On the Relationships between Topological Coindices

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Abstract: ***The equations for a graph G having n vertices and m edges with $\overline{M_1}(G)$, $M_1(\overline{G})$, $\overline{M_1}(\overline{G})$, $\overline{M_2}(G)$, $M_2(\overline{G})$, $\overline{M_2}(\overline{G})$ and $\overline{F}(G)$ were established by Gutman et al.[1]. In this paper the relationships between topological coindices and coindices of complement graphs of Dutch windmill, wheel and helm graph are studied.***

Keywords: Complement graph, Dutch windmill graph, forgotten coindex, helm graph, multiplicative Zagreb coindex, wheel graph, Y-coindex, Zagreb coindex

1. Introduction

Let G be a simple, finite, connected graph with vertex set V(G) and edge set E(G). The degree of a vertex $u \in V(G)$ is denoted by $d_G(u)$ and is the number of vertices adjacent to u. The edge connecting the vertices u and v is denoted by uv. A topological index is a numerical parameter mathematically derived from the graph structure. A complement $\overline{G}$ of a graph G consist of same set of vertices, where two vertices v and w are adjacent by an edge vw if and only if they are not adjacent in G [2]. Hence $vw \in E(\overline{G}) \Leftrightarrow vw \notin E(G)$.A complement graph consists of a number of edges and the degree of vertex v which are represented as $\overline{m} = \binom{n}{2} - m$ and $d_{\overline{G}}(v) = n\text{-}1\text{-}d_G(v)$ respectively. Edges of the complement graph $\overline{G}$ are exactly the non-edges of G.So studying non-edges of G is equivalent to studying the edges of $\overline{G}$. The Zagreb coindices of $\overline{G}$ count over non-edges in $\overline{G}$ (that is edges in G) using degree values from $\overline{G}$.This means coindices are computed on different sets of vertex pairs. The degrees used in the calculation are also different due to being from G versus $\overline{G}$.The relations between some Zagreb indices and Zagreb coindices of graphs have been studied in [3]. For a simple graph G, $\overline{M_1}(G) = \overline{M_1}(\overline{G})$ [4-6].Some topological indices such as $M_1(H_n)$, $M_2(H_n)$, $H(H_n)$ and $F(H_n)$ were computed in [7].The multiplicative Zagreb coindices of graph operations were studied in [8] and Gourava coindices of a molecular graph in [9]. The M-polynomials of some cycle related graphs were studied in [10]. Topological indices and M-polynomials of wheel graph are studied in many papers as [11-15]. Some topological indices of molecular structure in anticancer drugs were studied in [16].For the wheel graph $W_{1,n}$, $n \geq 4$,first, second and third leap Zagreb coindices were computed in [17].For any (n,m) graph, $\overline{M_1}(G) = 2m(n\text{-}1)\text{-}M_1(G)$ and $\overline{M_1}(\overline{G}) = 2m\text{-}(n\text{-}1)\text{-}M_1(G)$ [18] and $HZ+\overline{HZ} = (n\text{-}2)M_1+4m^2$ [19]. Let G be a simple graph then $\overline{M_3}(G) = M_3(\overline{G})$.

A helm graph H_n is a graph constructed from a wheel W_n by adding n vertices of degree one adjacent to each terminal vertex. A helm graph H_n has edges 3n and 2n+1 vertices among which 1 vertex is of degree n, n vertices of degree 4 and n pendant vertices (a vertex of degree 1).

A wheel graph W_n is the join of K_1 and C_n. Wheel graphs W_n has n+ 1 vertices and 2n edges with central and rim vertices. The central vertex has degree n-1, while other n-1 vertices on the cycle have degree 3.Wheel graphs are used to model certain types of networks such as a star network with a central hub and a ring of nodes. The edge set of W_n can be partitioned as $|E_{(3,3)}| = n$, $|E_{(3,n)}| = n$ [20].For topological coindex of complement graph of wheel graph, the vertex pairs are center-peripheral with degrees (0, n-4) and peripheral-peripheral with degrees (n-4,n-4).

