquad 0 < t < dT \tag{22}$$

$$V_0 = C_1 x \qquad 0 < t < dT \tag{23}$$

when switch is off

$$\dot{x} = A_2 x + B_2 V_d \qquad 0 < t < (1-d)T \tag{24}$$

$$V_0 = C_2 x \qquad 0 < t < (1-d)T \tag{25}$$

To yield an average state space model of the converter over a switching period, the equations equivalent to the two previous states are time weighted and averaged as follows,

$$\dot{x} = [A_1 d + A_2(1-d)]x + [B_1 d + B_2(1-d)]V_d \tag{26}$$

$$y = V_0 = [C_1 d + C_2(1-d)]x \tag{27}$$

Table 2: Parameters of the Cuk converter [71].

Input Voltage V_d	12 Volts
Output Voltage V_0	24 Volts
Load Resistance	12 Ohm
L_1	68.7 μH
L_2	2.2 mH
C_1	3.7 μF
C_2	984
Switching frequency	100 kHz

5. Numerical Examples

To check the effectiveness and the accuracy of the proposed technique with other popular model reduction techniques, integral square error (ISE), relative integral square error (RISE), integral absolute error (IAE) and integral time absolute error (ITAE) in between the original system and lower order model have been calculated and which are defined as [37], [38]

$$\begin{cases} ISE = \int_0^\infty [y(t_i) - y_r(t_i)]^2 dt \\ RISE = \int_0^\infty [y(t_i) - y_r(t_i)]^2 dt \,/\, \int_0^\infty [\hat{y}(t_i)]^2 dt \end{cases} \tag{28}$$

$$\begin{cases} IAE = \int_0^\infty |y(t_i) - y_r(t_i)| dt \\ ITAE = \int_0^\infty t|y(t_i) - y_r(t_i)| dt \end{cases} \tag{29}$$

where $y(t_i)$ and $y_r(t_i)$ are step responses of original system and reduced order model respectively at t_i time. The final time for simulation is taken as 100 s with a sampling interval of 0.1 s.

Example 1. The value of the parameters of dc-dc converter from Table 2, are substituted in Equations (18) and (19), converting the state space model into transfer function as

$$G(s) = \frac{N(s)}{D(s)} = \frac{-8.148\times10^2 s^3 + 2.456\times10^7 s^2 - 1.232\times10^{12} s + 2.154\times10^{16}}{s^4 + 1.494\times10^2 s^3 + 4.922\times10^8 s^2 + 6.25\times10^{10} s + 2.02\times10^{14}} \tag{30}$$

By applying the reciprocal transformation in the denominator polynomial of original system, it gives $\tilde{D}(s)$ as

$$\tilde{D}(s) = s^n D\left(\frac{1}{s}\right) = 1 + 1.494\times10^2 s + 4.922\times10^8 s^2 + 6.25\times10^{10} s^3 + 2.02\times10^{14} s^4 \tag{31}$$

By using Equation (37), the alpha array is formed as

	2.02×10^{14}	4.922×10^8	1
$\alpha_1 = 3232$	6.25×10^{10}	1.494×10^2	
$\alpha_2 = 127.032$	4.92×10^8	1	
	22.4	0	

By using α_1 and α_2, the denominator polynomial of second order reduced model is obtained as

$$\tilde{P}_2(s) = 1 + \alpha_2 s + \alpha_1\alpha_2 s^2 = 1 + 1.271\times10^2 s + 4.108\times10^5 s^2 \tag{32}$$

For computing the denominator polynomial of the desired reduced system, taking the reciprocal transformation of Equation (32), and it gives

$$P(s) = s^2 + 1.271\times10^2 s + 4.108\times10^5 \tag{33}$$

By using denominator polynomial of the Equation (33), the numerator polynomial of the lower order system is calculated by using proposed technique discussed in section 3.2 and it is

$$Q(s) = -2.5059\times10^3 s + 4.3805\times10^7 \tag{34}$$

Hence, the transfer function of desired lower order systems is obtained as

$$R_2(s) = \frac{Q(s)}{P(s)} = \frac{-2.5059\times10^3 s + 4.3805\times10^7}{s^2 + 1.271\times10^2 s + 4.108\times10^5} \tag{35}$$

