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Study of Intuitionistic Fuzzy Closed Sets and Intuitionistic Fuzzy Continuity

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Abstract: ***The aim of this paper is to introduce the new class of intuitionistic fuzzy closed sets called intuitionistic fuzzy closed sets inituitionistic fuzzy topological space. The class of all intuitionistic fuzzy closed sets lies between the class of all intuitionistic fuzzy closed sets and class of all intuitionistic fuzzy closed sets. We also introduce the concepts of intuitionistic fuzzy open sets, intuitionistic fuzzy rw-continuity and intuitionistic fuzzy open and intuitionistic fuzzy closed mappings in intuitionistic fuzzy topological spaces.***

Keywords: Intuitionistic fuzzy closed sets, Intuitionistic fuzzy r open sets, Intuitionistic fuzzy connectedness, Intuitionistic fuzzy compactness, intuitionistic fuzzy continuous mappings, Intuitionistic fuzzy open mappings and intuitionistic fuzzy closed mappings.

1. Introduction

After the introduction of fuzzy sets by Zadeh [28] in 1965 and fuzzy topology by Chang [5] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets (IFS) was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for IFS. In 1997 Coker [6] introduced the concept ofIF topological spaces. Recently many fuzzy topological concepts has fuzzy compactness [8], fuzzy connectedness [24], fuzzy separation axioms [3], fuzzy continuity [9], fuzzy g-closed sets [16], fuzzy g- continuity [17], fuzzy closed sets [18] have been generalized for IF topological spaces. Recently authors of this paper introduced the concept of IF closed sets [20] in IF topology.

In the present paper we extend the concepts of rw-closed sets due to Benchalli and Walli [4] in IF topological spaces . The class of intuitionistic fuzzy closed sets is properly placed between the class of IF closed sets and IF closed sets. We also introduced the concepts of IF open sets, IF connectedness, compactness and continuity, and obtain some of their characterization and properties.

2. Preliminaries

Let X be a non- empty fixed set. An IFSA[1] In X is an object having the form $A=\{<x,\mu_A(x), \gamma_A(x)> : x \in X\}$, where the functions $\mu_A : X\rightarrow[0,1]$ and $\Upsilon_A : X\rightarrow[0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x\in X$ of the set A

Respectively and $0 \le \mu_A(x) + \gamma_A(x) \le 1$for each $x\in X$. The IF S s$\tilde{0}=\{<x,0,1>:x\in X\}$ and $\tilde{1}=\{<x,1,0>:x\in X\}$ are respectively called empty and whole IF Son X.An IF S $A = \{<x,\mu_A(x), \gamma_A(x)> : x \in X\}$ is called a subset of an IFS $B = \{<x,\mu_B(x), \gamma_B(x)> : x \in X\}$ (for short $A\subseteq B$) if$\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for each $x\in X$. The complement of an IFS $A = \{<x,\mu_A(x), \gamma_A(x)> : x \in X \}$ is the IFS $A^c = \{ <x,\gamma_A(x), \mu_A(x)>: x \in X\}$. The intersection (resp. union)of any arbitrary family of IFSs$A_i=\{<x,\mu_{Ai}(x), \gamma_{Ai}(x) >: x\in X,(i\in\wedge)\}$ X be the IF S$\cap A_i=\{<x , \wedge\mu_{Ai}(x) ,\vee\gamma_{Ai}(x) >: x\in X\}$ (resp.$\cup A_i=\{<x, \vee\mu_{Ai}(x) ,\wedge\gamma_{Ai}(x) >:x \in X\}$). Two IFSs$A=\{<x,\mu_A(x), \gamma_A(x)>:x\in X\}$ and $B=\{<x,\mu_B(x), \gamma_B(x)>:x\in X\}$ are said be q-coincident (A_qB for short) if and only if there exists an element $x\in X$, such that $\mu_A(x) >\gamma_B(x)$ or $\gamma_A(x)<\mu_B(x)$. A family $\Im$ of IFSs on a nonempty set X is called an intuitionistic fuzzy topology [6] on X if the intuitionistic fuzzy sets $\tilde{0},\tilde{1}\in\Im$, and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair$(X,\Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set. The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It is denoted by cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted by int(A) [6].

Lemma 2.1[6]**:** Let A and B be any two IF Sof an IF topological space $(X,\Im)$. Then:

a) $(A_qB)\Leftrightarrow A\subseteq B^c$.
b) A is an intuitionistic fuzzy closed set in X $\Leftrightarrow$ cl(A)= A
c) A is an intuitionistic fuzzy open se tin X$\Leftrightarrow$ int(A) = A.
d) $cl(A^c) = (int(A))^c$.
e) $int(A^c) = (cl(A))^c$.
f) $A\subseteq B\Rightarrow int(A) \subseteq int(B)$.
g) $A\subseteq B\Rightarrow cl(A) \subseteq cl(B)$.
h) $cl(A\cup B)=cl(A) \cup cl(B)$.
i) $int (A \cap B)=int(A)\cap int(B)$

Definition2.1[7]**:**Let X is a nonempty set and $c\in X$ a fixed element in X .If $\alpha\in(0,1]$ and $\beta\in [0, 1)$ are two real number such that $\alpha+ \beta \le 1$ then,

a) $c(\alpha,\beta) = <x,c_\alpha,c_{1-\beta}>$ is called an IF point in X, where α denotes the degree of membership of $c(\alpha,\beta)$, and β denotes the degree of non membership of $c(\alpha,\beta)$.
b) $c(\beta) =<x,0,1-c_{1-\beta}>$ is called a vanishing intuitionistic fuzzy point in X, where β denotes the degree of non membership of $c(\beta)$.

Definition 2.2 [**8**]**:** A family $\{G_i: i\in\wedge\}$ of F in X is called an intuitionistic fuzzy open cover of X if $\cup\{G_i:i\in\wedge\}=1^{\sim}$ and a finite sub family of an intuitionistic fuzzy open cover $\{G_i:i\in\wedge\}$ of X which also an intuitionistic fuzzy open cover of X is called a finite sub cover of $\{G_i: i\in\wedge\}$.

Definition 2.3[8]: An intuitionistic fuzzy topological space $(X,\Im)$ is called intuitionistic fuzzy compact if every intuitionistic fuzzy open cover of X has a finite sub cover.

Definition 2.4[24]: An intuitionistic fuzzy topological space X is called intuitionistic fuzzy connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy open and intuitionistic fuzzy closed .

Definition 2.5[9]: An IFS A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

a) An intuitionistic fuzzy semi open of X if there is an intuitionistic fuzzy set O such that $O\subseteq A\subseteq cl(O)$
b) An intuitionistic fuzzy semi closed if the compliment of A is an intuitionistic fuzzy semi open set.
c) An intuitionistic fuzzy regular open of X if int(cl(A)) =A.
d) An intuitionistic fuzzy regular closed of X if c l(int(A))= A.
e) An intuitionistic fuzzy pre open if $A\subseteq int(cl(A))$.
f) An intuitionistic fuzzy pre closed if $cl (int(A))\subseteq A$

Definition2.6[22]: An intuitionistic fuzzy topological space $X,\Im)$ is called intuitionistic fuzzy regular semi open if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.