The Dutch windmill graph is obtained by taking $m \geq 1$ copies of the cycle C_n, $n \geq 3$,with a vertex in common [21-25].Let G be a Dutch windmill graph (D_n^m) with 1+m(n-1) vertices and mn edges, then there are two vertices V_2= m(n-1) and V_{2m}=1 and edges $E_{(2,2)}$= m(n-2) and $E_{(22m)}$ = 2m.

The first and second Zagreb coindices of graph G are defined in [26-27] as

$$\overline{M_1}(G) = \sum_{uv \notin E(G)} (d_G(u) + d_G(v)) \quad (1)$$

$$\overline{M_2}(G) = \sum_{uv \notin E(G)} (d_G(u) d_G(v)) \quad (2)$$

And the corresponding coindices for complement graph are

$$\overline{M_1}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u) + d_{\overline{G}}(v)) \quad (3)$$

$$\overline{M_2}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u) d_{\overline{G}}(v)) \quad (4)$$

The first, second multiplicative Zagreb indices, harmonic index, hyper Zagreb index and forgotten index were studied for vitamin D_3 by M.R.R.Kanna et al.[28].We define corresponding coindices for a graph G and its complement graph as

$$\overline{\prod_1}(G) = \prod_{uv \notin E(G)} (d_G(u) + d_G(v)) \quad (5)$$

$$\overline{\prod_2}(G) = \prod_{uv \notin E(G)} (d_G(u)\, d_G(v)) \quad (6)$$

$$\overline{H}(G) = \sum_{uv \notin E(G)} \frac{2}{(d_G(u)+d_G(v))} \quad (7)$$

$$\overline{HM}(G) = \sum_{uv \notin E(G)} (d_G(u) + d_G(v))^2 \quad (8)$$

$$\overline{F}(G) = \sum_{uv \notin E(G)} (d_G(u)^2 + d_G(v)^2) \quad (9)$$

And

$$\overline{\prod_1}(\overline{G}) = \prod_{uv \notin E(\overline{G})} (d_{\overline{G}}(u) + d_{\overline{G}}(v)) \quad (10)$$

$$\overline{\prod_2}(\overline{G}) = \prod_{uv \notin E(\overline{G})} (d_{\overline{G}}(u)\, d_{\overline{G}}(v)) \quad (11)$$

$$\overline{H}(\overline{G}) = \sum_{uv \notin E(\overline{G})} \frac{2}{(d_{\overline{G}}(u)+d_{\overline{G}}(v))} \quad (12)$$

$$\overline{HM}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u) + d_{\overline{G}}(v))^2 \quad (13)$$

$$\overline{F}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u)^2 + d_{\overline{G}}(v)^2) \quad (14)$$

Alsharafi et al. studied Y-index and coindex of nanotubes in [29-30], then $\overline{Y}(G)$ and $\overline{Y}(\overline{G})$ are defined as

$$\overline{Y}(G) = \sum_{uv \notin E(G)} (d_G(u)^3 + d_G(v)^3) \quad (15)$$

$$\overline{Y}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u)^3 + d_{\overline{G}}(v)^3) \quad (16)$$

The reduced second Zagreb coindex is defined as

$$\overline{RM_2}(G) = \sum_{uv \notin E(G)} (d_G(u) - 1)(d_G(v) - 1) \quad (17)$$

We define reduced second Zagreb coindex of a complement graph as

$$\overline{RM_2}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u) - 1)(d_{\overline{G}}(v) - 1) \quad (18)$$

The Nirmala coindices of a graph and its complement graph [31] are

$$\overline{N}(G) = \sum_{uv \notin E(G)} \sqrt{d_G(u) + d_G(v)} \quad (19)$$

$$\overline{N}(\overline{G}) = \sum_{uv \notin E(\overline{G})} \sqrt{d_{\overline{G}}(u) + d_{\overline{G}}(v)} \qquad (20)$$

All the symbols and notations used in this paper are standard and taken mainly from books of graph theory [32-34]. In this paper we study harmonic, first Zagreb, second Zagreb, reduced second Zagreb, hyper Zagreb, Nirmala, forgotten, Y, first, second multiplicative Zagreb coindices and corresponding coindices for complement graphs of Dutch windmill, wheel and helm graph.