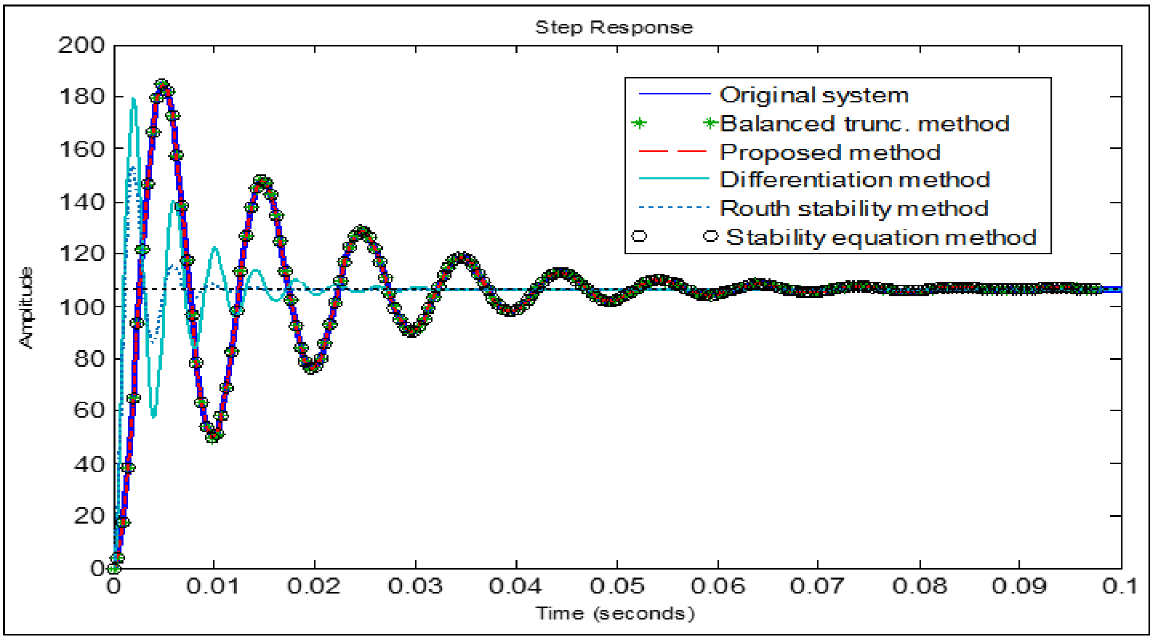

Figure 4: Comparison of time responses of original model and its various reduced order systems.

Table 3: Comparison of various model order reduction techniques for determining reduced order models.

Reduction Technique	Reduced Model	ISE	RISE	IAE	ITAE
Proposed method	$\frac{-2.5059\times10^3 s+4.3805\times10^7}{s^2+1.271\times10^2 s+4.108\times10^5}$	1.6154×10^{-5}	1.3312×10^{-11}	0.0077	0.0017
[18, 37]	$\frac{-1.232\times10^{12}s+2.154\times10^{16}}{4.9179\times10^8 s^2+6.25\times10^{10}s+2.02\times10^{14}}$	2.0657×10^{-5}	1.7023×10^{-11}	0.0084	0.0018
[40]	$\frac{-1.232\times10^{12}s+2.154\times10^{16}}{4.922\times10^8 s^2+6.25\times10^{10}s+2.02\times10^{14}}$	7.0213×10^{-5}	5.7862×10^{-11}	0.0126	0.0022
[24, 41]	$\frac{-2.505\times10^3 s+4.3805\times10^7}{s^2+1.271\times10^2 s+4.108\times10^5}$	8.5654×10^{-5}	7.0586×10^{-11}	0.2704	13.27
[24]	$\frac{-5.216\times10^{11}s+2.154\times10^{16}}{7.386\times10^7 s^2+6.209\times10^{10}s+2.02\times10^{14}}$	0.0158	1.3047×10^{-8}	0.1303	0.0139
[42]	$\frac{-1.2783\times10^{12}s+2.154\times10^{16}}{7.386\times10^7 s^2+6.209\times10^{10}s+2.02\times10^{14}}$	0.0158	1.3047×10^{-8}	0.1303	0.0139
[43]	$\frac{-5.4767\times10^8 s+2.5848\times10^{12}}{9.844\times10^3 s^2+3.75\times10^6 s+2.424\times10^{10}}$	0.0158	1.3047×10^{-8}	0.1303	0.0139
[44]	$\frac{-4.928\times10^7 s+2.5848\times10^{12}}{9.844\times10^3 s^2+3.75\times10^6 s+2.424\times10^{10}}$	0.0158	1.3047×10^{-8}	0.1303	0.0139
[15]	$\frac{-2.511\times10^3 s+4.378\times10^7}{s^2+1.27\times10^2 s+4.106\times10^5}$	0.08491	6.99752×10^{-8}	9.2126	4.6103×10^2
[25]	$\frac{-2.524\times10^3 s+4.382\times10^7}{s^2+1.271\times10^2 s+4.108\times10^5}$	1.4135	1.1649×10^{-6}	37.5969	1.8817×10^3

The error indices of the various reduced models are also presented in Table 3. From this table it is obvious that the proposed technique gives the lowest values of error compared to many other well-known existing model reduction techniques. It is also obvious that the proposed technique has the least error indices compared to some recent proposed techniques [37, 42, 43].