Theorem2.1[22]: Let$(X,\Im)$ be an intuitionistic fuzzy topological spaces and A be an IFS in X then, the following conditions are equivalent:

a) A is intuitionistic fuzzy regular semi open
b) Ai s both intuitionistic fuzzy semi open and intuitionistic fuzzy semi- closed.
c) A^c is intuitionistic fuzzy regular semi open

Definition 2.7[9] If A is an IFS in intuitionistic fuzzy topological space$(X,\Im)$ then

a) $scl(A) =\cap \{F:A\subseteq F,F$ is intuitionistic fuzzy semi closed$\}$
b) $pcl(A) =\cap \{F:A\subseteq F,F$ is intuitionistic fuzzy pre closed$\}$

Definition 2.8: An IF SA of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

a) Intuitionistic fuzzy g-closed if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy open.[16]
b) Intuitionistic fuzzy g-openifits complement A^c is intuitionistic fuzzy g-closed.[16]
c) Intuitionistic fuzzy rg-closed if $cl(A)\subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open.[18]
d) Intuitionistic fuzzy rg-open if its complement A^c is intuitionistic fuzzy rg-closed.[18]
e) Intuitionistic fuzzy w-closed if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.[20]
f) Intuitionistic fuzzy w-open if its complement A^c is intuitionistic fuzzy w-closed.[20]
g) Intuitionistic fuzzy gpr-closed if $pcl(A)\subseteq O$ whenever $A\subseteq O$ and O is intuitionistic fuzzy regular open.[21]
h) Intuitionistic fuzzy gpr-open if its complement A^c is intuitionistic fuzzy gpr-closed. [21]

Remark 2.1: Every intuitionistic fuzzy closed set is intuitionistic fuzzy g-closed but its converse may not be true.[16]

Remark 2.2: Every intuitionistic fuzzy g- closed set is intuitionistic fuzzy rg-closed but its converse may not be true.[18]

Remark 2.3: Every intuitionistic fuzzy w-closed (resp. Intuitionistic fuzzy w-open) set is intuitionistic fuzzy g-closed (intuitionistic fuzzy g-open) but its converse may not be true.[20]

Remark 2.4: Every intuitionistic fuzzy g-closed (resp. Intuitionistic fuzzy g-open) set is intuitionistic fuzzy gpr-closed (intuitionistic fuzzy gpr-open) but its converse may not be true[21]

Definition 2.9[6]: Let X and Y are two non empty sets and $f:X\to Y$ is a function:

a) If $B = \{<y,\mu_B(y), \gamma_B(y)> : y \in Y\}$is an intuitionistic fuzzy set in Y, then the preimage of B under f denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by $f^{-1}(B)=<x,f^{-1}(\mu_B)(x),f^{-1}(\gamma_B)(x)>:x\in X\}$.
b) If $A = \{<x, \lambda_A(x), \nu_A(x)> : x \in X\}$is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by $f(A)=\{<y,f(\lambda_A)(y),f(\nu_A)(y)>:y\in Y\}$ Where $f(\nu_A) = 1 – f(1- \nu_A)$.

Definition 2.10[9]: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: $X\to Y$ be a function. Then f is said to be

a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.
b) Intuitionistic fuzzy closed if the image of each intuitionisic fuzzy closed set in X is an intuitionistic fuzzy closed set in Y.
c) Intuitionistic fuzzy open if the image of each intuitionisic fuzz open set in X is an intuitionistic fuzzy open set in Y.

Definition 2.11[27]: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let f:$X\to Y$ be a function. Then f is said to be intuitionistic fuzzy almost continuous if inverse image of every intuitionistic fuzzy regular closed set of Y is intuitionistic fuzzy closed in X.

Definition 2.12[23]: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let f:$X\to Y$ be a function. Then f is said to be intuitionistic fuzzy almost irresolute if inverse image of every intuitionistic fuzzy regular semi openset of Y is intuitionistic fuzzy semi open in X.

Definition 2.13: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: $X\to Y$ be a function. Then f is said to be

a) Intutionistic fuzzy g-continuous if the pre image of

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every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g –closed in X.[17]

b) Intutionistic fuzzy w-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy w –closed in X.[20]
c) Intutionistic fuzzy w-open if image of every open set of X is intuitionistic fuzzy w- open in Y.[20]
d) Intutionistic fuzzy w-closed if image of every closed set of X is intuitionistic fuzzy w- closed in Y.[20]
e) Intutionistic fuzzy rg-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rg –closed in X.[19]
f) Intutionistic fuzzy gpr-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gpr –closed in X.[21]

Remark 2.5: Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true [17].

Remark 2.6: Every intuitionistic fuzzy w-continuous mapping is intuitionisticf fuzzy g-continuous, but the converse may not be true [20].

Remark 2.7: Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy rg- continuous, but the converse may not be true [16].

Remark 2.8: Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy gpr-continuous, but the converse may not be true [21].

3. Intuitionistic fuzzy closed set

Definition3.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,ℑ) is called an intuitionistic fuzzy closed if cl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy regular semi open in X.

First we prove that the class of intuitionistic fuzzy closed sets properly lies between the class of intuitionistic fuzzy closed sets and the class of intuitionistic fuzzy closed sets.

Theorem3.1: Every intuitionistic fuzzy closed set is intuitionistic fuzzy closed.

Proof: The proof follows from the

Definition3.1and the fact that every intuitionistic fuzzy regular semi open set is intuitionistic fuzzy semi open.

Remark 3.1: The converse of Theorem 3.1 need not be true as from the following example.

Example 3.1: Let X={a,b}andℑ={**0**□,U,**1**□}be an intuitionistic fuzzy topology on X, where U={<a,0.7,0.2>, <b,0.6,0.3>}. Then the IF SA={<a,0.7,0.2>, <b,0.8,0.1>}is Intuitionistic fuzzy closed but it is not intuitionistic fuzzy closed.

Theorem 3.2: Every intuitionistic fuzzy closed set is intuitionistic fuzzy closed.

Proof: The proof follows from the Definition3.1and the fact that every intuitionistic fuzzy regular open set is intuitionistic fuzzy regular semi open.

Remark 3.2: The converse of Theorem 3.2 need not be true as from the following example.