2. Materials and Methods

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. In computing $\overline{M_1}(G)$, we sum over all non-adjacent pairs of vertices in G. For topological coindices of complement graph $\overline{G}$, we take this sum over all non-edges of $\overline{G}$. The edge partition for non-adjacent pairs of complement graph of wheel graph is presented in table 1. All non-adjacent pairs of helm graph are given in table 2. The Dutch windmill; wheel and helm graphs are depicted in figures (1-3).

3. Results and Discussion

Dutch windmill graph

It is observed from figure 1 that there are $|E_{(2,2)}| = \binom{n-1}{2}$-(n-1) and $|E_{(2(m-n+1),2)}|$= (n-1)-2(m-n+1) non-adjacent edges in Dutch windmill graph.

Theorem 1.1a. Harmonic coindex of Dutch windmill graph is $\frac{(n-1)(n-4)}{4}+\frac{3n-2m-3}{m-n+1}$.

Proof. $\overline{H}(G) = \sum_{uv \notin E(G)} \frac{2}{(d_G(u)+d_G(v))}$
$= |E_{(2,2)}|\frac{2}{2+2} + |E_{(2(m-n+1),2)}|\frac{2}{2(m-n+1)+2}$
$= [\binom{n-1}{2}-(n-1)]\frac{2}{2+2} + (n-1-2(m-n+1))\frac{2}{2(m-n+1)+2}$
$= \frac{(n-1)(n-4)}{4}+\frac{3n-2m-3}{m-n+1}$.

Theorem 1.1b. Harmonic coindex of complement graph of Dutch windmill graph is $\frac{4n}{((m-3)n+(m-1)n-1)}+\frac{n(m-2)}{(m-1)n-1}$.

Proof. Harmonic coindex of complement graph
$\overline{H}(\overline{G}) = \sum_{uv \notin E(\overline{G})} \frac{2}{(d_{\overline{G}}(u)+d_{\overline{G}}(v))}$
$= |E_{((m-3)n,(m-1)n-1)}|\frac{2}{((m-3)n+(m-1)n-1)} + |E_{((m-1)n-1,(m-1)n-1)}|\frac{2}{((m-1)n-1+(m-1)n-1)}$
$= 2n\frac{2}{((m-3)n+(m-1)n-1)} + n(m-2)\frac{2}{((m-1)n-1+(m-1)n-1)}$
$= \frac{4n}{((m-3)n+(m-1)n-1)}+\frac{n(m-2)}{(m-1)n-1}$.

Theorem 1.2a. First Zagreb coindex of Dutch windmill graph is $2(n-1)(n-4)$+ (3n-2m-3) $(2m-2n+4)$.

Proof. $\overline{M_1}(G) = \sum_{uv \notin E(G)} (d_G(u)+d_G(v))$
$= |E_{(2,2)}|(2+2) + |E_{(2(m-n+1),2)}|(2(m-n+1)+2) = [\binom{n-1}{2}-(n-1)](2+2) + (n-1-2(m-n+1))(2(m-n+1)+2)$
$= (\frac{(n-1)(n-4)}{2})(2+2) + (n-1-2(m-n+1))(2(m-n+1)+2)$
$= 2(n-1)(n-4) + (3n-2m-3)(2m-2n+4)$.

Theorem 1.2b. First Zagreb coindex of complement graph of Dutch windmill graph is $2n[(2mn-4n-1)+(m-2)((m-1)n-1)]$.

Proof. First Zagreb coindex of complement graph
$\overline{M_1}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u)+d_{\overline{G}}(v))$
$= |E_{((m-3)n,(m-1)n-1)}|\big((m-3)n+(m-1)n-1\big) + |E_{((m-1)n-1,(m-1)n-1)}|((m-1)n-1+(m-1)n-1)$
$= 2n\big((m-3)n+(m-1)n-1\big) + n(m-2)((m-1)n-1+(m-1)n-1)$
$= 2n[(2mn-4n-1)+(m-2)((m-1)n-1)]$.

