Examining Free Disposal Hull (FDH) Data to Assess Efficiency: An Analysis of District Hospitals

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Abstract: ***Free Disposal Hull (FDH) serves as an alternative approach to Data Envelopment Analysis (DEA) for evaluating efficiency. This study applies the FDH method to assess the performance of district hospitals in the state of Madhya Pradesh, India. As a non-parametric technique commonly used in operations research and economics, FDH evaluates how effectively decision-making units (DMUs), such as hospitals, utilize resources. By analysing data from multiple district hospitals, this paper aims to identify inefficiencies and suggest actionable improvements to enhance healthcare services in the region. The findings highlight key areas for boosting efficiency and provide a benchmark for future performance assessments.***

Keywords: Free Disposal Hull, Data Envelopment Analysis, Efficiency, District Hospital

1. Introduction

Effective health service delivery depends on efficient healthcare, especially in environments with limited resources like Madhya Pradesh, India. With fewer resources, efficient hospitals can provide better patient outcomes. The Free Disposal Hull (FDH) method is used in this study to evaluate the effectiveness of Madhya Pradesh's district hospitals. FDH is a good fit for complex healthcare contexts because of its versatility in handling many inputs and outputs without requiring a predefined functional form.

This study uses the FDH Method, a non-parametric technique, to examine the relative effectiveness of Madhya Pradesh's district hospitals. The study topic that is being investigated is: how much input can be saved for a certain level of outputs, and did the hospital manage its resources efficiently? A significant portion of health care resources are consumed by hospitals, which are essential components of the Indian healthcare system. It is now essential to track and capture their inefficiencies.

The FDH technology is an alternative to DEA [11, 12] and [13]. Without assuming convexity, it is a comprehensive depiction of the production technology. Since the patient is the primary unit of measurement, efficiency will be assessed in relation to the production technology described by the patient's treatment procedure. By combining patient scores across hospitals, hospital efficiency can be evaluated at the patient level. Because FDH has a better fit to the data than DEA does, it reveals important aspects of the data that DEA does not understand [14]. With the use of the FDH techniques, the current research aims to assess the significance of this methodology, further demonstrate it with a typical case study, and perhaps add to the body of knowledge already available on hospital performance.

2. Efficiency in Healthcare

Healthcare efficiency refers to the ability of a healthcare system to maximize health outcomes using the least amount of resources possible. It involves optimizing the use of medical personnel, equipment, facilities, and funding to deliver high-quality care while minimizing waste and unnecessary procedures. Efficient healthcare systems reduce waiting times, improve patient satisfaction, and ensure better allocation of resources, ultimately leading to improved public health. Achieving efficiency also requires the integration of technology, evidence-based practices, and strong coordination among healthcare providers.

3. Free Disposal Hull (FDH)

A Free Disposal Hull (FDH) is a concept which assumed that Decision Making Units (DMUs) can freely dispose of any unwanted inputs or outputs without incurring additional costs. This means that if a DMU has excess resources or produces more output than desired, they can simply discard the surplus without negative consequences. FDH is a non-parametric technique that builds a piecewise linear frontier across the data points to assess the performance of DMUs. In contrast to DEA, FDH does not need convexity of the production possibility set and accepts varying returns to scale.

The potential of FDH was presented in the 1999 publication by Kerstens and Vanden Eckaut, wherein the FDH technique was given particular returns to scale assumptions (RTS) [27]. The application of Returns to Scale (RTS) assumptions to infer the characterisation of returns to scale in an FDH VRS (Variable Returns to Scale) model serves as the primary driving force behind the methodology demonstrated by Briec et al. (2000) [28]. The FDH models are solved using two computational techniques. The first one is predicated on enumeration techniques by Briec et al. (2004) [30], Cherchye et al. (2001) [31], and Tulkens (1993) [32]. Second one is by using mathematical programming on the technological definitions provided by Vanden Eeckaut and Kerstens (1999) [33]. Using an LP framework, Agrell and Tind (2001) introduced RTS in FDH models [34].

FDH has been utilized in the healthcare industry to examine hospital efficiency in several geographic areas, demonstrating its adaptability in a variety of situations. Briec, Kerstens, and Vanden Eeckaut (2004) [30] incorporate conventional returns to scale assumptions into the non-convex FDH model to provide a range of nonparametric, deterministic non-convex technologies. Among other things, they demonstrate how this technology specifications allow for the analytical derivation of the conventional technical input efficiency metric [35].