Example 2: Consider the 5th order SISO linear time invariant system

$$G(s)=\frac{s^4+7s^3+42s^2+142s+156}{s^5+25s^4+258s^3+930s^2+1441s+745} \quad (36)$$

The second order reduced model obtained by Routh approximation method is

$$R_2(s)=\frac{0.1782s+0.1957}{s^2+1.8089s+0.9352} \quad (37)$$

By applying the proposed method, the second order reduced model is

$$R_2(s)=\frac{0.1783s+0.1958}{s^2+1.8089s+0.9352} \quad (38)$$

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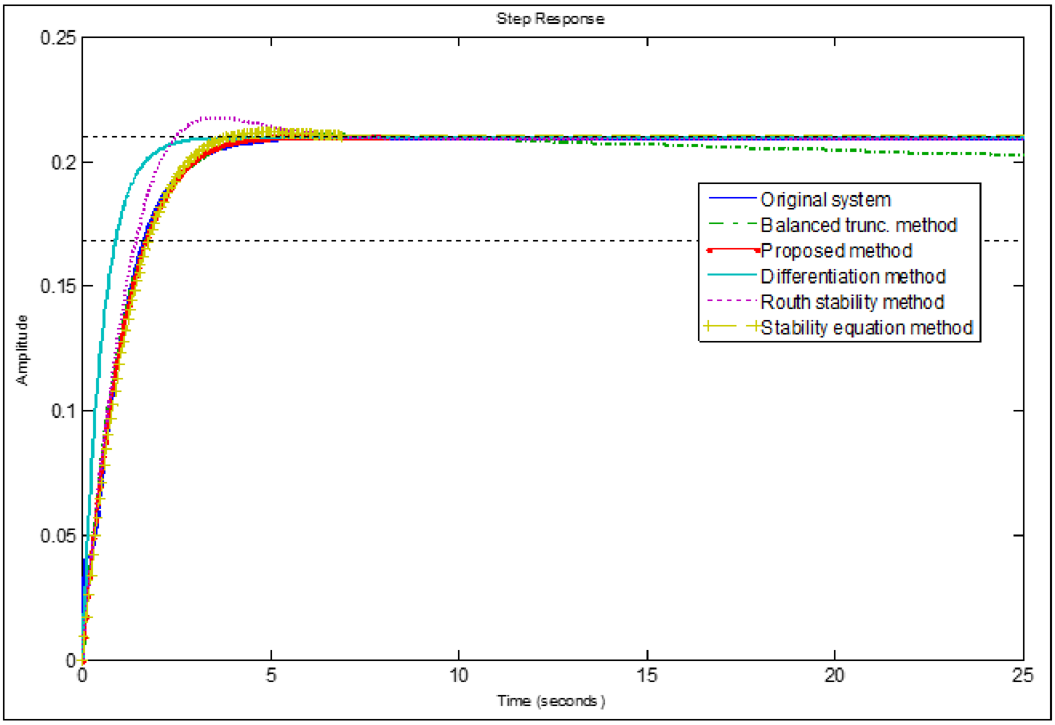

Figure 5: Comparison of unit step responses of original model with its reduced systems obtained by various reduction techniques.

The time responses of the original model and reduced models obtained by the proposed technique as well as some other existing reduction techniques are shown in Figure 5. The response of the reduced system computed by proposed technique is closely matched with the response of original system compared to the responses of reduced models obtained by some other reduction techniques. A comparative analysis of proposed techniques with other existing reduction methods in terms error indices is tabulated in Table 4. This table indicates that the reduced system obtained by the proposed technique gives the lowest values of error indices compared with other well-known model reduction techniques.