Example 3.2: Let X= {a, b, c, d}and intuitionistic fuzzy sets O, U, V, W defined as follows O = {<a, 0.9, 0.1>, <b,0, 1>, <c,0, 1>, <d,0, 1>}
U={<a,0,1,>,<b,0.8,0.1>,<c,0,1>,<d,0,1>}
V={<a,0.9,0.1>,<b,0.8,0.1,>,<c,0,1>,<d,0,1>}
W= {<a, 0.9, 0.1>, <b,0.8, 0.1>, <c,0.7, 0.2>, <d,0, 1>}

ℑ={**0**□,O,U, V, W **1**□,} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a,0, 1>, <b,0, 1>, <c,0.7, 0.2>, <d,0, 1>} is intuitionistic fuzzy closed but it is not intuitionistic fuzzy closed.

Theorem 3.3: Every intuitionistic fuzzy closed set is intuitionistic fuzzy closed.

Proof: Let A is an intuitionistic fuzzy closed set in intuitionistic fuzzy topological space (X,ℑ) Let A ⊆O where O is intuitionistic fuzzy regular open in X. Since every intuitionistic fuzzy regular open set is intuitionistic fuzzy regular semi open and A is intuitionistic fuzzy rw- closed set, we have cl(A)⊆O. Since every intuitionistic fuzzy closed set is intuitionistic fuzzy pre closed, pcl(A) ⊆cl(A). Hence pcl(A)⊆ O which implies that A is intuitionistic fuzzy gpr- closed.

Remark 3.3: The converse of Theorem 3.3 need not be true as from the following example.

Example 3.3: Let X={a, b, c, d, e} and intuitionistic fuzzy sets O, U, V defined as follows O = {<a,0.9, 0.1>, <b,0.8, 0.1>, <c,0, 1>, <d,0, 1>, <e, 0, 1>}
U= {<a,0, 1>, <b,0, 1>, <c,0.8, 0.1>, <d,0.7, 0.2>, <e, 0, 1>}
V={<a,0.9,0.1>,<b,0.8,0.1>,<c,0.8,0.1>,<d,0.7,0.2>,<e,0,1> }

Letℑ={**0**□, O, U, V, **1**□}be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A ={<a, 0.9, 0.1>, <b,0, 1>, <c,0, 1>, <d,0, 1>, <e, 0, 1>} is intuitionistic fuzzy gpr- closed but it is not intuitionistic fuzzy rw-closed.

Theorem 3.1: Let (X,ℑ) be an intuitionistic fuzzy topological space and A is an intuitionistic fuzzy set of X. Then A is intuitionistic fuzzy rw-closed if and only if¬(AqF) ⇒¬(cl(A)qF) for every intuitionistic fuzzy regular semi open set F of X.

Proof: Necessity: Let F be an intuitionistic fuzzy regular semi open set of X and¬(AqF). Then by Lemma 2.1(a) and Theorem 2.1, A ⊆ F^c and F^c intuitionistic fuzzy regular semi open in X. Therefore cl(A) ⊆F^c by Definition 3.1 because A is intuitionistic fuzzy rw-closed. Hence by Lemma 2.1(a),¬(cl(A)qF).

Sufficiency: Let O be an intuitionistic fuzzy regular semi open set of X such that A⊆O i.e. A⊆$(O)^c)^c$. Then by Lemma 2.1(a),¬(A_qO^c) and O^c is an intuitionistic fuzzy regular semi open set in X. Hence, by hypothesis ¬(cl$(A)_qO^c$). Therefore, by Lemma 2.1(a), cl(A) ⊆$((O)^c)^c$i.e. cl(A) ⊆ O. Hence, A is intuitionistic fuzzy rw-closed in X.

Theorem 3.2: Let A be an intuitionistic fuzzy rw-closed set in an IF topological space (X,ℑ) and c(α,β) be an IF point of X, such that c$(\alpha,\beta)_q$cl(int(A)) then cl(int(c(α,β))qA.

Proof: If¬cl(int(c$(\alpha,\beta))_q$ A then by Lemma2.1(a), cl(int(c(α,β)⊆A^c which implies that A ⊆ $(cl(c(\alpha,\beta)))^c$ and so cl(A) ⊆ $(cl(int(c(\alpha,\beta)))^c$⊆ $(c(\alpha,\beta))^c$, because A is intuitionistic fuzzy rw-closed in X. Hence by Lemma 2.1(a), ¬(c$(\alpha,\beta)_q$ (cl(int(A))), a contradiction.

Theorem3.3: Let A and B are two intuitionistic fuzzy rw-closed sets in an intuitionistic fuzzy topological space (X,ℑ), then A∪B is intuitionistic fuzzy rw-closed.

Proof: Let O be an intuitionistic fuzzy regular semi open set in X, such that A ∪ B ⊆ O. Then, A ⊆ O and B ⊆ O. So, cl(A) ⊆ O and cl(B) ⊆ O. Therefore, cl(A) ∪ cl(B) = cl(A∪B) ⊆ O. Hence A∪B is intuitionistic fuzzy rw-closed.

Remark 3.5: The intersection of two intuitionistic fuzzy rw-closed sets in an intuitionistic fuzzy topological space (X,ℑ) may not be intuitionistic fuzzy rw-closed. For,

Example 3.4: Let X={a, b, c, d}and intuitionistic fuzzysets O, U, V, W defined as follows O = {<a, 0.9, 0.1>, <b, 0, 1>, <c, 0, 1>, <d, 0, 1>}
U={<a,0,1,>,<b,0.8,0.1>,<c,0,1>,<d,0,1>}
V= {<a, 0.9, 0.1>, <b, 0.8, 0.1,>, <c, 0, 1>, <d, 0, 1>}
W= {<a, 0.9,0.1,>, <b, 0.8, 0.1>, <c, 0.7, 0.2>, <d, 0, 1>}
ℑ = {0,O, U, V, W 1}be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A={<a,0.9,0.1>, <b,0.8,0.1>, <c,0,1>, <d,0,1>}and B={<a,0.9, 0.1>,<b,0,1>, <c,0.7,0.2>, <d,0.9,0.1>} are intuitionistic fuzzy rw-closed in (X,ℑ) but A∩B is not intuitionistic fuzzy rw-closed.

Theorem 3.4: Let A be an intuitionistic fuzzy rw-closed set in an intuitionistic fuzzy topological space (X,ℑ) and A ⊆ B ⊆ cl(A). Then B is intuitionistic fuzzy rw-closed in X.

Proof: Let O be an intuitionistic fuzzy regular semi open set in X such that B⊆O. Then, A⊆O and since A is intuitionistic fuzzy rw- closed, cl(A)⊆O. Now B⊆cl(A)⇒cl(B)⊆ cl(A) ⊆ O. Consequently B is intuitionistic fuzzy rw-closed.

Theorem 3.5: If A is intuitionistic fuzzy regular open and intuitionistic fuzzy rw-closed set, then A is intuitionistic fuzzy regular closed and hence intuitionistic fuzzy open.