4. Methodology

FDH analysis provides a method of efficiency measurement without the assumption of convexity and relies on dominance relations between observed input–output bundles to measure efficiency. It is shown to be a special case of the BCC or CCR problems with additional {0, 1} constraints on the λj's.

Data Structure in general be, Let there be n DMUs, Each DMU uses m inputs and produce s outputs

- Inputs: $x_i = (x_{i1}, x_{i2}, \dots, x_{im})$
- Outputs: $(y_i = (y_{i1}, y_{i2}, \dots, y_{is})$

Production Possibility Set (PPS): The pairs of inputs and outputs that are producible—that is, where a given set of inputs can result in a given set of outputs.

FDH constructs the production possibility set as:

$$T_{FDH} = \{(x, y) \in \mathbb{R}_+^m \times \mathbb{R}_+^s : \exists j \in \{1, \dots, n\} such\ that\ x_j \leq x, y_j \geq y\}$$

This means a DMU is considered efficient if no other DMU can produce at least the same outputs with less or equal inputs. Efficiency Score

For a given DMU o, FDH efficiency is:

- Efficient if no other DMU dominates it, i.e., no DMU has inputs $\leq x_0\ and\ outputs\ \geq y_0$, with at least one strict inequality.
- Inefficient if such a dominating DMU exists.

Mathematically:

$$\theta_0 = \min\{\theta \geq 1 : \exists j\{1, \dots, n\}, x_j \leq \theta x_0, y_j \geq y_0\}$$

A second of $\theta_0 = 1$ implies FDH-efficiency

Input-oriented FDH model

The input-oriented FDH model under the variable returns-to-scale assumption is expressed as:

$$\theta^* = \min \theta$$

$$s.t. \sum_{j=1}^{N} \lambda_j x_{ij} \leq \theta x_{i0} \quad (i = 1,2, \dots, n)$$

$$\sum_{j=1}^{N} \lambda_j y_{rj} \geq y_{r0} \quad (r = 1,2, \dots, m);$$

$$\sum_{j=1}^{N} \lambda_j = 1;$$

$$\lambda_j \epsilon \{0,1\}; (j = 1,2, \dots, N); \qquad \theta\ unrestricted$$

Output-oriented FDH model:

The output-oriented FDH model under the variable returns-to-scale assumption is expressed as:

$$\max \phi$$

$$s.t. \sum_{j=1}^{N} \lambda_j y_{rj} \geq \phi y_{r0} \quad (r = 1,2, \dots, m)$$

$$\sum_{j=1}^{N} \lambda_j x_{ij} \geq x_{i0} \quad (i = 1,2, \dots, n);$$

$$\sum_{j=1}^{N} \lambda_j = 1;$$

$$\lambda_j \epsilon \{0,1\}; (j = 1,2, \dots, N); \qquad \phi\ unrestricted$$

5. Data Collection

The number of patient beds, physicians, and medical personnel including lab technicians and nurses were among the inputs. The number of deliveries, maternal deaths, and infant deaths were selected as the output variables.

Table I: Input-Output Variables

S. No.	Inputs	Outputs
1	x_1 = Number of Beds	y_1= Number of Deliveries
2	x_2 = Number of Doctors	y_2 = Number of Maternal Deaths
3	x_3 = Number of Health Personnel's (Like: nurses, Lab technician, etc.)	y_3 = Number of Infant Deaths

A sample of 51 district hospitals in Madhya Pradesh, India, were studied. Data was gathered from a number of sources, including the National Health Mission, the HMIS Health Bulletin, the Public Health and Family Welfare Department's 2023 Annual Statistical Reports, and the Directorate of Public Health.

Table II: Input-Output Variable Values for 2022-23

I/O	Mean	Standard Deviation	Maximum	Minimum
Input x_1	274.5098039	123.0335519	700	100
Input x_2	21.07843137	4.995370406	34	12
Input x_3	74.52941176	26.76591335	151	31
Output y_1	26968	11735.44876	66193	10144
Output y_2	39	32.84513872	177	2
Output y_3	626	306.0174389	1818	54

6. Results

Different district hospitals had different efficiency scores, indicating a range of performance levels, according to the FDH (Free Disposal Hull) research. It was discovered that a sizable fraction of these hospitals were not making the best use of their resources, or that they were functioning below the efficiency frontier. This underperformance suggests a significant discrepancy between their present level of operation and the highest level of efficiency that could be attained.