Theorem 1.3a. Second Zagreb coindex of Dutch windmill graph is $2(n-1)(n-4)$ +4(3n-2m-3) $(m-n+1)$.

Proof. $\overline{M_2}(G) = \sum_{uv \notin E(G)} (d_G(u)d_G(v))$
$= |E_{(2,2)}|(2\times 2) + E_{(2(m-n+1),2)}|(2(m-n+1)\times 2) = [\binom{n-1}{2}-(n-1)](2\times 2) + (n-1-2(m-n+1))(2(m-n+1)\times 2)$
$= (\frac{(n-1)(n-4)}{2})(2\times 2) + (n-1-2(m-n+1))(2(m-n+1)\times 2)$
$= 2(n-1)(n-4) + 4(3n-2m-3)(m-n+1)$.

Theorem 1.3b. Second Zagreb coindex of complement graph of Dutch windmill graph is $2n(m^2n^2-4mn^2-mn+3n^2+3n)$ +n(m-2)((m-1)n-1)^2.

Proof. Second Zagreb coindex of complement graph
$\overline{M_2}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u)\times d_{\overline{G}}(v))$
$= |E_{((m-3)n,(m-1)n-1)}|((m-3)n\times(m-1)n-1) + |E_{((m-1)n-1,(m-1)n-1)}|((m-1)n-1\times(m-1)n-1)$
$= 2n\big((m-3)n\times(m-1)n-1\big) + n(m-2)((m-1)n-1\times(m-1)n-1)$
$= 2n(m^2n^2-4mn^2-mn+3n^2+3n) + n(m-2)((m-1)n-1)^2$.

Wheel graph

Theorem 2.1a. Harmonic coindex of wheel graph is $\frac{(n-1)(n-4)}{6}$.

Proof. $\overline{H}(G) = \sum_{uv \notin E(G)} \frac{2}{(d_G(u)+d_G(v))}$
$= |E_{(3,3)}|\frac{2}{3+3} = \frac{(n-1)(n-4)}{2}\frac{2}{3+3}$
$= \frac{(n-1)(n-4)}{6}$.

Theorem 2.1b. Harmonic coindex of complement graph of wheel graph is $\frac{3(n-1)}{(n-4)}$.

Proof. Harmonic coindex of complement graph
$\overline{H}(\overline{G}) = \sum_{uv \notin E(\overline{G})} \frac{2}{(d_{\overline{G}}(u)+d_{\overline{G}}(v))}$
$= |E_{(0,n-4)}|\frac{2}{n-4} + |E_{(n-4,n-4)}|\frac{2}{n-4+n-4} = (n-1)\frac{2}{n-4} + (n-1)\frac{2}{n-4+n-4}$
$= \frac{3(n-1)}{(n-4)}$.

Theorem 2.2a. First Zagreb coindex of wheel graph is 3(n-1)$(n-4)$.

Proof. $\overline{M_1}(G) = \sum_{uv \notin E(G)} (d_G(u)+d_G(v))$
$= |E_{(3,3)}|(3+3) = \frac{(n-1)(n-4)}{2}(3+3)$
$= 3(n-1)(n-4)$.

Theorem 2.2b. First Zagreb coindex of complement graph of wheel graph is 3(n-1)$(n-4)$.

Proof. First Zagreb coindex of complement graph
$\overline{M_1}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u)+d_{\overline{G}}(v))$
$= |E_{(0,n-4)}|(0+n-4) + |E_{(n-4,n-4)}|(n-4+n-4) = (n-1)(0+n-4) + (n-1)(n-4+n-4)$
$= 3(n-1)(n-4)$.

Theorem 2.3a. Second Zagreb coindex of wheel graph is $\frac{9(n-1)(n-4)}{2}$.

Proof. $\overline{M_2}(G) = \sum_{uv \notin E(G)} (d_G(u)d_G(v))$
$= |E_{(3,3)}|(3\times 3) = \frac{(n-1)(n-4)}{2}(3\times 3)$
$= \frac{9(n-1)(n-4)}{2}$.

Theorem 2.3b. Second Zagreb coindex of complement graph of wheel graph is $(n-1)(n-4)^2$.