Proof: Suppose A is intuitionistic fuzzy regular open and intuitionistic fuzzy rw-closed set. As every intuitionistic fuzzy regular open set is intuitionistic fuzzy regular semi open and A ⊆ A, we have cl(A) ⊆ A. Also A ⊆ cl(A). Therefore cl(A) = A. That means A is intuitionistic fuzzy closed. Since A is intuitionistic regular open, them A is intuitionistic fuzzy open. Now cl(int(A)) = cl(A)) = A. Therefore, A is intuitionistic fuzzy regular closed and intuitionistic fuzzy closed.

Theorem3.6: If A is an intuitionistic fuzzy regular open and intuitionistic fuzzy rg-closed in intuitionistic fuzzy topological space (X,ℑ), then A is intuitionistic fuzzy rw-closed in X.

Proof: Let A is an intuitionistic fuzzy regular open and intuitionistic fuzzy rg-closed in X. We prove that A is an intuitionistic fuzzy rw-closed in X. Let U be any intuitionistic fuzzy regular semi open set in X such that A ⊆ U. Since A is intuitionistic fuzzy regular open and intuitionistic fuzzy rg-closed, we have cl(A) ⊆ A. Then cl(A) ⊆ A ⊆ U. Hence A is intuitionistic fuzzy rw-closed in X.

Theorem3.7: If A is an intuitionistic regular semi open and intuitionistic fuzzy rw-closed in intuitionistic fuzzy topological space (X,ℑ), then A is intuitionistic fuzzy closed in X.

Proof: Suppose A is both intuitionistic fuzzy regular semi open and intuitionistic fuzzy rw- closed set in X. Now A ⊆ A. Then cl(A) ⊆ A. Hence A is intuitionistic fuzzy closed in X.

Corollary3.1: If A is an intuitionistic fuzzy regular semi open and intuitionistic fuzzy rw- closed in intuitionistic fuzzy topological space (X,ℑ). Suppose that F is intuitionistic fuzzy closed in X then A ∩ F is intuitionistic fuzzy rw-closed in X.

Proof: Suppose A is both intuitionistic fuzzy regular semi open and intuitionistic fuzzy rw- closed set in X and F is intuitionistic fuzzy closed in X. By Theorem 3.7, A is intuitionistic fuzzy closed in X . So A ∩ Fis intuitionistic fuzzy closed in X. Hence A ∩ Fis intuitionistic fuzzy rw-closed in X.

Theorem 3.8: If A is both intuitionistic fuzzy open and intuitionistic fuzzy g-closed in intuitionistic fuzzy topological space (X,ℑ). Then A is intuitionistic fuzzy rw-closed set in X.

Proof: Let A is both intuitionistic fuzzy open and intuitionistic fuzzy g-closed in X. Let A⊆U, where U is intuitionistic fuzzy regular semi- open in X. Now A ⊆ A. By hypothesis cl(A) ⊆ A. That is cl(A) ⊆ U. Thus, A is intuitionistic fuzzy rw-closed in X.

Definition3.2: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,ℑ)is called intuitionistic fuzzy rw-open if and only if its complement $A^c$is intuitionistic fuzzy rw- closed.

Remark3.6: Every intuitionistic fuzzy w-open set is intuitionistic fuzzy rw-open but its converse may not be true.

Example3.5: Let X = {a, b} andℑ = {**0**,U, **1**,}be an intuitionistic fuzzy topology on X, where U={<a,0.7,0.2>,

<b,0.6,0.3>}. Then the intuitionistic fuzzy set A={<a,0.2,0.7>,
<b, 0.1, 0.8>} is intuitionistic fuzzy rw-open in (X, ℑ) but it is not intuitionistic fuzzy w-open in (X, ℑ).

Theorem3.9: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,ℑ) is intuitionistic fuzzy rw-open if F$\subseteq$ int(A) whenever F is intuitionistic fuzzy regular semi open and F $\subseteq$ A.

Proof: Follows from Definition 3.1andLemma 2.1

Theorem3.10: Let A be an intuitionistic fuzzy rw-open set of an intuitionistic fuzzy topological space (X,ℑ) and int(A) $\subseteq$ B $\subseteq$ A. Then B is intuitionistic fuzzy rw-open.

Proof: Suppose A is an intuitionistic fuzzy rw-open in Xand int(A) $\subseteq$ B $\subseteq$A. $\Rightarrow A^c \subseteq B^c \subseteq (int(A))^c \Rightarrow A^c \subseteq B^c \subseteq cl(A^c)$ by Lemma 2.1(d) and A^c is intuitionistic fuzzy rw-closed it follows from Theorem 3.4 that B^c is intuitionistic fuzzy rw-closed. Hence, B is intuitionistic fuzzy rw-open.

Theorem3.11: Let A be an intuitionistic fuzzy w-closed set in an intuitionistic fuzzy topological space (X,ℑ) and f: (X,ℑ) → (Y,$ℑ^*$) is an intuitionistic fuzzy almost irresolute and intuitionistic fuzzy closed mapping then f (A) is an intuitionistic rw-closed set in Y.

Proof: Let A be an intuitionistic fuzzy w-closed set in X and f: (X,ℑ) → (Y,$ℑ^*$) is an intuitionistic fuzzy almost irresolute and intuitionistic fuzzy closed mapping. Let f(A) $\subseteq$ G where G Is intuitionistic fuzzy regular semi open in Y then A $\subseteq f^{-1}$(G) and f^{-1}(G) is intutionistic fuzzy semi open in X because f is intuitionistic fuzzy almost irresolute. Now A be an intuitionistic fuzzy w-closed set in X, cl(A) $\subseteq f^{-1}$(G). Thus, f(cl(A)) $\subseteq$ G and f(cl(A)) is an intuitionistic fuzzy closed set in Y (since cl(A) is intuitionistic fuzzy closed in X and f is intuitionistic fuzzy closed mapping). It follows that cl(f (A) $\subseteq$ cl(f (cl(A))) = f (cl(A)) $\subseteq$ G. Hence cl(f (A)) $\subseteq$ G whenever f (A) $\subseteq$ G and G is intuitionistic fuzzy regular semi open in Y. Hence f (A) is intuitionistic fuzzy rw-closed set in Y.

Theorem3.12: Let(X,ℑ) be an intuitionistic fuzzy topological space and IFRSO(X) (resp. IFC(X)) be the family of all intuitionistic fuzzy regular semi open (resp. intuitionistic fuzzy closed) sets of X. Then IFRSO(X) $\subseteq$ IFC(X) if and only if every intuitionistic fuzzy set of X is intuitionistic fuzzy rw-closed.

Proof: Necessity: Suppose that IFRSO(X)$\subseteq$ IFC(X) and let A be any intuitionistic fuzzy set of X such that A$\subseteq$U$\in$IFRSO(X) i.e. U is intuitionistic fuzzy regular semi open. Then, cl(A) $\subseteq$cl(U)=U because eU$\in$IFRSO(X)$\subseteq$ IFC(X). Hence cl(A) $\subseteq$ U whenever A$\subseteq$U and U is intuitionistic fuzzy regular semi open. Hence A is rw-closed set.