Table 4: Comparison of various model order reduction techniques with respect to ISE, RISE, IAE and ITAE

Model order reduction technique	Reduced order model	ISE	RISE	IAE	ITAE
Proposed method	$\frac{0.1783s+0.1958}{s^2+1.8089s+0.9352}$	9.4491×10^{-4}	8.2749×10^{-4}	0.1692	1.7423
[17]	$\frac{0.1782s+0.1957}{s^2+1.8089s+0.9352}$	9.5694×10^{-4}	8.3805×10^{-4}	0.2611	7.0126
[18]	$\frac{142s+156}{909.5238s^2+1441s+745}$	0.0017	0.0015	0.2643	0.7717
[40]	$\frac{1142s+156}{930s^2+1441s+745}$	0.0021	0.0018	0.2926	0.8615
[42]	$\frac{91.0389s+156}{770.2174s^2+1197.6291s+745}$	0.0049	0.0043	0.4844	1.2745
[16]	$\frac{133.0285s+156}{770.2174s^2+1197.6291s+745}$	0.0078	0.0069	0.6404	1.7791
[43]	$\frac{-0.002341s+9360.18}{5580s^2+34584s+44700}$	0.0125	0.0110	0.6470	1.3224
[44]	$\frac{2130.041s+9360.18}{5580s^2+34584s+44700}$	0.0381	0.0333	1.0222	1.6766
[39]	$\frac{0.007s+156}{909.5238s^2+1441s+745}$	0.10534	0.0923	1.9481	3.5683
[25]	$\frac{0.1961s+0.0019}{s^2+0.9295s+0.0114}$	0.30663	0.2685	15.1664	1007.9

Example 3: Consider a multivariable sixth-order original system [45] having two inputs and two-outputs defined by the following transfer matrix as

$$[G(s)] = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix} \quad (39)$$

$$= \frac{1}{D(s)}\begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

where

$$D(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$
$$= s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$

and

$$A_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$$
$$A_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$
$$A_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$
$$A_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

The second order reduced system computed by using Routh Approximation method is

$$[R_2(s)] = \frac{\begin{bmatrix} 0.9101s+0.7091 & 0.4917s+0.2837 \\ 0.4373s+0.3546 & 1.076s+0.7091 \end{bmatrix}}{s^2+1.548s+0.7091} \quad (40)$$

Hence the denominator of the desired reduced model obtained by Routh approximation technique [25] is

$$P_2(s) = s^2 + 1.548s + 0.7091 \quad (41)$$

The numerator of the reduced model is obtained by proposed method discussed in section 3.2, and transfer matrix of the 2nd order reduced model computed by proposed technique is

$$[R_2(s)] = \frac{\begin{bmatrix} 0.9098s+0.7091 & 0.4916s+0.2836 \\ 0.4373s+0.3545 & 1.0753s+0.7091 \end{bmatrix}}{s^2+1.548s+0.7091} \quad (42)$$

The transfer matrix of second order reduced model obtained by Narwal and Prasad [36] is

$$[R_2(s)] = \frac{\begin{bmatrix} 0.8930s+0.6181 & 0.4517s+0.2472 \\ 0.4314s+0.3091 & 1.0579s+0.6181 \end{bmatrix}}{s^2+1.34952s+0.6181} \quad (43)$$

The reduced order model obtained by Sikander and Prasad [37] is

$$[R_2(s)] = \frac{\begin{bmatrix} 0.7938s+0.6181 & 0.4273s+0.2472 \\ 0.37952s+0.309 & 0.93382s+0.6181 \end{bmatrix}}{s^2+1.34952s+0.6181} \quad (44)$$

The transfer matrix of second order reduced model obtained by Parmar, Prasad and Mukherjee [46] is

$$[R_2(s)] = \frac{\begin{bmatrix} 0.8503s+0.6171 & 0.4617s+0.2466 \\ 0.4093s+0.3086 & 0.9977s+0.6171 \end{bmatrix}}{s^2+1.34952s+0.6181} \quad (45)$$

The reduced order model obtained by Parmar, Prasad and Mukherjee [48] is

$$[R_2(s)] = \frac{\begin{bmatrix} 6.0429s+8.4707 & 3.9419s+3.3883 \\ 2.8097s+4.2354 & 8.0195s+8.4707 \end{bmatrix}}{s^2+13.6666s+8.4707} \quad (46)$$

Table 5: Comparison of ISE with different reduction techniques

Reduction method	$r_{11}(s)$	$r_{12}(s)$	$r_{21}(s)$	$r_{22}(s)$
[45]	0.225	0.0682	0.0613	0.6780
[39]	0.1672	0.0958	0.0312	0.2004
[37]	0.1615	0.0897	0.0296	251.3574
[46]	0.1471	0.0884	0.0258	0.1598
[17]	0.0764	0.0596	0.0116	0.0808
Proposed method	0.0765	0.0595	0.0115	0.0812