Proof. Second Zagreb coindex of complement graph
$\overline{M_2}(\overline{G}) = \sum_{uv \notin E(\overline{G})} (d_{\overline{G}}(u)d_{\overline{G}}(v))$

$=|E_{(0,n-4)}|(0\times(n-4))+|E_{(n-4,n-4)}|((n-4)\times(n-4))=(n-1)(n-4)\times(n-4)$
$=(n-1)(n-4)^2$.

Helm graph

Let H_n be a helm graph of order 2n+1 and size 3n then from figure 3, we can say that the edge partition is divided into $|E_{(1,4)}|=n$, $|E_{(4,4)}|=n$ and $|E_{(4,n)}|=n$. All non-adjacent pairs are pendant-pendant, pendant-cycle and pendant-center and cycle-cycle vertices.

Theorem 3.1a. Harmonic coindex of helm graph is $\frac{21n^3+10n^2+49n-20}{40(1+n)}$.

Proof. $\overline{H}(G)=\sum_{uv\notin E(G)}\frac{2}{(d_G(u)+d_G(v))}$
$=|E_{(1,1)}|\frac{2}{1+1}+|E_{(1,4)}|\frac{2}{1+4}+|E_{(4,4)}|\frac{2}{4+4}+|E_{(1,n)}|\frac{2}{1+n}$
$=\frac{n(n-1)}{2}\frac{2}{1+1}+n(n-1)\frac{2}{1+4}+\frac{n(n-3)}{2}\frac{2}{4+4}+\frac{2n}{1+n}=\frac{21n^3+10n^2+49n-20}{40(1+n)}$.

Theorem 3.1b. Harmonic coindex of complement graph of helm graph is $\frac{7n^3-2n^2+3n}{6(n+1)}$.

Proof. Harmonic coindex of complement graph
$\overline{H}(\overline{G})=\sum_{uv\notin E(\overline{G})}\frac{2}{(d_{\overline{G}}(u)+d_{\overline{G}}(v))}$
$=|E_{(1,1)}|\frac{2}{1+1}+|E_{(1,3)}|\frac{2}{1+3}+|E_{(1,n)}|\frac{2}{1+n}+|E_{(3,3)}|\frac{2}{3+3}$
$=\frac{n(n-1)}{2}\frac{2}{1+1}+n(n-1)\frac{2}{1+3}+n\frac{2}{1+n}+\frac{n(n-3)}{2}\frac{2}{3+3}=\frac{7n^3-2n^2+3n}{6(n+1)}$.

Theorem 3.2a. First Zagreb coindex of helm graph is $11n^2-17n$.

Proof. $\overline{M_1}(G)=\sum_{uv\notin E(G)}(d_G(u)+d_G(v))$
$=|E_{(1,1)}|(1+1)+|E_{(1,4)}|(1+4)+|E_{(1,n)}|(1+n)+|E_{(4,4)}|(4+4)$
$=\frac{n(n-1)}{2}(1+1)+n(n-1)(1+4)+n(1+n)+\frac{n(n-3)}{2}(4+4)$
$=11n^2-17n$.

Theorem 3.2b. First Zagreb coindex of complement graph of helm graph is $9n^2-13n$.

Proof. First Zagreb coindex of complement graph
$\overline{M_1}(\overline{G})=\sum_{uv\notin E(\overline{G})}(d_{\overline{G}}(u)+d_{\overline{G}}(v))$
$=|E_{(1,1)}|(1+1)+|E_{(1,3)}|(1+3)+|E_{(1,n)}|(1+n)+|E_{(3,3)}|(3+3)$
$=\frac{n(n-1)}{2}(1+1)+n(n-1)(1+3)+n(1+n)+\frac{n(n-3)}{2}(3+3)$
$=9n^2-13n$.

Theorem 3.3a. Second Zagreb coindex of helm graph is $\frac{27n^2-57n}{2}$.