Sufficiency: Suppose that every intuitionistic fuzzy set of X is intuitionistic fuzzy rw-closed. Let U $\in$ IFRSO(X), then since U $\subseteq$ U and U is intuitionistic fuzzy rw-closed, cl(U) $\subseteq$ U then U $\in$ IFC(X).Thus IFRSO(X)$\subseteq$ IFC(X).

Definition 3.3: An intuitionistic fuzzy topological space (X ℑ) is called intuitionistic fuzzy rw-connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy open and intuitionistic fuzzy closed.

Theorem 3.13: Every intuitionistic fuzzy rw-connected space is intuitionistic fuzzy connected.

Proof: Let (X, ℑ) be an intuitionistic fuzzy rw-connected space and suppose that (X, ℑ) is not intuitionistic fuzzy connected. Then there exists a proper IFS A (A≠ **0̃**, A≠ **1**) such that A is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Since every intuitionistic fuzzy open set (resp. intuitionistic fuzzy closed set) is intuitionistic r w-open(resp. intuitionistic fuzzy rw-closed), X is not intuitionistic fuzzy rw-connected, a contradiction.

Remark3.7: Converse of Theorem3.13may not be true for

Example3.6: Let X = {a, b} andℑ = {0̃, U, 1̃} be an intuitionistic fuzzy topology on X, where U ={<a, 0.5, 0.5>, < b, 0.4, 0.6 >}.Then, intuitionistic fuzzy topological space (X, ℑ) is intuitionistic fuzzy connected but not intuitionistic fuzzy rw-connected because there exists a proper intuitionistic fuzzy set A={<a,0.5,0.5>,<b, 0.5,0.5>} which is both intuitionistic fuzzy rw-closed and intuitionistic rw-open in X.

Theorem3.14: An intuitionistic fuzzy topological (X,ℑ) is intuitionistic fuzzy rw-connected if and only if there exists no non zero intuitionistic fuzzy rw-open sets A and B in X such that A=B^c.

Proof: Necessity: Suppose that A and B are intuitionistic fuzzy rw-open sets such that A≠ **0̃**≠B and A = B^c. Since A=B^c, B is an intuitionistic fuzzy rw-open set which implies that B^c = Ais intuitionistic fuzzyrw-closed set and B≠ **0̃** this implies that B^c≠ **1**ĩ.e. A≠**1̃.** Hence, there exists a proper intuitionistic fuzzy setA(A≠ **0̃**, A≠ **1̃**) such that A is both intuitionistic fuzzy rw-open and intuitionistic fuzzy rw-closed. But this is contradiction to the fact that X is intuitionistic fuzzy rw-connected.

Sufficiency: Let (X,ℑ) is an intuitionistic fuzzy topological space and A is both intuitionistic fuzzy rw-open set and intuitionistic fuzzy rw-closed set in X such that**0̃** ≠ A ≠ 1**̃.** Now take B=A^c. In this case B is an intuitionistic fuzzy rw-open set andA≠**1̃.**Thisimpliesthat B = A^c ≠ 0̃, which is a contradiction. Hence there is no proper IFS of X which is both intuitionistic fuzzy rw-open and intuitionistic fuzzy rw-closed. Therefore, intuitionistic fuzzy topological (X,ℑ) is intuitionistic fuzzy rw-connected

Definition3.3: Let (X,ℑ) be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set X. Then rw-interior and rw-closure of A are defined as follows. rwcl (A)=∩{K: K is an intuitionistic fuzzy rw- closed set in X and A$\subseteq$K} rwint(A) =∪ {G: G is an intuitionistic fuzzy rw-open set in X and G$\subseteq$ A}

Remark 3.8: It is clear that A $\subseteq$wcl(A)$\subseteq$ cl(A)for any intuitionistic fuzzy set A.

Theorem3.15: An intuitionistic fuzzy topological space (X, ℑ) is intuitionistic fuzzy rw- connected if and only if there exists no non zero intuitionistic fuzzy rw-open sets A and B in X such that $B = A^c$, $B = (rwcl(A))^c$, $A = (rwcl(B))^c$.

Proof: Necessity: Assume that there exists intuitionistic fuzzy sets A and B such that $A \neq \tilde{0} \neq B$ in X such that $B = A^c$, $B = (rwcl(A))^c$, $A = (rwcl(B))^c$. Since $(rwcl(A))^c$ and $(rwcl(B))^c$ are intuitionistic fuzzy rw-open sets in X, which is a contradiction.

Sufficiency: Let A is both an intuitionistic fuzzy rw-open set and intuitionistic fuzzy rw-closed set such that $\tilde{0} \neq A \neq 1^{\sim}$. Taking $B= A^c$, we obtain a contradiction.

Definition 3.4: An intuitionistic fuzzy topological space(X,ℑ) is said to be intuitionistic fuzzy rw-$T_{1/2}$if every intuitionistic fuzzy rw-closed set in X is intuitionistic fuzzy closed in X.

Theorem3.16: Let (X,ℑ) be an intuitionistic fuzzy rw-$T_{1/2}$ space, then the following conditions are equivalent:

(a) X is intuitionistic fuzzy rw-connected.
(b) X is intuitionistic fuzzy connected.

Proof:(a)⇒(b) follows fromTheorem3.13
(b) ⇒ (a): Assume that X is intuitionistic fuzzy rw-$T_{1/2}$ and intuitionistic fuzzy rw-connected space. If possible, let X be not intuitionistic fuzzy rw-connected, then there exists a proper intuitionistic fuzzy set A such that A is both intuitionistic fuzzy rw-open and rw-closed. Since X is intuitionistic fuzzy rw-$T_{1/2}$, A is intuitionistic fuzzy open and intuitionistic fuzzy closed which implies that X is not intuitionistic fuzzy connected, a contradiction.

Definition 3.4: A collection $\{A_i: i\in \Lambda\}$ of intuitionistic fuzzy rw-open sets in intuitionistic fuzzy topological space(X,ℑ) is called intuitionistic fuzzy rw-open cover of intuitionistic fuzzy set B of X if $B \subseteq \cup\{A_i: i\in \Lambda\}$

Definition 3.5: An intuitionistic fuzzy topological space(X,ℑ) is said to be intuitionistic fuzzy rw-compact if every intuitionistic fuzzy rw-open cover of X has a finite sub cover.

Definition 3.6: Anintuitionistic fuzzy set B of intuitionistic fuzzy topological space (X, ℑ) is said to be intuitionistic fuzzy rw-compact relative to X, if for every collection $\{A_i: i\in \Lambda\}$ of intuitionistic fuzzy rw-open subset of X such that $B \subseteq \cup\{A_i: i \in \Lambda\}$ there exists finite subset Λ_o of Λ such that $B \subseteq \cup\{A_i: i \in \Lambda_o\}$.