Table III: Efficiency Score and Target Values

S. No.	District Hospitals Name	Efficiency Score	Targets Values					
			Inputs			Outputs		
			No. of Beds (x_1)	No. of Doctors (x_2)	No. of Health Personnel's (x_3)	No. of Deliveries (y_1)	No. of Maternal Deaths (y_2)	No. of Infant Deaths (y_3)
1	AGAR MALWA	1	100	12	32	11173	13	547
2	ALIRAJPUR	1	100	16	40	18639	31	385
3	ANUPPUR	0.88889	100	16	45	15996	55	602
4	ASHOKNAGAR	1	100	16	45	15996	55	602
5	BALAGHAT	0.95238	100	20	45	44737	62	544
6	BARWANI	1	300	27	82	40544	55	1151
7	BETUL	1	300	22	72	25300	26	752
8	BHIND	1	300	18	66	27293	28	296
9	BHOPAL	1	300	34	114	51411	177	1818
10	BURHANPUR	0.88889	100	16	45	15996	55	602
11	CHHATARPUR	1	300	27	85	40786	38	960
12	CHHINDWARA	1	400	23	78	40235	49	1180
13	DAMOH	0.86364	200	19	55	31435	38	486
14	DATIA	1	200	14	47	20150	37	967
15	DEWAS	0.75	300	15	78	40479	25	954
16	DHAR	1	300	15	78	40479	25	954
17	DINDORI	1	100	16	31	12774	13	390
18	GUNA	1	400	23	78	40235	49	1180
19	GWALIOR	1	200	22	69	41375	94	398
20	HARDA	1	100	12	32	11173	13	547
21	HOSHANGABAD	0.69231	200	18	50	23864	51	880
22	INDORE	1	300	27	116	66193	89	1048
23	JABALPUR	1	500	31	105	37713	131	1006
24	JHABUA	1	200	17	42	34124	19	529
25	KATNI	1	200	18	50	23864	51	880
26	KHANDWA	0.88	300	22	72	25300	26	752
27	KHARGONE	1	300	24	84	35428	58	920
28	MANDLA	0.81034	200	14	47	20150	37	967
29	MANDSAUR	0.60714	200	17	42	34124	19	529
30	MORENA	0.90909	100	20	45	44737	62	544
31	NARSINGHPUR	1	300	23	82	18575	109	441
32	NEEMUCH	1	200	14	47	20150	37	967
33	PANNA	1	200	14	47	20150	37	967
34	RAISEN	1	200	18	41	20448	40	416
35	RAJGARH	1	300	15	78	40479	25	954
36	RATLAM	0.625	300	15	78	40479	25	954
37	REWA	1	100	20	45	44737	62	544
38	SAGAR	1	300	26	82	37117	64	551
39	SATNA	1	400	23	78	40235	49	1180
40	SEHORE	1	200	18	50	23864	51	880
41	SEONI	0.81818	200	18	50	23864	51	880
42	SHAHDOL	0.85714	200	18	50	23864	51	880
43	SHAJAPUR	1	200	14	47	20150	37	967
44	SHEOPUR	1	100	18	43	14526	20	656

45	SHIVPURI	1	300	25	98	37799	27	644
46	SIDHI	1	300	18	57	25387	70	503
47	SINGROLI	1	200	19	55	31435	38	486
48	TIKAMGARH	1	200	17	42	34124	19	529
49	UJJAIN	0.51656	300	15	78	40479	25	954
50	UMARIA	1	100	12	32	11173	13	547
51	VIDISHA	1	300	24	84	35428	58	920

Target values are predetermined standards or objectives pertaining to the resources that a system or organization uses. Target values define the intended levels or upper bounds on the number of beds, physicians, and other health workers that a hospital hopes to employ in order to meet specific output targets. These objectives, which may change based on the hospitals or particular process under study, are established in accordance with strategic objectives, operational requirements, or efficiency criteria.