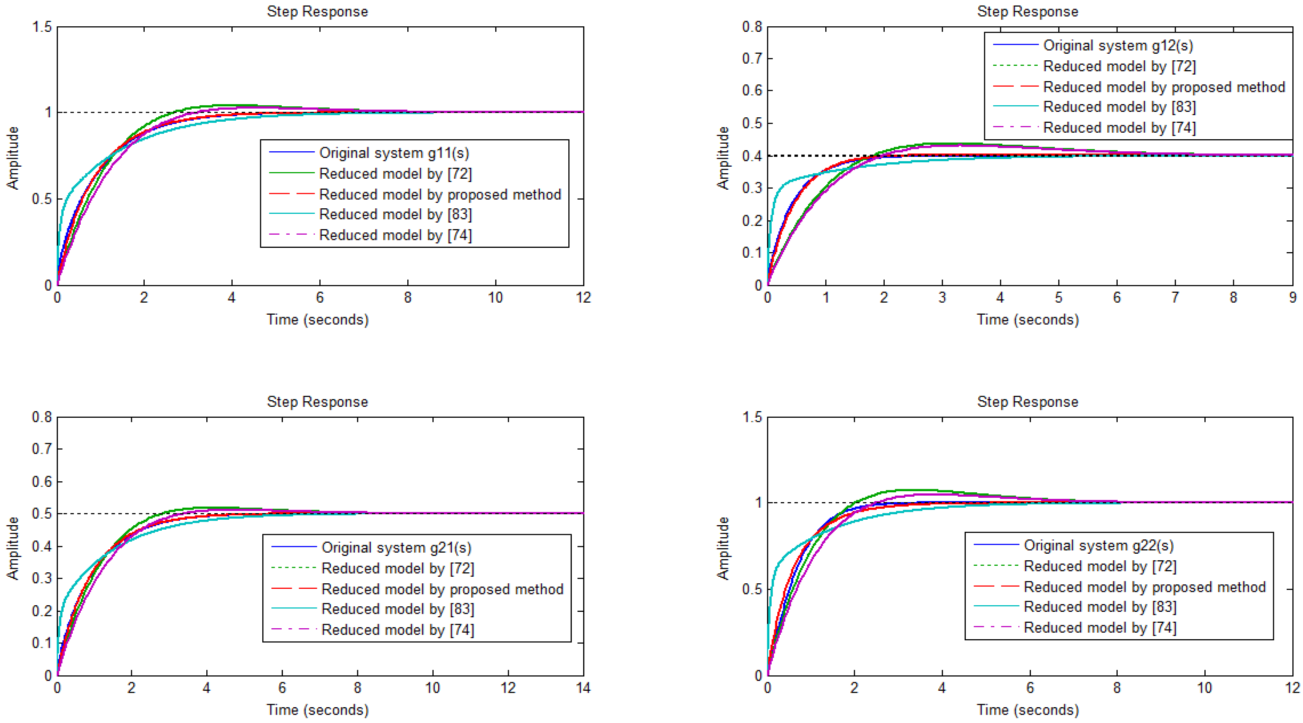

Figure 6: Comparison of step responses of the original system with the reduced models

The comparison of step responses of reduced models with original system is shown in Figure 6, which demonstrates the superior performance of the proposed technique. From this figure, the response of the lower order system computed by the proposed technique is approximately matched to the response of the original system. The ISE values of various reduced order systems are tabulated in Table 5, which demonstrates that the proposed technique has the lower ISE value compared with some other existing methods. From this table it is also obvious that the proposed method has least ISE compared to some recent [37, 39] and optimization [37, 46] methods.

6. Conclusion

By using a new model reduction technique, the denominator of the reduced model is obtained by Routh approximation method, and the numerator is computed by simple mathematical algorithm discussed in sub-section 3.2. The comparison of the step responses in Figures 4, 5 and 6 demonstrate that the reduced models obtained by proposed technique are closer approximations of the original systems. Furthermore, the accuracy, efficiency and the better performance of the proposed technique were evaluated by comparing the error indices values for the various lower order systems in Tables 3, 4 and 5. It is observed that the lower order systems obtained by the proposed technique showed better performance and they preserved most of the fundamental features of the original system such as stability, transient and steady state responses. The proposed method has been applied on two standard examples taken from literature. The first example is the transfer function of Cuk converter, and its lower order model may be used to undertake controller design and study of the closed loop performance.

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