Definition 3.7: A crisp subset B of intuitionistic fuzzy topological space (X, ℑ) is said to be intuitionistic fuzzy rw-compact if B is intuitionistic fuzzy rw-compact as intuitionistic fuzzy subspace of X.

Theorem 3.16: A intuitionistic fuzzy rw-closed crisp subset of intuitionistic fuzzy rw-compact space is intuitionistic fuzzy rw-compact relative to X.

Proof: Let A be an intuitionistic fuzzy rw-closed crisp subset of intuitionistic fuzzy rw- compact space (X,ℑ). Then A^c is intuitionistic fuzzy rw-open in X. Let M be a cover of A by intuitionistic fuzzy rw-open sets in X. Then the family $\{M, A^c\}$ is intuitionistic fuzzy rw-open cover of X. Since X is intuitionistic fuzzy rw-compact, it has a finite subcoversay

$\{G_1, G_2, ..., G_n\}$. If this sub cover contains A^c, we discard it. Otherwise, leave the sub cover as it is. Thus we obtained a finite intuitionistic fuzzy rw-open sub cover of A. Therefore A is intuitionistic fuzzy rw-compact relative to X.

4. Intuitionistic fuzzy rw-continuity

Definition 4.1: A mapping f: (X,ℑ). →(Y, σ) is intuitionistic fuzzy rw-continuous if inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy rw-closed set in X.

Theorem4.1: A mapping f: (X,ℑ). →(Y,σ) is intuitionistic fuzzy rw-continuous if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy rw-open in X.

Proof: It is obvious because $f^{-1}(U^c)=(f^{-1}(U))^c$ for every intuitionistic fuzzyset U of Y.

Remark 4.1: Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy rw-continuous, but converse may not be true. For,

Example4.1 LetX={a, b}, Y={x, y} and intuitionistic fuzzysets U and V are defined as follows:
U= {<a,0.7, 0.2>, <b, 0.6, 0.3>}
V= {<x, 0.7, 0.2>, <y, 0.8, 0.1>}

Letℑ={0˜,**U,1˜**} and σ={**0˜, V, 1˜**} be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping f : (X,ℑ). →(Y, σ) defined by f (a) = x and f (b) = y is intuitionistic fuzzy rw-continuous but not intuitionistic fuzzy continuous.

Remark4.2 Every intuitionistic fuzzy rw-continuous mapping is intuitionistic fuzzy rg- continuous, but converse may not be true. For,

Example4.2: Let X = {a, b, c, d } Y ={p, q r, s} and intuitionistic fuzzy sets O,U,V,W,T are defined as follows:
O={<a,0.9,0.1>, <b,0,1>, <c,0,1>, <d,0,1>}
U={<a,0,1,>, <b,0.8,0.1>, <c,0,1>, <d,0,1>}
V={<a,0.9,0.1>, <b,0.8,0.1,>, <c,0,1>, <d,0,1>}
W={<a,0.9,0.1,>, <b,0.8,0.1>, <c,0.7,0.2>, <d,0,1>}
T= {<p,0,1>, <q,0,1><r,0.7,0.2>, <s,0, 1>}

Letℑ= {**0˜**, O, U, V, W, **1˜**}and σ = {**0**,T, **1**}be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping f:(X,ℑ)→ (Y, σ) defined by f(a)=p, f(b)=q,f(c)=r, f (d) = s is intuitionistic fuzzy rg-continuous but not intuitionistic fuzzy rw-continuous.

Remark 4.3 Every intuitionistic fuzzy rw-continuous mapping is intuitionistic fuzzy gpr- continuous, but converse may not be true. For,

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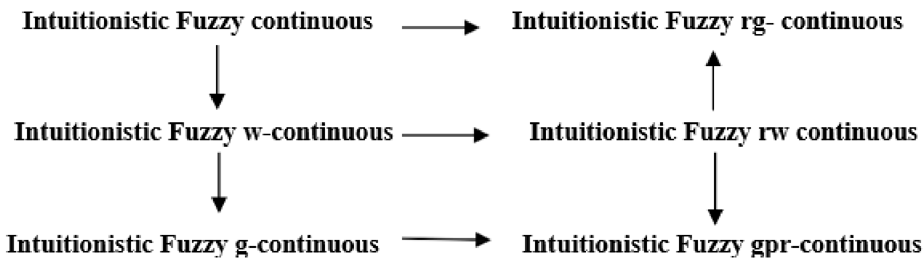
Example 4.3: Let X = {a, b, c, d, e} Y = {p, q r, s, t} and intuitionistic fuzzy sets O, U, V, W are defined as follows:
O={<a,0.9,0.1>, <b,0.8,1>, <c,0,1>, <d,0,1>, <e,0,1>}
U={<a,0,1,>, <b,0,1>, <c,0.8,1>, <d,0.7,0.2>, <e,0,1>}
V= {<a, 0.9,0.1>, <b, 0.8,0.1,>, <c, 0.8,0.1>, <d, 0.7,0.2>, <e,0,1>}
W={<a,0.9,0.1,>, <b,0,1>, <c,0.8,0.1>, <d,0.7,0.2>, <e,0,1>}

Let $\Im$ = {**0˜**, O, U, V,**1˜** } and σ ={**0˜**, W, **1˜**}be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,$\Im$).→(Y, σ) defined by *f*(a)= p, *f*(b)=q, *f*(c)=r, *f* (d) = s, f(e) = t is intuitionistic fuzzy gpr-continuous but not intuitionistic fuzzy rw- continuous.

Remark4.3: From the Remarks 2.5, 2.6, 2.7, 2.8, 4.1, 4.2, 4.3 we reach the following diagram of implications:



Theorem4.2: If *f* : (X,$\Im$) → (Y, σ) is intuitionistic fuzzy rw-continuous then for each intuitionistic fuzzy pointc(α,β) of X and each intuitionistic fuzzy open set V of Y such that *f*(c(α,β)) ⊆ V there exists a intuitionistic fuzzy rw-open set U of X such that c(α,β) ⊆ U and *f*(U) ⊆ V.

Proof: Let c(α,β) be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that *f*(c(α,β)) ⊆ V. Put U = f^{-1}(V). Then by hypothesis U is intuitionistic fuzzy rw- open set of X such that c(α,β) ⊆ U and f (U) = *f* (f^{-1}(V)) ⊆ V.

Theorem4.3: Let *f* : (X,$\Im$) → (Y, σ) is intuitionistic fuzzy rw-continuous then for each intuitionistic fuzzy point c(α,β) of X and each intuitionistic fuzzy open set V of Y such that *f*(c(α,β)) qV, there x is a intuitionistic fuzzy rw-open set U of X such that c(α,β) qU and f(U) ⊆ V.