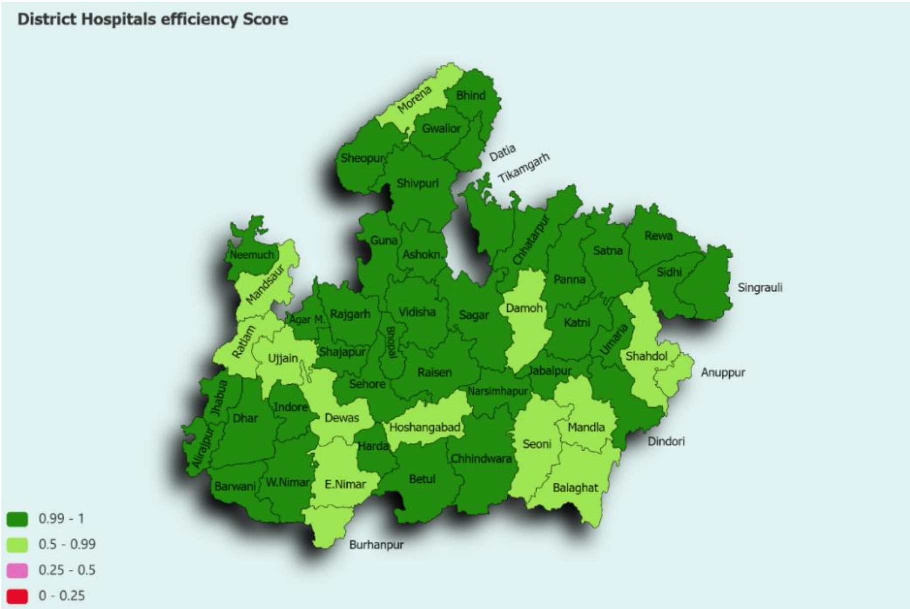

Figure I: Efficiency Values of District Hospitals of Madhya Pradesh, India

A wide variety of performance is revealed by the analysis of district hospitals' FDH efficiency scores. A considerable proportion of hospitals attained an impeccable efficiency score of 1.0, signifying that they are functioning at the forefront of efficiency and making the most efficient use of their available resources. Nonetheless, it was discovered that a number of hospitals, including those in Damoh (0.86364), Mandla (0.81034), and Ujjain (0.51656), had efficiency scores lower than 1.0. This suggests that these hospitals are not making the best use of their inputs, such as the number of beds, doctors, and medical staff, than other hospitals. The possibility for resource optimization and operational enhancements is highlighted by these lower ratings. Targeted initiatives are required for hospitals with substandard efficiency in order to improve their performance and bring them closer to the efficiency frontier.

In this study, the differences between the inputs and targets for each resource across the hospital had been computed in order to present a thorough results analysis. This will assist in determining which hospitals are not performing up to par and where enhancements are needed. Agar Malwa represents a minor overstretch in terms of medical professionals (+10) and doctors (+3) relative to the intended numbers. The hospital Alirajpur satisfies all of its resource targets, demonstrating ideal resource distribution. At the Anuppur district hospital, there is a slight overabundance of doctors (+2) but a significant over allocation of beds (+100) and medical staff (+30). Balaghat is a significant oversupply of beds (+200) and medical staff (+49), which points to possible inefficiencies in the use of available resources.

7. Conclusion

Based on the data supplied, a district hospital analysis shows notable differences in resource use and productivity amongst various establishments. Some hospitals exhibit noticeable over-allocation, while others, like Ashoknagar and Alirajpur, have achieved optimal resource allocation with no deviations from goal values. For example, hospitals with extra beds and medical staff, like as those in Anuppur and Balaghat, etc. may be inefficient. These variations imply that in order to improve overall efficiency, certain hospitals might profit from reallocating resources or making operational changes. The results highlight the significance of focused interventions to maximize resource utilization and enhance overall hospital performance.

Hospitals like Anuppur and Balaghat show a notable over-allocation in important areas. Balaghat displays a surplus of 200 beds and 49 health staff, but Anuppur has 100 more beds and 30 more health personnel than required. These differences imply that these hospitals are not using their resources as efficiently as they could be and that some of them should be reallocated. While some district hospitals have been able to effectively maximize their use of resources, there is still space for development. Improving the overall efficacy and efficiency of healthcare services throughout the district may be possible by addressing these inefficiencies through smart resource management and reallocation. This strategy maximizes the impact of the healthcare system by ensuring that resources are used as efficiently as feasible while also improving patient care.

Conflict of Interests

With regard to the publishing of this work, the authors declare that they have no conflicts of interest.

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