Proof. $\overline{M_2}(G)=\sum_{uv\notin E(G)}(d_G(u)d_G(v))$
$=|E_{(1,1)}|(1\times1)+|E_{(1,4)}|(1\times4)+|E_{(1,n)}|(1\times n)+|E_{(4,4)}|(4\times4)$
$=\frac{n(n-1)}{2}(1\times1)+n(n-1)(1\times4)+n(1\times n)+\frac{n(n-3)}{2}(4\times4)$
$=\frac{27n^2-57n}{2}$.

Theorem 3.3b. Second Zagreb coindex of complement graph of helm graph is $9n^2-17n$.

Proof. Second Zagreb coindex of complement graph
$\overline{M_2}(\overline{G})=\sum_{uv\notin E(\overline{G})}(d_{\overline{G}}(u)d_{\overline{G}}(v))$
$=|E_{(1,1)}|(1\times1)+|E_{(1,3)}|(1\times3)+|E_{(1,n)}|(1\times n)+|E_{(3,3)}|(3\times3)$
$=\frac{n(n-1)}{2}(1\times1)+n(n-1)(1\times3)+n(1\times n)+\frac{n(n-3)}{2}(3\times3)$
$=9n^2-17n$.

The computed values of reduced second Zagreb, hyper Zagreb, Nirmala, forgotten, Y, first, second multiplicative Zagreb coindices and corresponding coindices for complement graphs of Dutch windmill, wheel and helm graph are given in table 3.

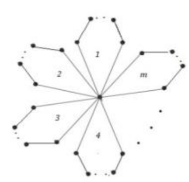

Figure 1

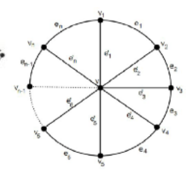

Figure 2

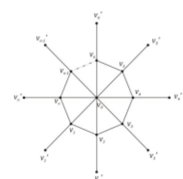

Figure 3

Figure 1. Dutch windmill graph (D_n^m), figure 2. wheel graph W_n and figure 3. helm graph H_n.

Table 1: The edge partition for non-adjacent pairs of complement graph of wheel graph

Vertices	center-peripheral	peripheral-peripheral
$(d_G(u), d_G(v))$	(0, n-4)	(n-4, n-4)
Number of edges	n-1	n-1

Table 2: All non-adjacent pairs for helm graph.

Pairs	pendant-pendant	pendant-cycle	cycle-cycle	pendant-center
$(d_G(u), d_G(v))$	(1,1)	(1,4)	(4,4)	(1,n)
Number of edges	$\frac{n(n-1)}{2}$	$n(n-1)$	$\frac{n(n-3)}{2}$	n

Table 3: Topological coindices and coindices of complement graphs of Dutch windmill, wheel and helm graph.

Topological index	reduced second Zagreb	hyper Zagreb	Nirmala
Dutch windmill graph	$\overline{RM_2}(G)=\frac{(n-1)(n-4)}{2}+(2m-2n-1)(3n-2m-3)$, $\overline{RM_2}(\overline{G})=2n[((m-3)n-1)((m-1)n-2)]+n(m-2)((m-1)n-2)^2$	$\overline{HM}(G)=8(n-1)(n-4)+(3n-2m-3)(2m-2n+4)^2$, $\overline{HM}(\overline{G})=2n[n(m-3)+((m-1)n-1)]^2+n(m-2)[2((m-1)n-1)]^2$	$\overline{N}(G)=(n-1)(n-4)+(3n-2m-3)(2m-2n+4)^{\frac{1}{2}}$, $\overline{N}(\overline{G})=2n(2mn-4n-1)^{\frac{1}{2}}+(nm-2n)(2mn-2n-1)^{\frac{1}{2}}$
Wheel graph	$\overline{RM_2}(G)=2(n-1)(n-4)$, $\overline{RM_2}(\overline{G})=(n-1)(n-5)^2$	$\overline{HM}(G)=18(n-1)(n-4)$, $\overline{HM}(\overline{G})=5(n-1)(n-4)^2$	$\overline{N}(G)=\frac{(n-1)(n-4)\sqrt{6}}{2}$, $\overline{N}(\overline{G})=(1+\sqrt{2})(n-1)(n-4)^{\frac{1}{2}}$
Helm graph	$\overline{RM_2}(G)=\frac{9}{2}n(n-3)$, $\overline{RM_2}(\overline{G})=2n(n-3)$	$\overline{HM}(G)=18n(n-1)+16n(n-3)+n(1+n^2)$, $\overline{HM}(\overline{G})=n(20n-37+n^2)$	$\overline{N}(G)=0.7n(n-1)+2.23n(n-1)+1.4n(n-3)+n(1+n)^{\frac{1}{2}}$, $\overline{N}(\overline{G})=0.7n(n-1)+2n(n-1)+n(1+n)^{\frac{1}{2}}+1.22n(n-3)$