Proof: Let c(α,β) be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that *f* (c(α,β))q V. Put U = f^{-1}(V). Then by hypothesis U is intuitionistic fuzzy rw-open set of X such thatc(α,β)q U and *f*(U)= *f*(f^{-1}(V)) ⊆ V.

Theorem 4.4: If *f*:(X,$\Im$)→(Y,σ) is intuitionistic fuzzy rw-continuous, then *f*(rwcl(A)⊆cl(*f*(A))forever y intuitionistic fuzzy set A of X.

Proof: Let A be an intuitionistic fuzzy set of X. Then cl(*f*(A)) is an intuitionistic fuzzy closed set of Y. Since *f* is intuitionistic fuzzy rw-continuous, f^{-1}(cl(*f*(A))) is intuitionistic fuzzy rw- closed in X. Clearly A ⊆f^{-1}(cl((A)). Therefore wcl(A)⊆wcl(f^{-1}(cl(*f*(A)))) = f^{-1}(cl(*f*(A))). Hence *f* (rwcl(A) ⊆ cl(*f* (A)) for every intuitionistic fuzzy set A of X.

Theorem 4.5: If *f* : (X,$\Im$) →(Y, σ) is intuitionistic fuzzy rw-continuous and *g*: (Y, σ) →(Z, μ) is intuitionistic fuzzy continuous. Then *gof*: (X,$\Im$) →(Z, μ) is intuitionistic fuzzy rw- continuous.

Proof: Let A is an intuitionistic fuzzy closed set in Z. then g^{-1}(A) is intuitionstic fuzzy closed in Y because *g* is intuitionistic fuzzy continuous. Therefore $(gof)^{-1}$(A) =f^{-1}(g^{-1}(A)) is intuitionistic fuzzy rw-closed in X. Hence *g of* is intuitionistic fuzzy rw-continuous.

Theorem4.6: If *f*: (X,$\Im$) → (Y, σ) is intuitionistic fuzzy rw-continuous and *g*: (Y, σ) → (Z, μ) is intuitionistic fuzzy g-continuous and (Y, σ) is IF $T_{1/2}$ then *gof* : (X,$\Im$)→ (Z, μ) is intuitionistic fuzzy rw-continuous.

Proof: Let A is m an intuitionistic fuzzy closed set in Z, then g^{-1}(A) is intuitionistic fuzzy g-closed in Y. Since Y is $T_{1/2}$, then g^{-1}(A) is intuitionstic fuzzy closed in Y. Hence, $(gof\,)^{-1}$(A) = f^{-1}(g^{-1}(A)) is intuitionistic fuzzy rw-closed in X. Hence *g of* is intuitionistic fuzzy w-continuous.

Theorem4.7: If *f*: (X,$\Im$) → (Y, σ) is intuitionistic fuzzy rg-irresolute and *g*: (Y, σ) → (Z, μ) is intuitionistic fuzzy rw-continuous. Then *gof*: (X,$\Im$) → (Z, μ) is intuitionistic fuzzy rg- continuous**.**

Proof: Let A is an intuitionistic fuzzy closed set in Z, then g^{-1}(A) is intuitionistic fuzzy rw- closed in Y, because *g* is intuitionistic fuzzy rw-continuous. Since every intuitionistic fuzzy rw-closed set is intuitionistic fuzzy rg-closed set, therefore g^{-1}(A) is intuitionistic fuzzy rg- closed in Y .Then $(gof\,)^{-1}$(A) = f^{-1}(g^{-1}(A)) is intuitionistic fuzzy rg-closed in X, because *f* is intuitionistic fuzzy rg-irresolute. Hence *gof* : (X,$\Im$) → (Z,μ) is intuitionistic fuzzy rg- continuous**.**

Theorem 4.8: An intuitionistic fuzzy rw-continuous image of a intuitionistic fuzzy rw-compact space is intuitionistic fuzzy compact.

Proof: Let. *f*: (X,$\Im$). →(Y, σ) is intuitionistic fuzzy rw-continuous map from a intuitionistic fuzzy rw-compact space (X,$\Im$) onto an intuitionistic fuzzy topological space (Y, σ). Let {Ai: i ∊ Λ} be an intuitionistic fuzzy open cover of Y then {f^{-1}(Ai): i ∊ Λ} is a intuitionistic fuzzy rw-open cover of X. Since X is intuitionistic fuzzy rw-compact it has finite intuitionistic fuzzy sub cover say {f^{-1}(A_1),f^{-1}(A_2),----f^{-1}(An)}. Since *f* is onto {A_1, A_2, …, A_n}is an intuitionistic fuzzy open cover of Y and so (Y, σ) is intuitionistic fuzzy compact.

Theorem 4.9: If f: $(X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-continuous surjection and X is intuitionistic fuzzy rw-connected then Y is intuitionistic fuzzy connected.

Proof: Suppose Y is not intuitionistic fuzzy connected. Then there exists a proper intuitionistic fuzzy set G of Y which is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Therefore $f^{-1}(G)$ is a proper intuitionistic fuzzy set of X, which is both intuitionistic fuzzy rw- open and intuitionistic fuzzy rw-closed, because f is intuitionistic fuzzy rw-continuous surjection. Hence, X is not intuitionistic fuzzy rw-connected, which is a contradiction.

Definition4.2: A mapping $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-open if the image of every intuitionistic fuzzy open set of X is intuitionistic fuzzy rw-open set in Y.

Remark4.5: Every intuitionistic fuzzy w-open map is intuitionistic fuzzy rw-open but converse may not be true. For,

Example4.4: Let X = {a, b}, Y = {x , y} and the intuitionistic fuzzy set U and V are defined as follows:
U= {<a,0.7, 0.2 >, <b, 0.8, 0.1>}
V= {<x, 0.7, 0.2 >, <y, 0.6, 0.3>}

Then $\Im = \{\tilde{0}, U, 1^{\sim}\}$ and $\sigma = \{\tilde{0}, V, 1\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then, the mapping f: $(X,\Im) \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy rw-open but it is not intuitionistic fuzzy w-open.

Theorem 4.10: A mapping f: $(X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-open if and only if for every intuitionistic fuzzy set U of X$f(int(U)) \subseteq rwint(f(U))$.

Proof: Necessity: Let f be an intuitionistic fuzzy rw-open mapping and U is an intuitionistic fuzzy open set in X. Now $int(U) \subseteq U$ which implies that $f\,(int(U) \subseteq f(U)$. Since f is an intuitionistic fuzzy rw-open mapping, $f(Int(U)$ is intuitionistic fuzzy rw-open set in Y such that $f(Int(U) \subseteq f(U)$ therefore $f(Int(U) \subseteq rwint\, f(U)$.

Sufficiency: Forth e converse suppose that U is an intuitionistic fuzzy open set of X. Then $f\,(U) = f\,(Int(U) \subseteq rwint\ f(U)$. But $wint(f\,(U)) \subseteq f\,(U)$. Consequently $f\,(U) = wint(U)$ which implies that $f(U)$ is an intuitionistic fuzzy rw-open set of Y and hence f is an intuitionistic fuzzy rw-open.