	forgotten	Y	first multiplicative Zagreb	second multiplicative Zagreb
Dutch windmill graph	$\overline{F}(G)=4(n-1)(n-4)+4(3n-2m-3)((m-n+1)^2+1)$ $\overline{F}(\overline{G})=2n[((m-3)n)^2+((m-1)n-1)^2]+2n(m-2)((m-1)n-1)^2$	$\overline{Y}(G)=8(n-1)(n-4)+8(3n-2m-3)((m-n+1)^3+1)$ $\overline{Y}(\overline{G})=2n[((m-3)n)^3+((m-1)n-1)^3+(m-2)((m-1)n-1)^3]$	$\overline{\prod_1}(G)=4^{\frac{(n-1)(n-4)}{2}}\times(2m-2n+4)^{(3n-2n-3)}$, $\overline{\prod_1}(\overline{G})=((m-3)n+(m-1)n-1)^{2n}\times(2(m-1)n-1)^{n(m-2)}$	$\overline{\prod_2}(G)=4^{\frac{(n-1)(n-4)}{2}}\times(4(m-n+1))^{(3n-2m-3)}$, $\overline{\prod_2}(\overline{G})=[(m-3)n\times((m-1)n-1)]^{2n}\times((m-1)n-1)^{2n(m-2)}$

Wheel graph	$\bar{F}(G)=9(n-1)(n-4)$, $\bar{F}(\bar{G})=3(n-1)(n-4)^2$	$\bar{Y}(G)=27(n-1)(n-4)$, $\bar{Y}(\bar{G})=3(n-1)(n-4)^3$	$\overline{\prod_1}(G)=6^{\frac{n(n-1)(n-4)}{2}}$, $\overline{\prod_1}(\bar{G})=(n-4)^{(n-1)}\times(2(n-4))^{(n-1)}$	$\overline{\prod_2}(G)=9^{\frac{(n-1)(n-4)}{2}}$, $\overline{\prod_2}(\bar{G})=0$
Helm graph	$\bar{F}(G)=18n(n-1)+16n(n-3)+n(1+n^2)$, $\bar{F}(\bar{G})=0.5n(31n-47+2n^2)$	$\bar{Y}(G)=66n(n-1)+64n(n-3)+n(1+n^3)$, $\bar{Y}(\bar{G})=n(38n-55+n^3)$	$\overline{\prod_1}(G)=2^{\frac{n(n-1)}{2}}\times 5^{n(n-1)}\times 8^{\frac{n(n-3)}{2}}\times(1+n)^n$, $\overline{\prod_1}(\bar{G})=2^{\frac{n(n-1)}{2}}\times 4^{n(n-1)}\times 6^{\frac{n(n-3)}{2}}\times(1+n)^n$	$\overline{\prod_2}(G)=1^{\frac{n(n-1)}{2}}\times 4^{n(n-1)}\times 16^{\frac{n(n-3)}{2}}\times n^n$, $\overline{\prod_2}(\bar{G})=1^{\frac{n(n-1)}{2}}\times 3^{n(n-1)}\times 9^{\frac{n(n-3)}{2}}\times n^n$

4. Conclusion

The relationships between topological coindices of a graph and coindices of complement graphs of Dutch windmill, wheel and helm graph are obtained. The equation $\overline{M_1}(G)=\overline{M_1}(\bar{G})$ is satisfied for wheel graph. The equality/inequality between these relationships is established. Our result supports remark $\bar{F}(G)\neq\bar{F}(\bar{G})$ between forgotten coindex and forgotten coindex of complement graph for these graphs.

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