Theorem 4.11: If f: $(X,\Im) \to (Y, \sigma)$ is an IF rw-open map then $int(f^{-1}(G) \subseteq f^{-1}(rwint(G)$ for every IFSG of Y.

Proof: Let G is an intuitionistic fuzzy set of Y. Then int $f^{-1}(G)$ is an intuitionistic fuzzy open set in X. Since f is intuitionistic fuzzy rw-open $f(int f^{-1}(G))$ is intuitionistic fuzzy rw-open in Y and hence $f(Int\, f^{-1}(G)\,) \subseteq rwint(f(\,f^{-1}(G)) \subseteq rwint(G)$. Thus int $f^{-1}(G) \subseteq f^{-1}(rwint(G)$.

Theorem4.12: A mapping $f : (X,\Im) \to (Y,\sigma)$ is intuitionistic fuzzy rw-open if and only if for each IFS S of Y and for each intuitionistic fuzzy closed set U of X containing $f^{-1}(S)$ there is a intuitionistic fuzzy rw-closed V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Necessity: Suppose that f is an intuitionistic fuzzy rw-open map. Let S be the intuitionistic fuzzy closed set of Yand U is an intuitionistic fuzzy closed set of X such that $f^{-1}(S) \subseteq U$. Then $V = (f^{-1}(U^c))^c$ is intuitionistic fuzzy rw-closed set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: For the converse suppose that F is an intuitionistic fuzzy open set of X. Then $f^{-1}((f(F))^c \subseteq F^c$ and F^c is intuitionistic fuzzy closed set in X. By hypothesis there is an intuitionistic fuzzy rw-closed set V of Y such that$(f(F))^c \subseteq V$and$f^{-1}(V) \subseteq F^c$. Therefore $F \subseteq (\,f^{-1}(V))^c$. Hence $V^c \subseteq f\,(F) \subseteq f((\,f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is intuitionistic fuzzy rw-open set of Y. Hence $f\,(F)$ is intuitionistic fuzzy rw-open in Y and thus f is intuitionistic fuzzy rw-open map.

Definition4.3: A mapping $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-closed if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy rw-closed set in Y.

Remark 4.6: Every intuitionistic fuzzy closed map is intuitionistic fuzzy rw-closed but converse may not be true. For,

Example 4.5: Let X= {a, b}, Y={x,y}

Then the mapping f: $(X,\Im) \to (Y, \sigma)$ defined in Example 4.4 is intuitionistic fuzzy rw-closed but it is not intuitionistic fuzzy w-closed.

Theorem 4.13: A mapping $f:(X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-closed if and only if f or each intuitionistic fuzzy set S of Y and for each intuitionistic fuzzy open set U of X containing $f^{-1}(S)$ there is a intuitionistic fuzzy rw-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Necessity: Suppose that f is an intuitionistic fuzzy rw-closed map. Let S be the intuitionistic fuzzy closed set of Yand U is an intuitionistic $f^{-1}(S) \subseteq U$. Then $V = Y - f^{-1}(U^c)$is intuitionistic fuzzy rw-open set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: Forth converse suppose that F is an intuitionistic fuzzy closed set of X. Then $(f(F))^c$ is an intuitionistic f fuzzy set of Y and F^c is intuitionistic fuzzy open set in X such that $f^{1}((f(F))^c) \subseteq F^c$. By hypothesis there is an intuitionistic fuzzy rw-open set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$. Therefore, $F \subseteq (f^{-1}(V))^c$. Hence, $V^c \subseteq f(F) \subseteq f(\,(\,f^{-1}(V))^c) \subseteq V^c$ which implies $f(F)=V^c$. Since V^c is intuitionistic fuzzy rw-closed set of Y. Hence $f(F)$ is intuitionistic fuzzy rw-closed in Y and thus f is intuitionistic fuzzy w-closed map.

Theorem 4.14: If f: $(X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy almost irresolute and intuitionistic fuzzy rw-closed map and A is an intuitionistic fuzzy w-closed set of X, then $f\,(A)$ intuitionistic fuzzy rw-closed.

Proof: Let $f(A) \subseteq O$ where O is an intuitionistic fuzzy regular semi open set of Y. Since f is intuitionistic fuzzy almost irresolute therefore $f^{-1}(O)$ is an intuitionistic fuzzy semi open set of X such that $A \subseteq f^{-1}(O)$.Since A is intuitionistic fuzzy w-closed of X which implies that $cl(A) \subseteq (f^{1}(O))$ and hence $f(cl(A)) \subseteq O$ which implies that $cl(f(cl(A))) \subseteq O$ therefore $cl(f((A))) \subseteq O$ whenever $f(A) \subseteq O$ where O is an intuitionistic fuzzy regular semi open set of Y. Hence $f(A)$ is an intuitionistic fuzzy rw- closed set of Y.

Theorem4.15: If $f : (X,\Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy regular semi irresolute and intuitionistic fuzzy rw-closed map and A is an intuitionistic fuzzy rw-closed set of X, then $f(A)$ is intuitionistic fuzzy rw-closed.

Proof: Let $f(A) \subseteq O$ where O is an intuitionistic fuzzy regular semi open set of Y. Since f is intuitionistic fuzzy regular semi irresolute therefore $f^{-1}(O)$ is an intuitionistic fuzzy regular semi open set of X such that $A \subseteq f^{1}(O)$.Since A is intuitionistic fuzzy rw-closed of X which implies that $cl(A) \subseteq (f^{-1}(O))$ and hence $f(cl(A) \subseteq O$ which implies that $cl(f(cl(A))) \subseteq O$ therefore $cl(f((A))) \subseteq O$ whenever $f(A) \subseteq O$ where O is an intuitionistic fuzzy regular semi open set of Y. Hence $f(A)$ is an intuitionistic fuzzy rw-closed set of Y.

Theorem4.16: If f: $(X,\Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy closed and g: $(Y, \sigma) \rightarrow (Z, \mu)$ is intuitionistic fuzzy rw-closed. Then $gof : (X,\Im) \rightarrow (Z,\mu)$ is intuitionistic fuzzy rw-closed.

Proof: Let H be an intuitionistic fuzzy closed set of intuitionistic fuzzy topological space$(X,\Im)$. Then $f(H)$ is an intuitionistic fuzzy closed set of (Y, σ) because f is an inuituionistic fuzzy closed map. Now $(gof)(H) = g(f(H))$ is an intuitionistic fuzzy rw-closed set in the intuitionistic fuzzy topological space Z because g is an intuitionistic fuzzy rw-closed map. Thus, gof: $(X,\Im) \rightarrow (Z, \mu)$ is intuitionistic fuzzy rw-closed.

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